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Abstract

In this paper we develop a closed economy DSGE model of Pakistan with informality both in the labor and product markets. We try to remain consistent with the micro-foundations of Pakistan's economy for the purpose of estimation of the model parameters. However a couple of them have been calibrated to match the long-run features of the Pakistan economy. We introduce exogenous shocks of technology, fiscal spending and nominal interest rate in our model. Despite having to rely on annual data our model performs relatively better than existing DSGE literature on emerging markets. We find good degree of evidence of crowding out of private investment due to increases in government spending as well as spillover effects in the informal sector due to relatively lower intensity hinting at its possible shock absorbing role at the level of sustenance. The impulse response functions, in general, exhibit theoretically correct and empirically sound results.

Key Words: *DSGE, Informality, Closed Economy, Emerging Markets, Monetary Policy.*
JEL Classification*: D58, E32, E52.*

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1 Introduction

Dynamic Stochastic General Equilibrium (DSGE) models have become a workhorse for policy makers and central bankers for comparing possible impact of different policy scenarios. The proponents of these models claim that DSGE models are capable of replicating a number of stylized business cycle facts, for different developed and emerging economies, and are also not subject to the Lucas Critique [Pichler, (2007)]. In the last two decades significant progress has been made regarding specification and estimation of these models according to the need and features of the economy at hand. The most important recent contributions in terms of specification and standardization of modeling procedures involved in DSGE modeling are due to Smets and Wouters (2003) and Christiano, et al. (2005). As a result of this significant improvement in DSGE modeling literature many central banks of advanced countries have already developed their DSGE models recognizing the usefulness of these models for policy analysis and forecasting. Following the example of developed countries many of the emerging economies are also now focusing on constructing DSGE models for their countries encompassing relevant features of their respective economies.

The original DSGE models are actually the extension of real business cycle (RBC) models. Kydland and Prescott (1982) laid the foundation of DSGE modeling in the spirit of RBC theory. Real Business Cycle theory, assuming price flexibility and rationality of optimizing agents facing some constraints, investigates quarterly fluctuations when economy is hit by a real shock (the most common one being a technology shock). The earlier RBC models were criticized because economic policies had no role to play in these models. Furthermore these models failed to replicate some of the empirical regularities such as liquidity effects, co-movement of productivity and employment or the co-movement of real wages and output (Kremer et al., 2006). However, over time, there has been extensive work done that has helped in making these models theoretically parsimonious and empirically sound.

The New-Keynesian approach to DSGE models extended the RBC literature by introducing rigidities in price and wage setting. Rotemberg and Woodford (1997) were the first to introduce such a framework. The synthesis of New-Keynesian and RBC models proved to be successful in terms of matching economic theory to empirical evidence. For example, the idea of sticky prices broke down the neutrality of money hypothesis, giving a new dimension and life to the usefulness of monetary policy. Despite all the progress, these models failed to generate persistence of monetary policy shocks [Chari et al., (1998)], which led to the emergence of a large literature [see for example Andersen (1998), Huang and Liu (2002), and Edge (2002)] in order to develop an alternative approach. With the passage of time a wider set of possible as-

sumptions were introduced into DSGE models. For example, financial sector rigidities (Christensen and Dib, 2008), asymmetric information (Collard and Dellas, 2004), habit persistence in consumption (Fuhrer, 2000), adjustment costs in investment and variable capital utilization (Smets and Wouters, 2003 and 2005), and customer hold-up effects (Aksoy et al., 2009).

These models have been reasonably successful in replicating business cycles features of developed economies and have gained considerable importance for policy analysis and forecasting at central banks around the western world. However, for developing countries like Pakistan the adoption of such models requires a significant amount of groundwork and customization i.e. to be consistent with relevant micro evidence. However, any information about even the basic micro foundations of Pakistan economy is difficult to obtain as there is an inherent lack of micro-based surveys and even appropriate frequency data of major macroeconomic variables is mostly unavailable. Furthermore the lack of forward looking variables available in developing countries further complicates the situation. These challenges related to unavailability and consistency of micro-macro data tend to be understated when it comes to developing DSGE models for developing countries. For most of the existing literature on DSGE models for emerging economies, key parameters are borrowed from the literature and data transformation remains inadequate. One major contribution of this paper is that it overcomes some of these issues to a certain extent. This has been done through conducting wage and price setting surveys of manufacturing firms both in the formal and infomal sectors representing all the subsectors under manucturing in Pakistan. Besides, we have tried to use respresntative micro level datasets made available by Federal Bureau of Statistics (e.g., Labor Force Survey) and compiled effectively by us at the Research Department.

In addition, certain features of an economy like Pakistan cannot be anchored on the results of similar features from the developed world due to distinctly different developing nature of emerging economies. So the blueprint of models borrowed from advanced economies would not work for these economies without bringing them in line with relevant economic structure prevailing in a given economy. Some of the common features of emerging economies which differentiate them from developed economies need to be embedded in the economic models for meaningful policy implications include:

i) Existence of a large informal sector;

ii) Small open economies vulnerable to external shocks;

iii) Weak financial sector;

iv) Weak economic and political institutions.

Keeping in view the significance of above issues, the objective of this project is to develop a preliminary DSGE model of Pakistani economy consistent with some of the key economic features of Pakistan. Since this is the very first attempt to do so we are prioritizing the inclusion of only one feature in the otherwise standard DSGE model on the basis of our subjective degree of preference. As a result we incorporate informality in labor and product markets in a simple DSGE model. Future versions of the model would incorporate other relevant economic features of Pakistan.

2 Modeling with Informality: Literature on Developing Economies

Most of the literature on DSGE models has attempted to explain business cycle fluctuations of advanced economies with well established business cycles features, but literature on business cycles of emerging economies is very limited. However, in the past few years some efforts have been made in developing DSGE models capable of capturing the business cycle fluctuations for emerging economies. DSGE modeling for developing countries poses a number of challenges where not only the economic environment is different but is also less well-known. To exacerbate the situation further important features (even the stylized facts) of such economies are also not well established.

Batini et al. (2011) has recently developed a DSGE model for Indian economy with informality in goods market in the presence of credit constraints. They also introduced labor market frictions in the formal sector a la Zenou (2008). They used Bayesian technique for estimating parameters and have shown that the inclusion of informal sector and financial frictions improved the fit of their model. Peiris and Saxegaard (2007) introduced credit frictions in the presence of informality with an assumption that part of the inputs used in the production process are financed through borrowing at a premium over deposits from the informal sector. The study was aimed at evaluating monetary policy trade-offs in low-income countries with informal lending sources. The model was estimated using data for Mozambique.

Conesa et al. (2002) incorporated informal goods producing sector with differentiated technology in a simple RBC model. In this model sector trade-off is allowed through the presence of a wage premium in the formal sector. Furthermore, labor is assumed to be indivisible in the formal sector. Households can choose working between the two sectors with a given probability. A worker in the informal sector can enjoy more leisure but at the cost of lower wage.

Some studies (e.g. Antunes and Cavalcanti (2007) and Koreshkova (2006)) introduced cash-in-advance constraint faced by the informal sector, where most of the transactions are made through cash in order to analyze the role of

money, in terms of an inflation tax, on the informal sector. They tried to show the effect of the informal sector on optimal policy choice by government.

Aruoba (2010) and Aruoba and Schorfheide (2011) also introduced cash-in-advance constraint to differentiate informal sector from the formal sector by assuming that money is the only medium of exchange used in the informal sector. In their paper, using a very specific search-based monetary model, they found that large informal sector gets smaller in size and overall tax collection becomes higher under rising inflation.

Mattesini and Rossi (2010) analyzed the monetary policy in a dual economy in the New Keynesian framework with one competitive (informal) and one unionized (formal) sector. They came up with the conclusion that level of output is associated with relative size of the two sectors.

