SBP Research Bulletin
Volume 14, Number 1, 2018

Multiple Equilibria in Investments Financed by Debt

An extension to Gertler, Rogoff (1990)

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Abstract: *This paper offers the extension of Gertler and Rogoff (1990), and illustrates that in presence of imperfect capital markets, non-linear relationship exists between level of investment in low-income countries and cost of external debt. This non-linearity can be attributed to concessional loans and financing at early stage of development; adjustments with external supply shocks; and the disparities of capital efficiency in different countries*

JEL Classification:F21, F34, E22,

Keywords: Imperfect Capital Market, External Debt, Investment

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1. Introduction

This paper presents a modification to the original model developed by Gertler and Rogoff (1990); and captures multiple equilibria in the relationship between level of investment and cost of debt in case of low-income countries. Across countries, capital movement is usually associated with the country's level of income and net worth, higher the net worth higher would be the capital inflows and vice versa. This also signifies that the spread in the world's riskless interest rate and loan rate varies across countries and negatively relates with their income level. High-income countries enjoy low riskless rates, whereas low-income countries suffer due to relatively higher rates even if the assumption of risk free rates holds true across countries. The endogeniety of domestic capital market imperfection has remained a source of concern for the policy makers, especially in low-income countries. Capital imperfections may create information asymmetries and introduce an incentive problem between lenders and borrowers often referred as 'moral hazard' problem.

In contrast to Gertler and Rogoff (1990), who established a linear association between a country's per capita income and external debt, we found that these variables experience a non-linear relationship. Specifically, this study found a non-linear association of a country's total and private external debt with income level. This non-linearity can be attributed to number of reasons for instance domestic financial frictions, concessional loans and financing at early stage of development, adjustments with external supply shocks and the disparities of capital efficiency in different countries. To confirm our findings, we present an extension of the model and substantiate our proposal based on the data of 39 developing economies over different years.

The remainder of the paper is organized as follows; section 2 presents a brief literature review, following which a formal model, as well as an extension of that model is developed in section 3. Section 4 examines robustness of the said framework and checks whether our findings are backed up by factual information, and finally section 5 presents some concluding remarks.

2. Literature Review

The imperfect capital mobility between rich and low-income countries and asymmetries in credit market remained as a central point of discussion in economic literature. According to the neoclassical growth model, developing countries with high productivity growth should receive more capital flows; however, empirical evidence suggests that the capital has been moving from developing to developed countries such as US. The academic and research community has produced a substantial amount of contributions covering this topic for academic purposes and policy recommendations. We cover some of those within the framework of our paper.

Gertler and Rogoff (1989) developed a model of international finance under asymmetric information to explain the phenomena of why in low-income economies, stocks of foreign debt of private borrowers experience higher growth relative to their income. In this framework, asymmetric information available only to individual debtors creates frictions in investment markets. The study explained that due to problem of moral hazard, marginal product of capital and spread between borrowing and lending rates were larger in low-income countries.

Atkenson (1991), presented a model of international lending in which retorted why risk sharing between creditors and debtors is incomplete and stated two main reasons i.e., moral hazard and the risk of repudiation that hinders risk sharing in the international lending relationship. He also discussed the role of constrained optimal pattern of capital flows between lenders (who cannot observe borrower's behavior) and borrower (who may renounce his debt) in creating difficulties for evaluation of project efficiency and debt repayment capacities. In the end, he discussed the property of optimal contract and suggested that for incentive reason borrowing country experiences capital outflow and fall in both consumption and investment.

Jeanneau and Marian (2002), could not find any evidence of an asymmetric behavior of determinants with respect to capital inflows or outflows in international lending to emerging economies, suggests that pull and push factors are both responsible for booms and cutbacks in international lending to emerging economies.

Gourinchas and Jeanne (2012), found that capital does not flow more to countries having high growth and investment rate as predicted in neoclassic growth model. The study termed this phenomena as 'allocation puzzle' possibly related to saving (not from investment) and to the behavior of publicly originated capital flows. It tried to explain this puzzle via different explanations for example 1) positive association between saving and growth and role of domestic frictions in distorting this relationship; 2) contribution of low domestic financial development in constraining domestic demand and; 3)part of government policies. However, the study could not find the consistent answer to this puzzle and concluded this puzzle as an open question.

