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### **Household Preferences in an Emerging Economy: Robust Estimates of Discount Factor and Coefficient of Relative Risk Aversion for Pakistan**

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**STATE BANK OF PAKISTAN**

## SBP Working Paper Series

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## Abstract

This study attempts to robustly estimate the two most often needed key household preference parameters for Pakistan: the discount factor ( $\beta$ ) and coefficient of relative risk aversion ( $\gamma$ ). We estimate the plausible range for these parameters such as  $\beta$  between 0.95–0.99 and  $\gamma$  between 0.7–1.1, which are close to values observed in other developing economies, using richer data, alternative utility function specifications, recursive estimates, and estimation at different frequencies (quarterly vs. annual). In doing so, we revisit the earlier work of Ahmed et al. (2012). Additionally, we also examine how these estimates influence the economy's response to shocks, such as the impact of a domestic labor productivity shock on output, inflation, interest rate and exchange rate, within a DSGE framework. Our results suggest that any values of  $\beta$  and  $\gamma$  between their ranges given above show limited deviations except for output. In this regard, for calibrated models, using the average of these ranges is a reasonable approximation, while for Bayesian estimation; priors may take any values between them.

**JEL Classification:** C51, D11, D15

**Key Words:** Coefficient of Relative Risk Aversion, Discount Factor, Non-linear Euler Equation, Generalized Method of Moments

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## Non-Technical Summary

The study focuses on the quantification of two important traits of household spending behavior. One feature is how much people in Pakistan value future consumption relative to current consumption. We simply called this patience or in technical terms discount factor ( $\beta$ ). The second trait is how willing households are to take risks. In technical terms this is known as the coefficient of relative risk aversion ( $\gamma$ ). These two parameters are very important for the transmission of shocks in a DSGE framework.

We estimated these parameters on both quarterly and annual data with multiple techniques. Results were almost consistent across different data frequency and estimation techniques. Furthermore, these estimates align with findings from other developing economies.

The results show that household in Pakistan give value to future consumption. With different methodology the discount factor ranges between 0.95 to 0.99. The quantification of the coefficient of relative risk aversion shows that households in Pakistan are moderately risk takers, as the value of this parameter ranges between 0.7 and 1.1.

Since the estimated values of the discount factor and the coefficient of relative risk aversion are in ranges rather than a single estimate. Therefore, we tested how the model's responses to shocks are sensitive to different values of these parameters.

Our results show that model response of macroeconomic variables significantly changes in magnitude, with the changing value of the risk aversion parameter. However, as far as the direction of responses are concerned, they remain consistent with the theory. Hence, using average value of CRRA may provide relatively better approximations.

## 1. Introduction

People's choices about whether to spend or save are shaped by two fundamental factors: how much they value the future and how willing they are to take risks. The first is measured by the discount factor ( $\beta$ ), which captures the weight placed on future consumption compared to consumption today. The second is measured by the coefficient of relative risk aversion ( $\gamma$ ), which reflects the extent to which households are prepared to face uncertainty in their consumption. Accurate estimates of these parameters are essential, since they strongly influence the transmission of shocks in a DSGE framework.

Earlier work by Ahmed et al. (2012) provided initial estimates for Pakistan ( $\beta = 0.98$  and  $\gamma = 0.57$ ). However, data limitations and restrictive assumptions left scope for improvement. In particular, issues such as time variation, frequency of data, alternative utility specifications, and richer datasets were not fully addressed, leaving the estimates uncertain.

This paper revisits and improves those estimates in several important ways:

1. In Ahmed et al. (2012), reliable private consumption data were unavailable, so the estimations relied on annual total consumption figures that combined both public and private spending. We now use quarterly per capita private consumption, which aligns more closely with the Euler equation, as this equation is primarily derived from household consumption rather than aggregate spending.
2. Instead of relying on a single specification of the utility function, we estimated the parameters using several commonly used specifications from the DSGE literature, allowing us to establish a plausible range for  $\beta$  and  $\gamma$ . We also checked robustness by varying the data frequency (quarterly vs. annual) and examining how parameter estimates evolve over time.
3. Where Ahmed et al. (2012) used the historical average real interest rate as a proxy for the steady-state rate, we now draw on newly available estimates of Pakistan's natural rate (Ahmed et al., forthcoming) to refine the calculation of  $\beta$ .
4. We explore the macroeconomic implications of our estimates within a DSGE framework.