Castillo and Montoro (2008) modeled their economy with frictions in the labor market by introducing formal and informal labor contracts and analyzed the interaction between the two sectors and monetary policy. They introduced informality through hiring costs owing to labor market conditions (degree of tightness). In their model firms in the wholesale sector are assumed to balance the high productivity in formal sector with the lower hiring costs faced by the informal sector. The main finding of this theoretical framework is the cyclical behavior of informal sector i.e. it expands with rising aggregate demand because of lower hiring costs. Through this channel a link between informality, the inflation dynamics and monetary policy is established. This study supports the idea of informal labor market being a buffer for an economy.

Our model is closest to Conesa et al. (2002) in the sense that we introduce an informal goods producing sector with differentiated technology, as well as informality in labor market in a simple RBC model. However, we differentiate formal and informal sectors' labor on the basis of skill from the same household rather than assuming that households can choose between working for the two sectors with a given probability. In this way we have segmented the labor market with flexibility that households can decide on working hours of labor supplied to each of the two sectors, formal and informal, which maximizes overall utility of the households. A wage premium is charged in the formal sector over wage in the informal sector due to monopoly of households on skilled labor.

The main reasons for developing a RBC model with informality in product market and to some extent in the labor market is twofold. First to keep the model simple with minimum number of parameters to be estimated or calibrated under a data scarce situation. Secondly, it is a fact that a significant part of Pakistan's GDP (about 30 percent) is produced by the informal sector (Arby et al., 2010). Similarly,70 percent of our labor force is engaged in the

informal sector as well (various Labor Force Surveys, FBS).

Furthermore, the frequency of our model is annual making a stronger case for RBC type model. This is further supported by finding that annual price rigidity does not hold for Pakistan. Choudhary et al. (2011) find that all the firms in their sample of Pakistani formal firms adjust prices within a year irrespective of the competition they face. They also show that at least in the manufacturing sector 58 percent firms are connected with the informal sector either through demand or supply channels where demand channels tend to dominate. This motivated us to model the production of goods and services in the informal sector along with the formal sector.

3 Model

The model incorporates the informal sector through production as informality is one of the striking features of the developing countries. In our model, economy consists of households, firms, government and a monetary authority. There are two types of firms; formal and informal. These firms are further classified as intermediate and final good producing firms. Intermediate good producing firms sell their products to the final good producer, where the final good producing firms are only the retailers. Households derive utility from leisure, real money balances and consumption. They also supply labor and rent capital to firms. Each household has a unit of labor which is a composite of formal (skilled) and informal (unskilled) labor. The formal (skilled) labor is further divided into different types. Households have monopoly over each type "r" of labor which gives them the market power in wage setting process of such types of formal labor.

The final goods are produced by using intermediate goods. Differentiated goods produced by the formal intermediate firms, employing hired labor and capital, are sold to formal final good producers in a monopolistically competitive market. Labor is the only input used in the production of informal intermediate goods, which are sold to informal final good producers in a perfectly competitive market, which we assume for simplification.

In both formal and informal sector, final goods are produced by packaging intermediate goods under different technologies. Final output of both sectors is sold in a perfectly competitive environment. Aggregate price index is combination of sector prices. Government finances its consumption partly through taxes on formal sector and partly through printing money. Monetary authority follows Taylor type interest rate rule while setting its policy rate which then is the rate at which formal firms and government borrows from the household. We have three exogenous shocks in our model i.e., technology shock;

government spending shock/fiscal shock; and an interest rate shock.

3.1 The Household

The representative household's utility function consists of consumption, c_t, real money balances, $\frac{M_t}{P_t}$, and leisure $1 - h_t$. The total time endowment to the household is normalized to one. The preferences of the representative household can be described by the following life time expected utility function:

$$U(.) = E_t \sum \beta^t U_t(c_t, \frac{M_t}{P_t}, 1 - h_t) \tag{1}$$

where h_t is total number of hours worked and $\beta \epsilon (0, 1)$ is the discount factor. This particular utility function is also called the money in the utility function (MIU). The main reason for adopting this particular functional form for utility is the size of the informal sector in Pakistan which is essentially based on cash transactions as in Arby et al. (2010). Therefore, preference for holding more cash is a reasonable assumption for our economy.

$$U(.) = \ln c(i) + \chi \ln \frac{M(i)}{P} - \frac{h(i)^{1+\phi}}{1+\phi} \tag{2}$$

where χ is the preference parameter on money holding. Here we would like to make it a point to note that upper-case letters would denote nominal variables and lower-case letter denote real variables. The household faces following budget constraint while making its decisions:

$$c_t + i_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = \frac{W_t}{P_t} h_t + \frac{R_t^k}{P_t} k_t + \frac{M_{t-1}}{P_t} + (1 + R_{t-1}) \frac{B_{t-1}}{P_t} + \frac{T_t}{P_t} + \frac{\Pi_t}{P_t} \tag{3}$$

The left hand side of (3) represents household's total expenditures, while its right hand side represents its total earnings where i_t , B_t, W_t, R_t^k, k_t, R_t, T_t , and Π represent investment, bond holdings, nominal wage rate, rent on capital, capital, interest rate on bonds, lump- sum transfers, and dividends/profit from firms respectively. Capital gets accumulated according to:

$$k_{t+1} = (1 - \delta) k_t + i_t \tag{4}$$

where $\delta \epsilon (0, 1)$ is the capital depreciation rate. Household chooses c_t, M_t, B_t, h_t, and k_t, given (3) and (4), so that its life time expected utility is maximized. First order conditions of the household's utility maximization problem are given as:

$$\frac{1}{c_t} = \beta(1 + R_t) E_t \frac{1}{\pi_{t+1} c_{t+1}} \tag{5}$$

$$E_t\left[\frac{\pi_{t+1}}{(1+R_t)}\left\{(1-\delta)+r^k_{t+1}\right\}\right]=1 \tag{6}$$

$$h_t=\left(\frac{w_t}{c_t}\right)^{\frac{1}{\phi}} \tag{7}$$

and

$$\frac{\chi}{M_t/P_t}=\frac{1}{c_t}-\beta E_t\frac{1}{\pi_{t+1}c_{t+1}} \tag{8}$$

where (5) is the typical Euler equation for consumption, (7) is the labor supply equation and money demand is given by (8). (6) equates real rate of return on bonds and capital.

3.1.1 Labor Supply Choice between Formal and Informal Sectors

Household's aggregate labor is a composite of both formal, h^F_t, and informal labour, h^I_t. It can be expressed as:

$$h_t=\left[\eta^{-\vartheta}\left(h^F_t\right)^{1+\vartheta}+(1-\eta)^{-\vartheta}\left(h^I_t\right)^{1+\vartheta}\right]^{\frac{1}{1+\vartheta}} \tag{9}$$

where fractions η and $(1-\eta)$ represent formal and informal labor division of the representative household respectively. Similarly the aggregate wage can be expressed as:

$$W_t=\left[\eta\left(W^F_t\right)^{\frac{\vartheta}{1+\vartheta}}+(1-\eta)\left(W^I_t\right)^{\frac{\vartheta}{1+\vartheta}}\right]^{\frac{1+\vartheta}{\vartheta}} \tag{10}$$

where η represents share of formal labor while ϑ is inverse of elasticity of substitution between formal and informal labor. The household optimizes her wage earnings, by choosing formal and informal labor hours, given her optimal set $\{c_t,M_t,B_t,h_t,k_t\}$. Household's conditional demands for formal and informal labor are:

$$h^F_t=\eta\left(\frac{w^F_t}{w_t}\right)^{\frac{1}{\vartheta}}h_t \tag{11}$$

and

$$h^I_t=(1-\eta)\left(\frac{w^I_t}{w_t}\right)^{\frac{1}{\vartheta}}h_t \tag{12}$$

equations (11) and (12) show that supply of each type of labor changes with relative wage of that type and aggregate labor supply change. Similarly formal and informal wages can be written as:

$$w_t^F = \eta \left(\frac{P_t^F}{P_t} \right)^{\vartheta} w_t \tag{13}$$

and

$$w_t^I = (1-\eta) \left(\frac{P_t^I}{P_t} \right)^{\vartheta} w_t \tag{14}$$

Here we assume that formal labor h_t^F is a composite of labor differentiated on basis of different levels of skill represented by r. On basis of that, the aggregate formal labor supply can take the following form:

$$h_t^F = \left[\int_0^{\eta} \left(h_t^F(r) \right)^{\frac{\rho-1}{\rho}} dr \right]^{\frac{\rho}{\rho-1}} \tag{15}$$

where ρ is elasticity of substitution between different labor types in the formal sector. Aggregate wage in the formal sector can be written as:

$$W_t^F = \left[\int_0^{\eta} \left(W_t^F(r) \right)^{\rho-1} dr \right]^{\frac{1}{\rho-1}} \tag{16}$$