Alfaro et al.,(2008), empirically investigated the role of different theories in explaining the why capital does not flow from rich countries to poor countries and highlighted the role of different policies in increasing capital inflows to poor economies including strengthening the protection of property rights, reducing corruption, increasing government stability, bureaucratic quality and law and order situation.

Koepke (2015), presented thorough review of literature on drivers of capital flows to emerging economies. He suggested that there is no explicit answer to what effects more to capital market imperfections in developing countries, since drivers of capital

flows changes across times and across different types of capital. The study showed that push factors like global risk aversion and external interest rates dominated in effecting portfolio debt and equity flows, but somewhat less for banking flows. Whereas, pull factors such as domestic output growth, asset returns and country risk explained all three capital flows components and banking flows.

3. The Gertler and Rogoff Theoretical Model

The model is an extension to the paper presented by Gertler and Rogoff (1990) that studies the capital flow movements and investment among poor and rich countries. This is a small open economy in which it cannot affect the international interest rate. This economy is having two periods, producing/consuming single good and possessing number of identical individuals. The representative individual is risk-neutral and care only about consuming in period 2:

$$U(c) = c \tag{1}$$

Where c is the consumption in the second-period. Also, the representative individual receives an endowment in period 1: W_1, and an endowment in period 2: W_2.

In that sense, there are saving W_1 which can be executed in two ways. The first option is to offer this to international markets at the world interest rate r (riskeless). The second option is to use these funds to finance a uncertain technology.

In this economy, each individual carries a project. All of these projects are identical in nature and yield following return: If they invest k units in period 1, it yield θ units of second-period output with probability $\pi(k)$, and zero units with probability 1 - $\pi(k)$. Suggested as:

$$y = \begin{cases} \theta \text{ with probability } \pi(k) \\ 0 \text{ with probability } 1\text{- } \pi(k) \end{cases} \tag{2}$$

Here, y is the second-period output. The probability has a concave function. This means that $\pi(.)$ Is increasing, strictly concave and twice continuously differentiable within the range of $\pi(0) = 0, \pi(\infty) = 1$. And, $\frac{r}{\theta} < \pi'(0) < \infty$. Thus, investing more k units, increases the probability that the individual's project will yield a high level of output but the marginal expected return to investment is negative (diminishing). One assumption is that output realizations are independent across individual's projects.

Also, the individual has a financial restriction. He only has endowment W_1 to invest. However, if he desires to capitalize more in his project, at that point he needs to get extra resources from capital market.
Thus, the new restriction is as follows:

$$W_1 + b \geq k \tag{3}$$

Where b is the extra funds that the individual borrows from the capital markets. However, against this amount, he issues a state-contingent security which pays z^g given the project yield a decent outcome (θ), and z^b if the project yield an undersirable outcome (0). In this framework, state-contingent security must offer the investors a market rate of return r.

Thus, the lender will expect the following return:

$$\pi(k)\, z^g + [1 - \pi(k)] z^b = rb \tag{4}$$

And, the individual's expected second-period consumption is given by the following equation:

$$E(c) = \pi(k)\, [\theta - z^g] - [1 - \pi(k)] z^b + r[W_1 + b - k] + W_2 \tag{5}$$

In equation (5), the first two terms represent expected net return on the project (see equation (4)), the subsequent term is the return from risk-free investments abroad, and the last term is the endowment in second period.

So, in this economy, there are lenders and borrowers. In this case, creditors possesses ability to perceive borrower's endowments (W_1, W_2), and their total borrowings (k). However, they cannot observe that how the borrowers uses these funds. For example, rather than invest in their specified project, borrowers can secretly lend these funds abroad.

Also, although investment cannot be observed, lenders can perceive realized output y, the production function $\pi(.)$, and the borrower's future endowment W_2.