For estimation, we adopt a method based on a Euler equation to obtain the values of  $\beta$  and  $\gamma$ , as alternative methods in the literature face significant limitations in the context of Pakistan<sup>1</sup>. One common approach is the Consumption-Based Capital Asset Pricing Model (CCAPM), including versions such as Hansen and Singleton (1983) and extensions like the Stochastic Discount Factor (SDF) models. These methods link consumption growth to asset returns and are widely used to estimate  $\beta$  and  $\gamma$ . However, they often produce unrealistically high risk aversion values (Weil, 1989). Another widely used approach is experimental and survey-based methods (Barsky et al. 1997), which measure how individuals react to risk in specific situations. While useful for

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<sup>1</sup> This approach follows the methodology used in Ahmed et al. (2012).

capturing short-term attitudes, these estimates only reflect the current conditions, fail to track how  $\beta$  and  $\gamma$  evolve over time, and do not scale well to the level of the whole economy.

A further common practice is to set  $\beta$  and  $\gamma$  based on averages from developed economies, a calibration strategy often used in Dynamic Stochastic General Equilibrium (DSGE) models. This can be convenient but risks ignoring the structural and behavioral differences of an emerging economy like Pakistan. To address such limitations, some studies use Bayesian estimation (An & Schorfheide, 2007), which combines data with prior beliefs. While flexible, this method introduces subjectivity in selecting priors and requires substantial computational resources.

In our study, we estimate a range of Pakistan's coefficient of relative risk aversion (CRRA) between 0.7 to 1.1 using annual data with different utility specification and methodologies. The estimated range of CRRA for Pakistan is consistent with developing economies such as India (0.92), and Sri Lanka (0.68) (Gadelman & Hernández-Murillo, 2015). These findings are also consistent with a large meta-study by Elminejad, Havranek, and Irsova (2022), which shows that, after adjusting for Methodological bias, the CRRA parameter for developing countries averages close to 1 (*Annexure-Table 4*). Given that, much higher values for this parameter are often reported in DSGE model studies, but these typically come from calibration rather than direct estimation. In practice, researchers adjust the parameter to make the model's simulated moments with that of observed data. In this way, parameter values reflect tuning choices for model fit rather than genuine structure of the economy.

We also estimated the discount factor range between 0.95 to 0.99. This range is also consistent with estimates reported for developing economies such as India - 0.986 (Gabriel et al. 2016), and Sri Lanka - 0.985 (Ehelepola, 2015).

The rest of the paper is structured as follows: Section 2 outlines the methodology. Section 3 presents the results. Section 4 discusses the sensitivity of macroeconomic dynamics to estimated parameters under DSGE setup, while section 5 concludes the study, respectively.

## 2. Methodology

This study estimates the discount factor ( $\beta$ ) and CRRA ( $\gamma$ ) using the Generalized Method of Moments (GMM) of Hansen (1982) and Singleton (1982) on a non-linear Euler equation. The Euler equation, derived from a Constant CRRA utility function, follows Ahmed et al. (2012). It links consumption growth to interest rates through  $\beta$  and  $\gamma$ , which are identified from observed data. Euler equation is as follows:

$$E \left[ \beta (1 + r_{t+1}) \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma} - 1 \right] = 0; \quad 0 < \beta < 1; \quad \gamma > 0; \quad (1)$$

In the above Euler equation,  $\beta$  (the discount factor) is a parameter that represents the household's time preference. It shows how much households prefer future consumption compared to current consumption. A high  $\beta$  (close to 1) shows that households are patient, valuing future consumption almost as much as present consumption.  $E_t$  is the expectations and  $c_t$  is consumption at time  $t$ .  $r_{t+1}$  is the real interest rate in the next period (period  $t + 1$ ), where  $\gamma$  (CRRA) measures the

household's risk tolerance with respect to changes in consumption. Higher values of  $\gamma$  indicate that households are more risk-averse, meaning they would prefer smoother consumption over time to avoid uncertainty.