Here we also assume that households have market power to set wages on basis of its type of skill r, so it maximizes following function:

$$\begin{pmatrix} \max \\ W_t^F(r) \end{pmatrix} W_t^F(r) h_t^F(r) - W_t^F h_t^F(r) \tag{17}$$

Differentiating the above (17) w.r.t $W_t^F(r)$ and simplifying we get

$$W_t^F(r) = \left(\frac{1}{1-\frac{1}{\rho}} \right) W_t^F \tag{18}$$

where expression in parentheses on the right hand side represent mark up of type r wage on average wage in the formal sector.

3.1.2 Choice between Formal and Informal Consumption

The representative household consumes goods produced in both formal and informal sector. Household's aggregate basket, which is a composite of both formal and informal consumption, can be expressed as:

$$c_t = \left[\omega^{\frac{1}{\mu}} \left(c_t^F \right)^{\frac{\mu-1}{\mu}} + (1-\omega)^{\frac{1}{\mu}} \left(c_t^I \right)^{\frac{\mu-1}{\mu}} \right]^{\frac{\mu}{\mu-1}} \tag{19}$$

where the FOC yields the following formal aggregate consumption:

$$c_t^F = \omega \left(\frac{P_t^F}{P_t} \right)^{-\mu} c_t \tag{20}$$

and the informal aggregate consumption:

$$c_t^I = (1-\omega) \left(\frac{P_t^I}{P_t} \right)^{-\mu} c_t \tag{21}$$

where ω and μ are share of formal consumption in total consumption and elasticity of substitution between formal and informal consumption respectively. The general price level, based on the shares of formal and informal aggregate consumption can be written as:

$$P_t^{1-\mu} = \omega \left(P_t^F \right)^{1-\mu} + (1-\omega) \left(P_t^I \right)^{1-\mu} \tag{22}$$

And, following the standard practice, we define gross inflation rate as:

$$\pi_t = \frac{P_t}{P_{t-1}} \tag{23}$$

3.2 Firms Behavior

3.2.1 Retailers

3.2.1.1 Formal Retailers Retailers are the buyers of intermediate goods which they just package and sell in perfectly competitive environment [1]. Final good producers (retailers) in the formal sector produce good, y_t^F, using a constant elasticity of substitution (CES) technology with a continuum of intermediate goods, $y_t^F(m)$, as inputs:

$$y_t^F = \left[\int_0^1 y_t^F(m)^{\frac{\varepsilon-1}{\varepsilon}} dm \right]^{\frac{\varepsilon}{\varepsilon-1}} \tag{24}$$

where ε is the elasticity of substitution between differentiated formal intermediate goods. Final goods formal producer's profit function is given by:

[1] The specification can be simpler following Gali (1994) where only one layer of production can produce and sell differentiated goods. However, both specifications, the one which we have used in our model, and the simpler one are equivalent. Our specification would, however, be more insightful when price stickiness gets involved in the formal manufacturing. At present there is no price stickiness due to its non-existence under annual frequency.

$$\Pi_t^F = (1-\tau)\, P_t^F y_t^F - \int_0^1 P_t^F(m) y_t^F(m) dm \tag{25}$$

where τ is the flat tax rate on final goods. Retailers optimize their profit while deciding on how much intermediate input m to purchase given its price and elasticity of substitution. This profit maximization yield the derived demand functions for each intermediate good m given by:

$$y_t^F(m) = \left[\frac{P_t^F(m)}{(1-\tau)\, P_t^F}\right]^{-\varepsilon} y_t^F \tag{26}$$

Then the formal retailer's aggregate price level is:

$$P_t^F = \frac{1}{(1-\tau)} \left[\int_0^1 P_t^F(m)^{1-\varepsilon} dm\right]^{\frac{1}{1-\varepsilon}} \tag{27}$$

3.2.1.2 Informal Retailers The informal retailers in our model are completely symmetric to the formal retailers with the exception that informal retailers pay no taxes and can only use intermediate goods produced in the informal sector. A final good y_t^I is produced in the informal sector by using informal intermediate goods as inputs with the following production technology:

$$y_t^I = \left[\int \left(y_t^I(j)\right)^{\frac{\upsilon-1}{\upsilon}} d_j\right]^{\frac{\upsilon}{\upsilon-1}} \tag{28}$$

where υ is elasticity of substitution between differentiated informal intermediate goods. The informal sector final good producing firm maximizes its profit as:

$$\begin{pmatrix}\max \\ y^I(j)\end{pmatrix} P^I y_t^I - \int p_t^I(j) y_t^I(j) dj \tag{29}$$

First order condition yields the derived demand functions for each informal intermediate good j which when aggregated is represented as following:

$$y_t^I(j) = \left(\frac{p_t^I(j)}{P_t^F}\right)^{-\upsilon} y_t^I \tag{30}$$

Hence, the aggregate price index P_t^I of final informal goods can be written as:

$$P_t^I = \left[\int \left(p_{(I)}^I\right)^{1-\upsilon} d_I\right]^{\frac{1}{1-\upsilon}} \tag{31}$$

3.2.2 The Intermediate Goods Producers

3.2.2.1 Formal Firm The formal sector intermediate producers have to decide about their demand for capital and labour for a given wage and capital

rent in a monopolistic labor market (based on type of skill r) and a competitive capital market. In addition, they set prices of their differentiated products while exploiting some degree of monopoly. This setup is supported by survey findings in Choudhary et al. (2011) where firms report that they remain in the formal sector for sake of their monopolistic position.

Before moving towards pricing decision by formal intermediate firms, we focus on their demand functions for factors and marginal cost. Formal sector intermediate producers employ the following technology for production:

$$y_t^F = a_t {k_t}^{\alpha} {h_t^F}^{1-\alpha} \tag{32}$$

where a_t is the exogenous level of technology embodied in the formal production process. The resulting first order conditions for wage and rent for capital are given by:

$$\frac{W_t^F}{P_t^F} = (1-\alpha)\, a_t \left(\frac{k_t}{h_t^F}\right)^{\alpha} \tag{33}$$

$$\frac{R_t^k}{P_t^F} = \alpha a_t \left(\frac{k_t}{h_t^F}\right)^{\alpha-1} \tag{34}$$

Combining (33) and (34) we can write the capital-labor ratio as:

$$\frac{r_t^k}{w_t^F} = \frac{\alpha}{1-\alpha}\frac{h_t^F}{k_t} \tag{35}$$

where r_t^k and $w_t^F$are real rent on capital and real wage in the formal sector respectively. This specification is reached by assuming the standard identical firms assumption for aggregation. Using (33), (34) and (35) we can write marginal cost as:

$$mc_t^F = \frac{1}{a_t}(\alpha)^{-\alpha}(1-\alpha)^{-(1-\alpha)}\left(w_t^F\right)^{1-\alpha}\left(r_t^k\right)^{\alpha} \tag{36}$$

The aggregate relative price of the formal intermediate goods production sector as compared to the aggregate price (which is a composite of formal and informal aggregate prices) is expressed as (after exploiting the marginal cost in 36):

$$\frac{P_t^F}{P_t} = \left(\frac{\varepsilon}{\varepsilon-1}\right) mc_t^F \tag{37}$$

Using (37) and the fact that none of the price components is firm-specific, we can write the aggregate price for the formal intermediaries as following:

$$P_t^F = \frac{1}{(1-\tau)}\left(\frac{\varepsilon}{\varepsilon-1}\right) mc_t^F \tag{38}$$

which depends on the marginal cost mc_t^F and tax rate τ.