Thus, under perfect information, the individual would maximize their investment in the point in which the expected marginal return on this project is equal to the world interest rate. This is the first-best optimum level of investment[2].

$$\pi'(k^*)\theta = r \tag{6}$$

However, in this economy, there is asymmetric information since what borrowers can do with capital (k) is non-observable. Therefore, it is not verifiable by the lender. Thus, contracts can be written by seeing output (y) (and not on k). Given any output-contingent payoffs (z^g, z^b) specified by the contract, the borrower will pick k to

[2] In this case, the borrower will invest all of the funds in his project

maximize her expected consumption in equation (5). Thus, she will equate her expected marginal gain from investing with her opportunity cost of (secretly) holding assets abroad:

$$\pi'(k)[\theta - (z^g - z^b)] = r \tag{7}$$

If $z^g - z^b = 0$, then we are in the case of perfect information. So, given that z^g varies from z^b, k will vary from its first-best optimum level (k^*) (see equation 6). Thus, two things will determine the marginal benefit of borrowers from investing: First, the marginal gain in expected output and second, on the adjustment in his expected commitment to lenders.

If the borrower could promise lenders a fixed payment across different outcomes, borrower would invest the first-best amount k^*. However, since the project yields zero in the bad state, this is not a feasible option. In that sense, since the borrower's consumption must be nonnegative, z^b cannot surpass the second period endowment W_2, which is defined in the following equation:

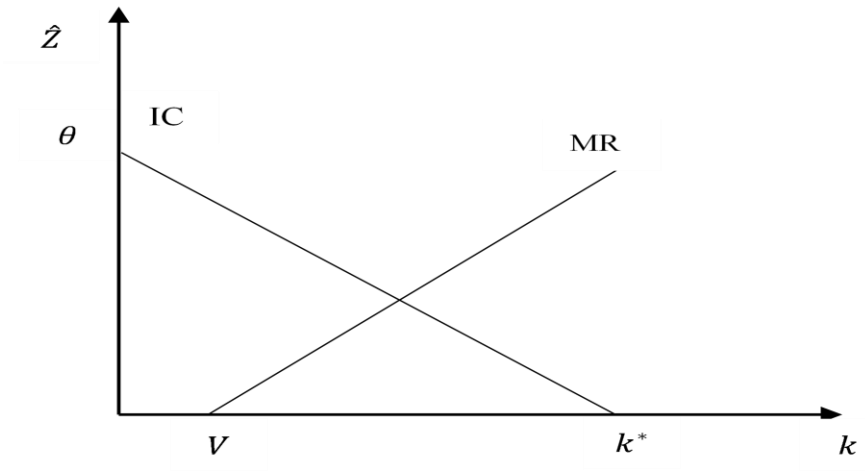
$$z^b \leq W_2 \tag{8}$$

The present value of the borrower's endowment is as follows:

$$V \equiv W_1 + W_2/r$$

So, if the present value of the endowment V is less than k^*, the borrower cannot offer lenders a riskless security.

Figure 1. Equilibrium in the small-country case



In a case, when $V < k^*$, the optimal incentive compatible contract is found by choosing z^g, z^b, b, and k to maximize the individual's expected second-period consumption (equation 5) subject to equations (3), (4), (7), and (8). So, the solution is as follows:

In a bad state, the contract pays creditors W_2, so equation (8) is binding. This reduces the spread between z^g and z^b, thereby minimizing the difference between the borrower's decision rule for k (equation (7)) and the socially efficient rule in equation (6). Likewise, equation (3) is another binding: $W_1 + b = k$. At the state of equilibrium, the borrower does not secretly lend abroad. In other words, more than necessary borrowing to finance k would increase the gap in $z^g \; and \; z^b$. The lender will offer more funding but a higher cost (higher gap).

For information constrained case, since (3) and (8) hold with equality, these equations can be used to eliminate b and z^b from (4) and (7). This results in following two equations which determine k and $\hat{Z} = z^g - z^b = z^g - W_2$:

Incentive Constrain Curve for the Borrowers:

$$\pi'(k)[\theta - \hat{Z}] = r \tag{9}$$

Market Rate of Return for the Lenders:

$$\hat{Z} = r(k - V)/\pi(k) \tag{10}$$

Equation (9) is derived from the incentive constraint (7) and can be drawn as a downward-sloping curve IC in Figure 1. An increase in $\hat{Z}$, lowers the borrower's expected marginal gain from investing and therefore must be offset by a decline in k. The IC curve intersects the vertical axis at a value of $\hat{Z}$, which lies between zero and θ (this is constraint by $\frac{r}{\theta} < \pi'(0) < \infty$). It intersects the horizontal axis at k^* due to equation (9) reduces to equation (6) when $\hat{Z}$ equals zero.

Equation (10) is the constraint that lenders must receive the market rate of return, and is labeled as the MR curve, which is an upward-sloping curve. Thus, when k rises, borrowing increases. In other words, z^g must increase due to z^b cannot adjust (see $\hat{Z} = z^g - z^b = z^g - W_2$). Hence, the curve intersects the horizontal axis at $k = V$.

Investment in the information-constrained case must be below its first-best value k^*. The result that k < k^* follows immediately from a comparison of (6) and (9), as well as from inspection in Figure 1. If k is below k^*, then ex post per-capita output, $\theta\pi(k)$ must lie below its first-best value, $\theta\pi((k^*)$. This means that per-capita investment and per-capita output will depend on per-capita wealth. A rise in V moves the MR curve downward in Figure 1, and leaves the IC curve unchanged, thereby increasing k and lowering $\hat{Z}$.

Also, the spread between the marginal product of capital and the world riskless interest rate will be determined by the income level of a country and in particular, it will be larger for the poorer country (the smaller the country). Though, the world capital market is perfectly integrated (the riskless rate is the same in all countries) the cross-country marginal products of capital may differ. Hence, in poor countries, the spread between the loan rate and the riskless rate will be higher. Thus, the loan (or risky debt) r^L is given by:

$$r^L = \frac{z^g - W_2}{k - V} = \frac{r}{\pi(k)} \tag{11}$$

This decreases in k. Note that r^L is the rate on the uncollateralized component of borrowing.

Figure 2A. Functional form of $\pi(.)$ - Gertler and Rogoff (1990)

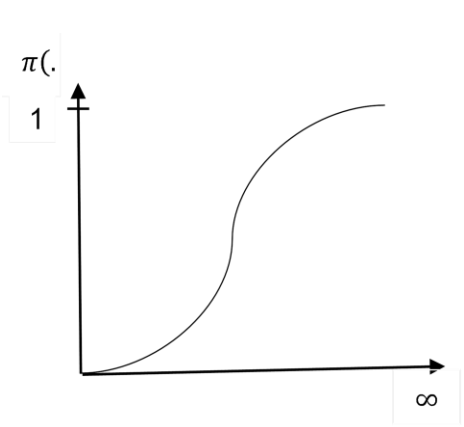
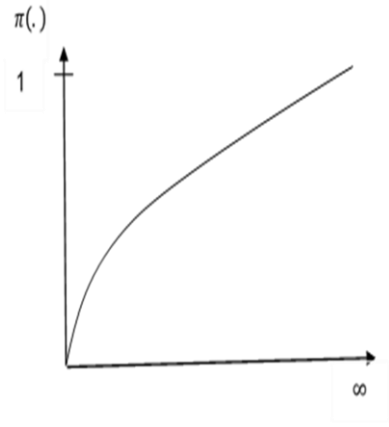


Figure 2B. New functional form of $\pi(.)$-Extension



This new form for the function $\pi^*(.)$ allows that the marginal rate return of capital is higher at some point but then the marginal return of capital is lower. Hence, this change in the functional form of $\pi(.)$ to $\pi^*(.)$ allows that MR curve to be a concave curve (see the appendix for the numerical math simulation).

Thus, in this extension of the model, the market rate of return for the lenders (MR Curve) is a concave function of the level of investment k. At initial levels of k, lenders can lend money at the market rate of return $\hat{Z}$. Nevertheless, there is an inflection point, in which at higher levels of k, the marginal rate of return is lower than $\hat{Z}$. This result contradicts what Gertler and Rogoff (1990) proposes, in which lenders charge higher levels of $\hat{Z}$ at higher levels of k since borrowers do not have more endowment in period 2. A possible explanation for this situation is that financial institutions (lenders) increase their exposure to risk (lending at higher levels of k at lower cost) when they know that they are insured by the Central Bank or the government is able to get support from donors, which evidences problems of moral hazard. In other

words, they allocate more and more money in the market since the Central Bank/IMF can act as a Lender of Last Resort and bail them out (see Figure 3).