3.2.2.2 Informal Firm The production function of the representative intermediate firm in the informal sector is given as:

$$y_t^I = \gamma h_t^I \tag{39}$$

where informal labor is the only input and there is no capital and technology. γ only provides information on informal labor productivity. Since the intermediate producers here are perfectly competitive, the profit maximization is straight forward and implies:

$$\frac{P_t^I}{P_t} = \frac{w_t^I}{\gamma} \tag{40}$$

The aggregate price of the informal intermediaries is given as:

$$p_t^I = mc_t^I = \frac{w_t^I}{\gamma}$$

We have modeled the informal sector intermediate firm in a very simple way in order to keep it easy to track. This modeling choice is further supported by findings in Choudhary et al. (2011).

3.3 Government Behavior

3.3.1 Monetary Policy

The central bank conducts monetary policy through a Taylor-rule type function. It manages the short-term nominal interest rate, R_t, in response to deviations of output y_t and inflation π_t from their respective potential/ target levels. The interest rate reaction function is given by:

$R_t = \xi_t r \left(\frac{\pi_t}{\pi^*}\right)^{\psi_1} \left(\frac{y_t}{y^*}\right)^{\psi_2}$where ξ_t is a nominal interest rate shock, r is the steady state nominal interest rate, $\pi^*$is targeted inflation level and y^* is potential output.

3.3.2 Fiscal Policy

Government finances its expenditures, consisting of its spending on goods and services and making lump-sum transfers to households by imposing a flat tax on final goods produced in the formal sector and also by printing money. It

does so partially by selling bonds to the households on which it pays interest back as well. Therefore, the government's budget constraint can be written as:

$$G_t + TR_t + \frac{(1 + R_{t-1})\, B_{t-1}}{P_t} = \tau Y^F + \frac{B_t}{P_t} + \frac{M_t - M_{t-1}}{P_t} \tag{41}$$

where $G_t = \bar{g} * g_t$ follows AR(1) process such as:

$$g_t = \varsigma_0 + \varsigma_1 g_{t-1} + \varepsilon_t \tag{42}$$

3.4 Aggregate Resource Constraints

The economy wide resource constraints are based on formal output y_t^F and informal output y_t^I, where the formal sector's resource constraint is the traditional closed economy resource constraint. On the other hand the informal sector's resource constraint equates informal output to informal consumption.

$$y_t^F = c_t^F + i_t + \bar{g}_t \tag{43}$$

where $\bar{g}_t$ is the real government spending.

$$y_t^I = c_t^I \tag{44}$$

The overall economy-wide resource constraint, with informal sector incorporated, looks like:

$$y_t = c_t^F + c_t^I + i_t + \bar{g}_t \tag{45}$$

4 Calibration

All parameters in our model are calibrated for annual frequency. There are 22 parameters in total with 16 structural and 6 shocks related parameters (see tables 1 and 2). Structural parameters can be categorized into four broad groups: (1) household related, (2) formal-informal consumption, (3) formal-informal labour supply and (4) production function parameters. Most of the parameters used in our model have been calibrated using partial estimation/computation approach. However, only very few of the parameters, for which estimation remained an issue throughout, are picked from existing DSGE models literature preferably for developing countries.

First of all, we discuss parameters related to household's utility function. The discount factor β is a benchmark of forward looking behavior has been computed to be 0.99 by taking inverse of average long term real interest rate (see Appendix A). This is in line with the estimated value of β in Ahmed, et al. (forthcoming). χ reflects household's preference for money holding and a value of 0.25 for this parameter is taken from DiCecio and Nelson (2007). The coefficient of labour supply in utility function ϕ is fixed at 1.5 following Fagan and Messina (2009). This value is consistent with the posterior mean reported by Smets and Wouters (2007).

The parameters $\frac{P^F}{P}$, $\frac{P^I}{P^F}$, ω and μ together govern the distribution of formal and informal consumption. The steady state share of formal sector in overall price level $\frac{P^F}{P}$ is set at 0.53 and share of formal consumption in total consumption ω is fixed at 0.55. These values have been taken from Khan and Khan (2011). $\frac{P^I}{P}$ is $1 - \frac{P^F}{P}$ and $\frac{P^I}{P^F}$ is simply ratio of $\frac{P^I}{P}$ to $\frac{P^F}{P}$. The value of μ; elasticity of substitution between formal and informal consumption has been calibrated to ensure that steady state ratios match with those observed in data.

The parameters η, ϑ and σ characterize the interaction of formal and informal sectors on the labour side. The share of formal labour supply in total labour supply $\eta = 0.29$ is computed by taking average of ratios of number of people employed in the formal sector to total number of people employed in the non-agricultural sector during 1990-1991 to 2008-2009. The relevant labour force data is collected from various issues of the Federal Bureau of Statistics Labor Force surveys. The elasticity of substitution between formal and informal labour supply ϑ is found to be 2 after estimation using before mentioned data. Using cross sectional data of Labour Force Surveys conducted between 1997-98 and 2008-09, we estimate this elasticity for each survey period and then take average of all estimated values to obtain final value of 2 (see Appendix A). The value of formal wage premium σ is set at 0.25. This value has been taken from the preliminary findings of the formal sector wage setting surveys conducted by the State Bank of Pakistan (see Choudhary et al. (Forthcoming)).

The two parameters α and δ are related to production. To calibrate the share of capital in production α, we took a value of 0.50 which is quite close to the average of capital shares of other less developed countries as reported by Liu (2008). The depreciation rate δ has been set at 0.15 which is in line with values used by other authors in the literature on DSGE models for developing countries such as $\delta = 0.1255$ as used by Garcia, et al. (2006). In addition, balance sheet analysis of joint stock companies listed at Karachi Stock exchange reveals that overall depreciation rate has been close to 10 percent. Since less well capitalized firms are expected to have higher depreciation rates, therefore we have adjusted this estimate upwards for our final value of 0.15.

The taylor rule responses of inflation and output have been estimated by

regressing nominal interest rate on deviations of inflation and output from their steady states. Following Ireland (2004), ψ_t^{π} and ψ_t^{Y} are estimated to be 0.48 and 0.52 respectively (see Appendix A). Steady state gross inflation $\bar{\pi}$ has been estimated to be 1.09 (see Appendix A).

Table 1: Structural Parameters

Parameter	Value	Parameter	Value
β	0.99	η	0.29
χ	0.25	ϑ	2
ϕ	1.5	σ	0.25
$\frac{P^F}{P}$	0.53	α	0.50
$\frac{P^I}{P}$	0.47	δ	0.15
$\frac{P^I}{P^F}$	0.89	ψ^{π}	0.48
ω	0.55	ψ^{y}	0.52
μ	0.7	$\bar{\pi}$	1.09

The parameters describing the three shock processes are estimated following King and Rebelo (2000). Persistence of technology shock ρ_A and standard deviation of technology shock σ_A are set at 0.9 and 0.02 respectively. Similarly ρ_G and σ_G are fixed at 0.78 and 0.14 respectively (see Appendix A). In the same manner interest rate persistence, coming from the Taylor Rule, ρ_R is set at 0.28 and its standard error σ_R is fixed at 0.016. The data for these estimations of shock related parameters has been acquired from Federal Bureau of Statistics (FBS) and State Bank of Pakistan (SBP).

Table 2: Shock Process Parameters

Sr.#	**Parameter**	**Description**	**Value**
1	ρ_A	Persistence of technology shock	0.9
2	ρ_G	Persistence of fiscal spending shock	0.78
3	ρ_R	Persistence of interest rate shock	0.28
4	σ_A	SD of technology shock	0.02
5	σ_G	SD of fiscal spending shock	0.14
6	σ_R	SD of interest rate shock	0.016

5 Results

5.1 Assessing the Simulation Performance of the DSGE Model

Before using DSGE model for policy analysis it is essential to test its capability of replicating the business cycle's stylized fact manifested by the Pakistani data. It is common in the literature to compare moments of the data from those generated by the model. Following the convention, we limit our moment comparison exercise to output as an indicator of growth, aggregate consumption and private investment. The data for these time series has been obtained from national accounts compiled by FBS. We use the annual data from 1980-81 to 2009-10.

Tables 3a and 3b below present some preliminary results. We can see that steady state results of the model are either very close or within the empirical range of the data. The empirical range here are the minimum and the maximum values of the series over a decade during the data period from 1980-81 to 2009-10. As far as the relative volatility of consumption and investment with output are concerned, our model overestimates these statistics as compared to the data.

Table 3a: Steady State Ratios and Relative Volatility [2]

Variables	Steady State Ratio with Y	Relative Volatility with Y
C	0.7 [0.68−0.84]	2.37 [1.24−1.56]
Pvt I	0.13 [0.10−0.16]	4.03 [3.73−3.85]

The cross-correlations of consumption and private investment with output obtained from the model are also higher vis-a-vis data. The autocorrelations of output and investment are lower while it is higher for consumption. However, all of these moments are not too far from the range obtained for the relevant moment from the data.

On the whole we can say that our model performs reasonably well on the basis of these criteria and is in line with acceptable performance for DSGE models in the literature.

We can see that most significant deviations from the data moments of relative volatility and cross correlation with output occur with respect to consump-

[2] *Numbers in parenthesis are minimum and maximum values over different decades between 1980-81 to 2009-10.*

tion. This can be partly explained by the fact that there are doubts over the accuracy of private consumption data for Pakistan.

Table 3b: Auto/Cross Correlations [3]

Variables	Y	C	Pvt.I
Y	0.56 [0.62−0.69]	–	–
C	0.97 [0.52−0.78]	0.47 [0.26−0.43]	–
Pvt.I	0.86 [0.56−0.70]	–	0.34 [0.46−0.57]

In the national income accounting system of Pakistan, private consumption is actually treated as a residual (also pointed out in Baqai, 1965). This point is discussed in detail in Choudhary and Pasha (forthcoming) and Malik (forthcoming).

5.2 Response of Economy to Structural Shocks

After the assessment of the simulation performance of the model, the next step is to use the model for policy analysis i.e. to analyze the impulse response functions generated in response to different exogenous structural shocks in our model. In our simulation exercises, we include three shocks namely technology; fiscal or government spending and interest rate shocks.

Figure 1, shows that following a positive technology shock in the formal sector (where technology lies by design), investment, output and consumption in the formal sector rise, while inflation falls. Since output rises, wages and working hours also rise and as a result there is an overall rise in household's income. On the other hand output in the informal sector also rises but this rise is insignificant and converges quickly to the steady state.

The consumption of informal goods decline first but then start increasing soon. This alternating behavior of fall and rise in informal consumption is due to the substitution and income effects dominating each other in turns. Since the technology shock results in lower inflation meaning that the prices of formal goods come down for the household (which now is earning more as well) substitutes its informal consumption with the formal consumption but as soon as the wages start coming down the substitution effect dominates resulting in improved demand for informal goods.

[3] *Numbers in parenthesis are minimum and maximum values over different decades between 1980-81 to 2009-10.*

Labor hours increase in both the formal and informal sectors but the magnitude is small i.e. due to the advancement in technology labor productivity increases which then cancels out the demand for extra labor due to higher demand for consumption. This impact on labor hours is much weaker in the informal sector as compared to the formal sector.

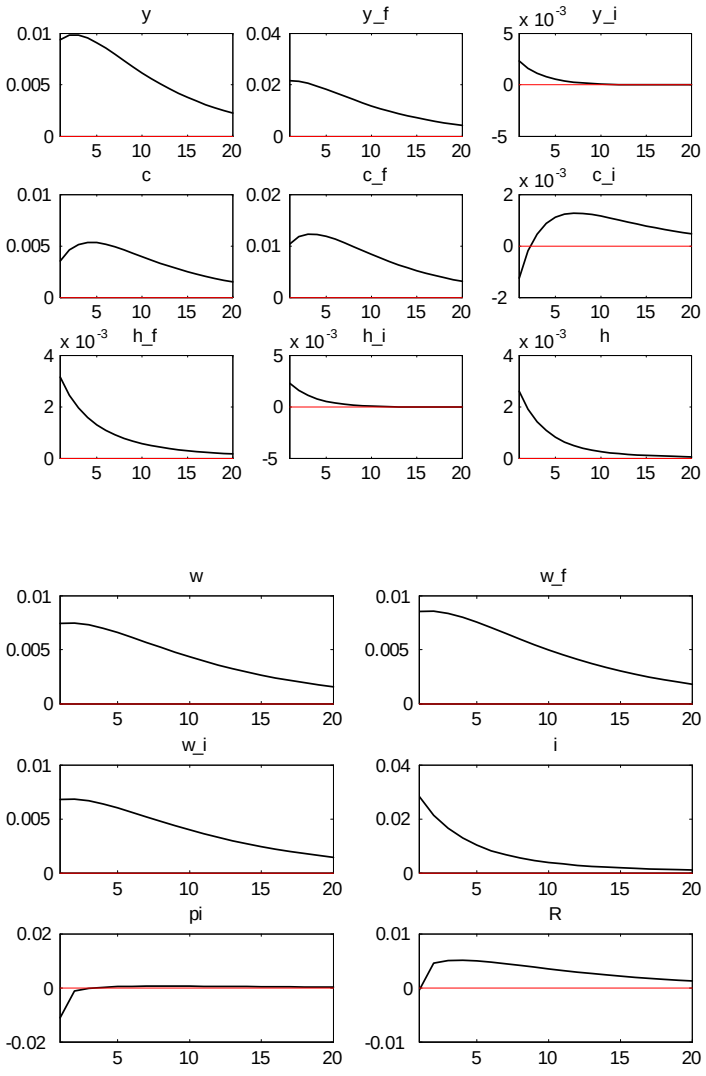

Figure 1: IRFs of the Technology Shock

Since in our model, the link between formal and informal sectors is modeled in a way that they interact mainly through consumption, and technology only effects the formal sector, the intensity of the impact of the technology shock on informal consumption and informal wage is minor and is only in form of spillover from the formal sector through income effect first and then through

a convergence oriented substitution effect of consumption.

In response to a positive shock to government spending (see figure 2) the nominal rate of return increases making it difficult for the private formal firms to invest in capital. This is the crowding-out effect of government spending on private investment and is experienced more intensely by the formal producers who need to raise capital on their own. To this extent the informal sector (i.e. informal production) is unaffected by crowding-out as their production process only utilizes labor and hence has no role for capital.