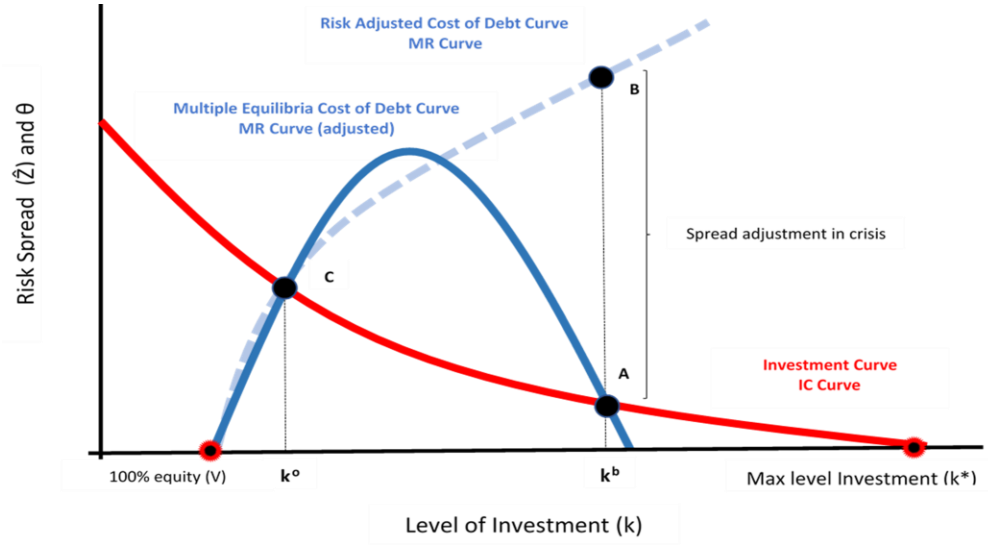
Then, equation (10) takes a new form as equation (12), and it can be represented together with the incentive constraint curve, equation (9), as shown in Figure 3.

(MR Adjusted) Market Rate of Return for the Lenders

$$\hat{Z} = r(k - V)/\pi^*(k) \qquad (12)$$

The concavity of the function representing the market rate of return for the lenders helps to find three important points, noted with A, B and C, in two levels of investment, k^o k^b, represented in Figure 3.

Figure 3: Concave Market Rate of Return for Lenders



First at the level of invest k^b, in point A, from the point of view of the investors, they are experiencing a bullish situation, the level of investment is high and the cost of the debt is low. From the point of view of the lender, they can lend more money at lower cost since they know that the government will bail them out. For instance, the situation could be similar to the behavior of investment in the Asian 5[3] during the early 90s, before the Asian financial crisis[4]. The average investment to GDP level was around 35% for the 5 countries and the EMBI Index was in average levels of 4%.

[3] Asian 5: Thailand, South Korea, Malaysia and the Philippines
[4] The Economist: Gold from the storm. https://www.economist.com/node/9401752

However, as the financial crisis struck in Asia, the EMBI shut up ten points, reaching levels of 14% by 1998[5], resembling the situation portrait in point B. As a result, the unsustainable level of investment resulted in non-performing loans reaching heights of 15%, and several corporations going bankrupt. Nevertheless, the economies started a deleveraging process, from point B to point C, accompanied by a contraction of investment from k^{b} to point k^{o}. The result was that the Asian 5 reached a sustainable level of investment k^{o}, by dropping more than 10% of GDP in investment by 1999. Although volatile the EMBI level also decreased during the next 5 years.

The modification to the original model of Gertler and Rogoff (1990) builds a framework to understand situations such as Sudden Stops. It is particularly interesting from a perspective that it captures the multiple equilibria in the relationship between the level of investment and the cost of debt. For instance, in Figure 3, point A is a typical example of a pre-crisis exuberance of low rates and high investment, while B could portrait a Sudden Stop ala Calvo, and C the consequences. From a microeconomic perspective, the net worth of the borrower, a proxy for the credit worthiness, is a determinant of the market return for the lenders ($\hat{Z}$) at initial levels of investment. However, there is an inflection point when the cost of debt can decay with an additional level of investment financed by extra debt, which will be shown in the next empirical section.