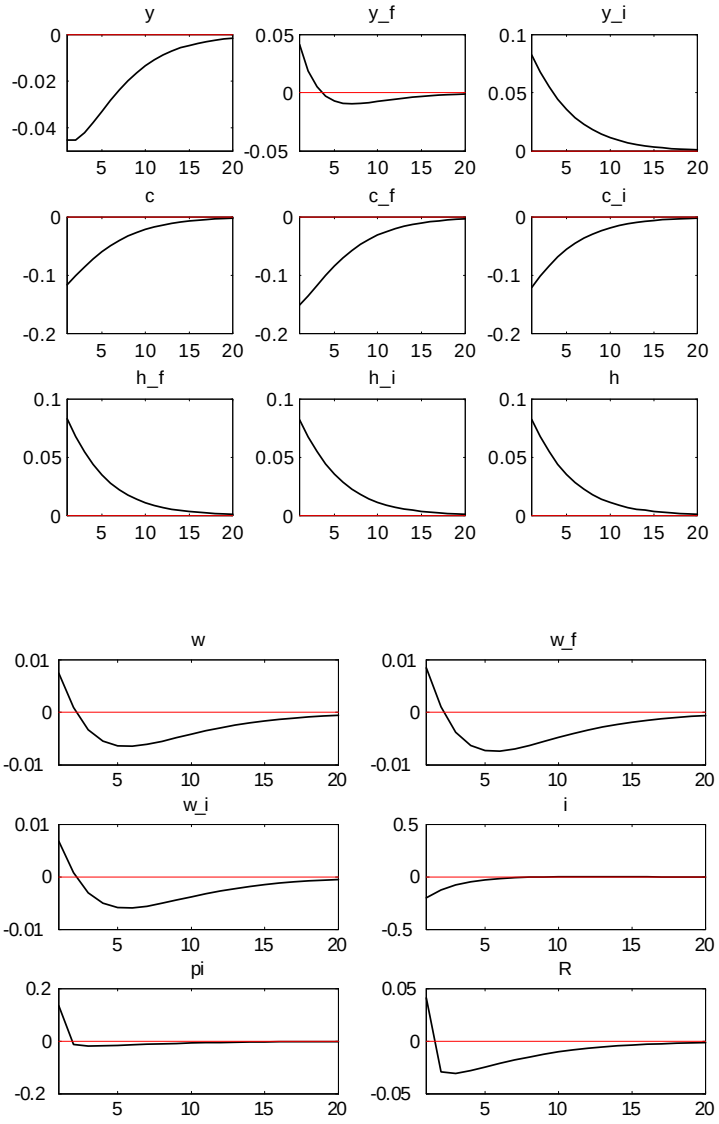

Figure 2: IRFs of the Fiscal Shock

This crowding-out results in lower level of aggregate output which then lowers aggregate wage with a certain lag of time. However, initially, hours worked and wages rise in both the sectors. This rise in wages is only for a short duration after which they fall below the steady state level.

In our model, since government both consumes and taxes the formal sector goods, higher government spending would lead to increased formal output production. This explains the initial positive jump in the formal output. This sudden rise is short-lived after which formal output falls below the steady state level due to the crowding-out effect.

Over time, the overall income of the household falls, consumption falls, and as a result output falls below the steady state, making demand for labor lesser than supply for labor. The inflationary rise in government spending makes formal goods more expensive.

The income effect thus results in relatively lesser effect on fall in consumption of the informal goods as compared to the formal goods. This is because households substitute some part of their expensive formal consumption with the informal counterparts.

The results of fiscal shock leading to crowding-out of private investment have also been empirically verified for Pakistan (see Khan and Khan, 2007; and Ahmad and Qayyum, 2008).

Figure 3 shows that, in response to a positive monetary policy shock, output, investment, labor hours, and wages decline on aggregate level which is a standard result of DSGE models. The inflation also decreases significantly but only for a short period. This immediate and strong response of inflation is due to flexible prices, a feature of Pakistan economy in an annual setting.

Aggregate consumption also declines. This behavior is rational since households substitute consumption for investment in bonds issued by the government, whereas private investment in the formal firms decline due to relatively higher interest rate.

Consumption on sector level increases with an insignificant size and for shorter period after which it starts decreasing and becomes negative. The sudden positive jumps in the level of formal and informal consumption in response to monetary policy shock are inconsistent with empirical evidence on consumption which generally is in the form of aggregate consumption only. We find no suitable explanation for such behavior of these sector specific impulse responses.

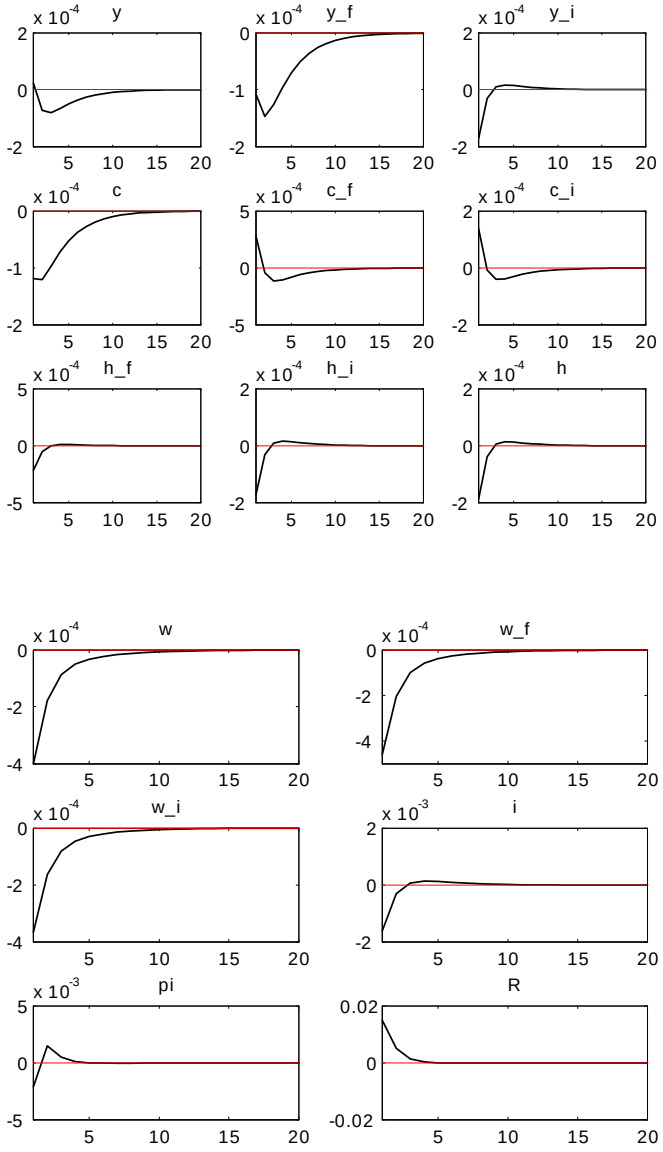

Figure 3: IRFs of the Interest Rate Shock

Our results are generally consistent with Batini, et al. (2011) for Indian economy, and especially for sector specific consumption. The policy impact on informal sector are small in size and shorter in duration as the policy rate is not directly linked with the production process in the informal sector. The response by the informal sector variables is only due to spillover effects from formal sector, mainly coming from the household's consumption decisions, as explained earlier. Generally, on aggregate level, the direction of impulse responses from all three shocks is in line with existing literature (see Smets and Wouters, 2003).

6 Conclusion

In this paper we develop a general equilibrium model with an informal sector and three types of shocks. The theoretical moments of the model perform reasonably well for private investment and less so for consumption. The latter is to be expected as private consumption and is treated as a residual in the aggregate resource constraint equation of the national accounting system of Pakistan. Further probing reveals that per capita consumption from aggregate data and surveys on household consumption do not match. Consequently, empirical claims on consumption have to be taken with a pinch of salt.

The direction and the propagation pattern of our impulse responses to shocks are theoretically sound and match patterns present in the existing literature. In particular, crowding-out effects on private investment are strong while there are relatively weak spillover effects of shocks to the informal economy implying that the informal economy can possibly serve as a shock absorber.

Two important extensions on the basis of the present paper are being sought. First is theoretical and entails micro-founding consumption and labor supply decisions when it comes to choosing between the formal and informal sectors as well as opening up the formal sector firms to foreign competition. Second is empirical and relates to developing quarterly time series of relevant macro variables so that general equilibrium models may be considered for short term policy analysis.