Although there are many reasons behind cycles, one possible explanation is the amplification that the longer finance cycle puts to economic cycles as claimed by Borio (2012). Nonetheless, that is outside the focus of the paper and further studies should try to incorporate the boom-bust story of emerging markets.

Furthermore, the model presented in the paper also shows the flaws respect to the relationship of the levels of debt and the wealth of a country. For example, in the original model, ($\hat{Z}$) the risk spread of debt, depends on the level of expected wealth that can be pledged in the future, which in turn depends on the current level of wealth. However, that is refuted from two angles. First, the level of debt and its cost is not linear as described in the first part of this section (see Figure 2B). And second, the empirical research of the paper shows that countries can accommodate larger levels of debt per-capita relative to income per-capita (see Figure 3 and 4). In other words, capital does flows into poorer countries, contrary to what Gertler and Rogoff (1990) claims, despite the fact that they have less sophisticated markets or even the less advanced technology.

4. Empirical Results

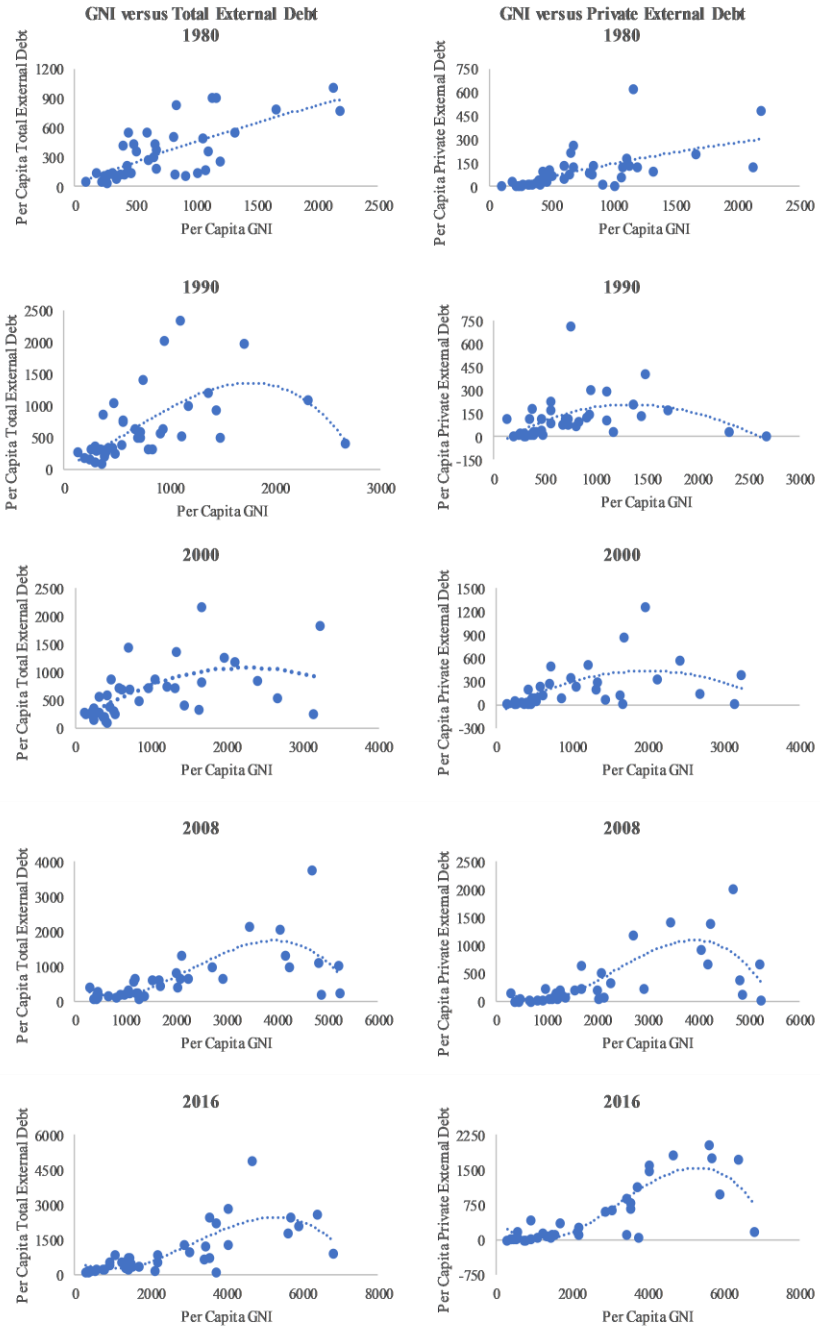
In contrast to Gertler and Rogoff (1990), who suggested that per-capita external borrowing should be increasing linearly in per-capita wealth, we found a positive but non-linear association between these two variables. We present scatter plots of total