Appendix

A Parameter Estimation

Discount Rate ($\boldsymbol{\beta}$)

The discount rate has been estimated using annual data from $1981 - 2011$. Return on government bonds and change in CPI have been used to measure long term interest rate and inflation respectively. To incorporate expectations, lagged inflation has been used to calculate the real interest rate.

Elasticity of Substitution Between Formal and Informal Labour ($\boldsymbol{\vartheta}$)

In order to estimate ϑ, we used the micro-level data from the annual Labor Force Surveys. We compiled labor force survey data from the last several waves available between 1997-98 and 2008-09. The survey was not conducted in 2000-01, 2002-03 and 2004-05.We estimated the elasticity of substitution between formal and informal sector separately for each wave (*as well as for all the data compiled together)* . We ran the following regression as in Psacharopoulos and Hinchliffe (1972):

$$\ln\left(\frac{W_t^F}{W_t^I}\right) = a + \vartheta \ln\left(\frac{L_t^F}{L_t^I}\right)$$

where W_t^F & W_t^I are the hourly wage rates of formal and informal sector employees in a household. L_t^F & L_t^I are the average hours worked in a week by employees in the formal and informal sector respectively. A caveat for our estimation of elasticity of substitution between formal and informal labor is that we were limited by the nature of LFS being a household survey. Our sample was reduced significantly for estimation by the fact that we could only use data from households that have more than one employee as well as at least on each in the formal and informal sector.

We considered an employee to be part of the formal sector if his/her response to the LFS question "what kind of enterprise?" about his/her work place was any one of Federal Government, Provincial Government, Local body Government, Public enterprise, Private limited company, Public limited company and Cooperative society. In addition, the respondents who answered the enterprise question with either Individual Ownership, Partnership or Other were considered part of the formal sector if and only if their enterprises kept written

accounts (as asked in the next question of the survey "Does the enterprise keep written accounts?"). On the other hand employees that responded to the enterprise question with either Individual Ownership, Partnership or Other and also answered the written accounts question with either "No" or "Don't know" were considered part of the informal sector.

*Taylor Rule (ψ_t^π **and** ψ_t^Y)*

To obtain response of policy interest rate to deviations of inflation and output from steady state, we regress log of interest rate on deviations of inflation and output from their trend values. We use average call money rate, GDP deflator and per capita real GDP for interest rate , inflation and output. Deviation of inflation from steady state is measured using residuals of following estimation:

$$\ln \pi_t = c + \mu_t^\pi$$

Results of this estimation show c =0.087331 implying steady state gross inflation equal to 1.09. For deviation of output from steady state, we regress log of per capita real GDP on constant and trend through following equation and take residuals:

$$\ln Y_t = c + \gamma t + \mu_t^Y$$

Furthermore, to estimate the response of interest rate to deviations in inflation and output, we estimate following equation:

$$\ln R_t = c + \upsilon^\pi \mu_{t-1}^\pi + \upsilon^Y \mu_{t-1}^Y$$

Estimated responses to inflation and output deviations are then normalized as $\frac{\upsilon^\pi}{\upsilon^\pi+\upsilon^Y}$ and $\frac{\upsilon^Y}{\upsilon^\pi+\upsilon^Y}$ to yield values of 0.48 for ψ^π and 0.52 for ψ^y.

Shock Process (ρ_A,ρ_G,ρ_R, σ_A,σ_G, σ_R)

The TFP series is obtained by using residuals of estimated neo- classical production function thorough following regression:

$$\ln Y_t = \alpha \ln K_t + (1-\alpha) \ln L_t + \ln A_t$$

To estimate ρ_A, we estimate the following equation:

$$\ln A_t = c + \rho_A \ln A_{t-1} + u_t^A$$

σ_A is calculated using residuals of above equation. Owing to unavailability of actual data, capital stock series has to be calculated using the perpetual

inventory method. There are different ways to calculate capital stock series and parameters of technology shock process are sensitive to variations in capital stock series. Using different series, we get a range of estimates for ρ_A between 0.85-0.95 and σ_A 0.0095-0.025. From these ranges, we choose the values of 0.9 and 0.02 for ρ_A and σ_A respectively. Similarly, to obtain ρ_G and σ_G, we estimate the following equation:

$$\ln g_t = c + \rho_G \ln g_{t-1} + \mu_t^g$$

Using log of real per capita government consumption, estimation of the above yield values of 0.78 for ρ_G. Standard deviation of residuals from above regression yields estimate of σ_G that is 0.14. The parameters of the interest rate shock, i.e., ρ_R and σ_R have been estimated in the similar manner as well. However, we have used the residuals of the Taylor Rule instead the simple AR-1 formulation followed for the other two shocks. The values obtained for ρ_R and σ_R are 0.28 and 0.016 respoectively.

B Complete Model

Financial assets optimization equation

$$\frac{1}{c_t} = \beta(1 + R_t)E_t \frac{1}{\pi_{t+1}c_{t+1}}$$

Physical assets optimization equation

$$E_t \left[\frac{\pi_{t+1}}{(1 + R_t)} \left\{ (1 - \delta) + r^k_{t+1} \right\} \right] = 1$$

Aggregate hours worked optimization equation

$$h_t = \left(\frac{w_t}{c_t} \right)^{\frac{1}{\phi}}$$

Money holding optimization equation

$$\frac{\chi}{M_t/P_t} = \frac{1}{c_t} - \beta E_t \frac{1}{\pi_{t+1}c_{t+1}}$$

Capital accumulation equation

$$k_{t+1} = (1 - \delta)k_t + i_t$$

Supply of formal labour

$$h^F_t = \eta \left(\frac{w^F_t}{w_t} \right)^{\frac{1}{\vartheta}} h_t$$

Supply of informal labour

$$h^I_t = (1 - \eta) \left(\frac{w^I_t}{w_t} \right)^{\frac{1}{\vartheta}} h_t$$

Composite wage rate

$$w_t = \left[\eta \left(w^F_t \right)^{\frac{1+\vartheta}{\vartheta}} + (1 - \eta) \left(w^I_t \right)^{\frac{1+\vartheta}{\vartheta}} \right]^{\frac{\vartheta}{1+\vartheta}}$$

Informal wage rate

$$\frac{P_t^I}{P_t} = \frac{w_t^I}{\gamma}$$

Formal wage rate

$$w_t^F = \left(w_t^I\right)^{\sigma}$$

where,

$$\sigma = \left(\frac{\rho}{\rho - 1}\right)$$

Formal price level

$$\frac{P_t^F}{P_t} = \frac{1}{(1-\tau)}\left(\frac{\epsilon}{\varepsilon - 1}\right) mc_t^F$$

General price equation

$$P_t^{1-\mu} = \omega\left(P_t^F\right)^{1-\mu} + (1-\omega)\left(P_t^I\right)^{1-\mu}$$

Formal consumption

$$c_t^F = \omega\left(\frac{P_t^F}{P_t}\right)^{-\mu} c_t$$

Informal consumption

$$c_t^I = (1-\omega)\left(\frac{P_t^I}{P_t}\right)^{-\mu} c_t$$

Gross general inflation rate

$$\pi_t = \frac{P_t}{P_{t-1}}$$

Gross sectoral inflation rates

$$\pi_t^F = \frac{P_t^F}{P_{t-1}^F} \text{ and } \pi_t^I = \frac{P_t^I}{P_{t-1}^I}$$

Formal production function

$$y_t^F = a_t {k_t}^{\alpha} h_t^{F1-\alpha}$$

Capital-labour ratio

$$\frac{k_t}{h_t^F} = \frac{\alpha}{1-\alpha}\frac{w_t^F}{r_t^k}$$

Marginal cost

$$mc_t^F = \frac{1}{a_t}(\alpha)^{-\alpha}(1-\alpha)^{-(1-\alpha)}\left(w_t^F\right)^{1-\alpha}\left(r_t^k\right)^{\alpha}$$

Informal production function

$$y_t^I = \gamma h_t^I$$

Taylor type rule

$$R_t = \xi_t r\left(\frac{\pi_t}{\pi_t^*}\right)^{\psi_1}\left(\frac{y_t}{y_t^*}\right)^{\psi_2}$$

Fiscal budget constraint

$$G_t + TR_t + \frac{(1+R_t)B_{t-1}}{P_t} = \tau Y^F + \frac{B_t}{P_t} + \frac{M_t - M_{t-1}}{P_t}$$

Formal and informal sectors' aggregate resource constraints

$$\begin{aligned} y_t^F &= c_t^F + i_t + g_t \\ y_t^I &= c_t^I \end{aligned}$$

Economy wide aggregate resource constraint

$$y_t = c_t^F + c_t^I + i_t + g_t$$

C Complete Model in Steady State

Financial assets optimization equation

$$R = \frac{\pi}{\beta} - 1$$

Physical assets optimization equation

$$\beta \left\{ (1 - \delta) + r^k \right\} = 1$$

Hours worked optimization equation

$$h = \left(\frac{w}{c} \right)^{\frac{1}{\phi}}$$

Money holding optimization equation

$$\frac{\chi}{M/P} = \frac{1}{c} - \beta \frac{1}{\pi c}$$

Capital accumulation equation

$$i = \delta k$$

Supply of formal labour

$$h^F = \eta \left(\frac{w^F}{w} \right)^{\frac{1}{\vartheta}} h$$

Supply of informal labour

$$h^I = (1 - \eta) \left(\frac{w^I}{w} \right)^{\frac{1}{\vartheta}} h$$

Composite wage rate

$$w = \left[\eta \left(w^F \right)^{\frac{1+\vartheta}{\vartheta}} + (1 - \eta) \left(w^I \right)^{\frac{1+\vartheta}{\vartheta}} \right]^{\frac{\vartheta}{1+\vartheta}}$$

Informal wage rate

$$w^I = \gamma \frac{P^I}{P}$$

Formal wage rate

$$w^F = \left(w^I\right)^{\sigma}$$

General price equation

$$P^{1-\mu} = \omega \left(P^F\right)^{1-\mu} + (1-\omega)\left(P^I\right)^{1-\mu}$$

Formal consumption

$$c^F = \omega \left(\frac{P^F}{P}\right)^{-\mu} c$$

Informal consumption

$$c^I = (1-\omega)\left(\frac{P^I}{P}\right)^{-\mu} c$$

Gross general inflation rate

$$\pi = 1$$

Formal production function

$$y^F = k^{\alpha} h^{F\,1-\alpha}$$

Capital-labour ratio

$$\frac{k}{h^F} = \frac{\alpha}{1-\alpha}\frac{w^F}{r^k}$$

Marginal cost

$$mc^F = (\alpha)^{-\alpha}(1-\alpha)^{-(1-\alpha)}\left(w^F\right)^{1-\alpha}\left(r^k\right)^{\alpha}$$

Formal price

$$P^F = \frac{1}{(1-\tau)}\left(\frac{\varepsilon}{\varepsilon-1}\right) MC^F$$

Informal production function

$$y^I = \gamma h^I$$

Demand for informal labour equation

$$\gamma \frac{P^I}{P} = W^I$$

Taylor type rule

$$R = r\left(\frac{\pi}{\pi^*}\right)^{\psi_1}\left(\frac{y}{y^*}\right)^{\psi_2}$$

Formal and Informal sectors' aggregate resource constraints

$$\begin{aligned} y^F &= c^F + i + g \\ y^I &= c^I \end{aligned}$$

Economy wide aggregate resource constraint

$$y = c^F + c^I + i + g$$

D Complete Model in the Log-linearized Form

Financial assets optimization equation

$$\widetilde{c_t} = -\beta R\widetilde{R_t} + \widetilde{c_{t+1}} + \widetilde{\pi_{t+1}}$$

Physical assets optimization equation

$$E_t\widetilde{\pi_{t+1}} - \beta R\widetilde{R_t} = -\beta r^k\widetilde{r^k_{t+1}}$$

Hours worked optimization equation

$$\widetilde{h_t} = \frac{1}{\phi}\left(\widetilde{w_t} - \widetilde{c_t}\right)$$

Money holding optimization equation

$$-\widetilde{c_t} = (1-\beta)\left(-\tilde{M}_t + \tilde{P}_t\right) - \beta E_t\left(\tilde{\pi}_{t+1} + \tilde{c}_{t+1}\right)$$

Capital accumulation equation

$$\widetilde{k_t} = (1-\delta)\,\widetilde{k_{t-1}} + \delta\widetilde{i_t}$$

Supply of formal labour

$$\widetilde{h_t^F} == \frac{1}{\vartheta}\left(\widetilde{w_t^F} - \widetilde{w_t}\right) + \widetilde{h_t}$$

Supply of informal labour

$$\widetilde{h_t^I} = \frac{1}{\vartheta}\left(\widetilde{w_t^I} - \widetilde{w_t}\right) + \widetilde{h_t}$$

Composite wage rate

$$\widetilde{w_t} = \frac{1}{w^{\frac{1+\vartheta}{\vartheta}}}\left(\eta\left(w^F\right)^{\frac{1+\vartheta}{\vartheta}}\widetilde{w_t^F} + (1-\eta)\left(w^I\right)^{\frac{1+\vartheta}{\vartheta}}\widetilde{w_t^I}\right)$$

Informal wage rate

$$\widetilde{w_t^I} = \widetilde{P_t^I} - \widetilde{P_t}$$

Formal wage rate

$$\widetilde{w_t^F} = (\frac{\rho}{\rho-1})\widetilde{w_t^I}$$

Formal price level

$$\tilde{P}_t^F - \widetilde{P_t} = \widetilde{mc_t^F}$$

Informal price level

$$\widetilde{w_t^I} = \widetilde{P_t^I} - \widetilde{P_t}$$

General price equation

$$\widetilde{P_t} = \omega\left(\widetilde{P_t^F}\right) + (1-\omega)\left(\widetilde{P_t^I}\right)$$

Formal consumption

$$\widetilde{c_t^F} = -\mu\left(\widetilde{P_t^F} - \widetilde{P_t}\right) + \widetilde{c}_t$$

Informal consumption

$$\widetilde{c_t^I} = -\mu\left(\widetilde{P_t^I} - \widetilde{P_t}\right) + \widetilde{c}_t$$

Gross general inflation rate

$$\widetilde{\pi_t} = \widetilde{P_t} - \widetilde{P_{t-1}}$$

Formal production function

$$\widetilde{y_t^F} = \alpha\widetilde{k}_t + (1-\alpha)\,\widetilde{h_t^F} + \widetilde{a}_t$$

Capital-labour ratio

$$\widetilde{k_t^F} = \widetilde{w_t^F} + \widetilde{h_t^F} - \widetilde{r_t^k}$$

Marginal cost

$$\widetilde{mc_t^F} = \alpha\widetilde{r_t^k} + (1-\alpha)\,\widetilde{w_t^F} - \widetilde{a}_t$$

Informal production function

$$\widetilde{y_t^I} = \widetilde{h_t^I}$$

Taylor type rule

$$\tilde{R}_t^c = \psi_1\widetilde{\pi_t} + \psi_2\widetilde{y_t} + \widetilde{\xi}_t$$

$$\widetilde{y_t^F} = \frac{1}{y^F}\left[c^F\widetilde{c_t^F} + i\widetilde{i}_t + \widetilde{g}_t\right]$$

$$y_t = \frac{1}{y}\left[c^F\widetilde{c_t^F} + c^I\widetilde{c_t^I} + i\widetilde{i}_t + \widetilde{g}_t\right]$$

Gross general inflation rate

$$\widetilde{\pi_t} = \widetilde{P_t} - \widetilde{P_{t-1}}$$

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