[5] IMF Finance and Development Magazine – sept 1999: Moderating Fluctuations in Capital Flows to Emerging Market Economies. http://www.imf.org/external/pubs/ft/fandd/1999/09/mussa.htm

income and total and private external borrowings for thirty-nine developing countries. To check robustness of our results, we examine the relationship over different years and present results in figure 3, for example 1980, 1990, 2000, 2008 and 20016. Scatter plots depicts non-unit relationship between total external debt and GNP, as well as private external debt and GNP.

With rising income scale, borrowing from private as well as overall system increases, however the rate of borrowing declines with higher GNP; this non-linearity could be explained by the number of factors. For instance, at an earlier stage of the development, countries more easily get the program loans or concessional financing, but at higher income level, these borrowing reduced sharply, simply countries are not eligible for these concessional and soft loans. Another explanation could be that the developing economies try to smooth out the external shock (such as increase in oil price) at the beginning, but eventually accustomed to these shocks and smoothed out their consumption and demanded less borrowing to fund their high cost imports. The presence of domestic financial frictions at the earlier stages of development may also explains constraints to domestic demand of foreign funds. The level of capital efficiency may contribute significantly in determining relationship between external debt trend income level, at earlier stage of development, countries tend to pose high capital efficiency, but marginal product of capital reduces with high level of capital employed. This argument is consistent with well-defined rule of diminishing marginal return and can be applied in the Gertler-Rogoff extension model and multiple equilibria. The results are consistent at different time points, and hold true for almost all of time periods, irrespective of high growth period (1990s), in recession (2008) and post-recession period (2016).

Figure 4: Empirical Evidence of concavity in $\hat{Z}$



Source: IMF,Author's Calculations

5. Conclusion

The capital market inefficiencies directly lead to the spread in riskless interest rates and loan rate, the level of disparity is significant in low income countries. We develop a model to identify this non- linear relationship and found that with increasing income level, countries tend to increase their level of external debt with rising income, however the level of debt falls down at higher income level.

We investigated the empirical evidence of this non-linearity and explained possible reasons for example, the concessional loans and financing at earlier stages of development, adjustments with external supply shock such as oil prices, the disparities of capital efficiency in different countries etc. The robustness has been checked with deriving results for different point in time which validates our claim regarding non-linearity of income and foreign debt levels.

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Appendix

This study runs a numerical simulation in Matlab to prove that MR curve can be concave.

Thus, assuming positives values of the world interest rate (r) equal to 1%, and the present value of the borrower's endowment (V) equal to 10 (this is positive since the borrowers have endowments in period 1 and in period 2) we obtain the new curve for MR in Figure 4.

This shows that at initial values of k, the value of $\hat{Z}$ is high. However, at this value of k increases, the value of $\hat{Z}$ decreases.

Code in Matlab

```
r = 0.01
V = 10
prob = @(k) (1/10)*(k/(k+1))
prob2 = @(k) (atan(1.5*k-1.5*20) + atan(20))/3
fplot(prob2, [0 100])
Zhat = @(k) r*(k-V)/prob2(k)
fplot(Zhat, [10 50])
```

Figure 5: New function based on a numerical simulation