For GMM estimation, moment conditions are required. Equation (1) provides the moment condition by linking parameters ( $\beta, \gamma$ ) to observed variables (consumption and interest rate).

By rearranging Equation (1), the moment condition is defined as

$$m(x_t, a_0) = \beta(1 + r_{t+1}) \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma} - 1 \quad (2)$$

where  $x_t = (r_t, c_t)$  represents the observed variables - real interest rate and real private consumption per capita and  $A = [\gamma, \beta]$  represent parameters to estimate.

For estimation, we employ a vector of instrumental variables, represented as  $\Phi_t$ , which includes variables from the information set (e.g., lagged consumption and interest rate) that have finite second moments.

## 2.1. GMM Estimation Process

1. **Moment Conditions:** The instrumental variables  $\Phi_t$  must be exogenous, ensuring that:

$$E[f(x_t, \Phi_t, A)] = E[m(x_t, A) \otimes \Phi_t] = 0 \quad (3)$$

where  $\otimes$  denotes the Kronecker product.<sup>2</sup> This allows us to construct the moments from the Euler equation.

2. **Objective Function:** The objective of GMM is to minimize the deviation of the moment conditions from zero. We define the expected value of the moments as:

$$\Gamma_o(A) = E[f(x_t, \Phi_t, A)] \quad (4)$$

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<sup>2</sup> The Kronecker product is an operator used to construct a larger matrix from two smaller ones by multiplying each element of one matrix by the entire second matrix. In context of Euler equation,  $m(x_t, A) \otimes \Phi_t$  make a larger matrix, by multiplying each element of the function  $m(x_t, A)$  (which represents our transformed Euler equation) by the entire instrument vector  $\Phi_t$ .

In line with Hansen and Singleton (1982) and Hall (1988), we employ following set of instrumental variables:

- Constant,  $(1 + r_{t-1}), (1 + r_{t-2}), c_{t-1}^*, c_{t-2}^*$

Here,  $c_t^* = \frac{c_{t+1}}{c_t}$  denotes the assumed expected future consumption.

To assess the validity of the instruments and the overall specification of our moment conditions, we tested the Hansen J-statistic.

If the model holds, then a sample estimator of  $\Gamma_0(a)$  is given by:

$$\Gamma_T(a) = \frac{1}{T} \sum_{t=1}^T f(x_t, \Phi_t, a) \quad (5)^3$$

3. **Weighted Sum of Squares:** We minimize the weighted sum of squared moments, defined as:

$$J_T(b) = \Gamma_T(a)' \Omega_T^* \Gamma_T(a) \quad (6)$$

Here,  $\Omega_T^*$  is a weighting matrix that optimally weights the moment conditions.

$$\Omega_T^* = \left( \frac{1}{T} \sum_{t=1}^T f(x_t, \Phi_t, a) f(x_t, \Phi_t, a)' \right)^{-1} \quad (7)$$

4. **Algorithm:**

To minimize equation (5), we need starting values for the weighting matrix  $\Omega_T$  and the parameter vector  $a$ . Initially we set  $\Omega_0 = I$  (*identity matrix*), and choose an arbitrary estimate of parameter vector  $a_0 = [\beta_0, \gamma_0]$ . This minimization yields the estimate  $a_1 = [\beta_1, \gamma_1]$ . The new parameter vector is then used to update the weighting matrix (equation (6)), denoted as  $\Omega_1$ . In the second stage,  $\Omega_1$  and  $a_1$  are substituted back into equation (5), and the minimization is performed again to obtain the final parameter vector  $a$ .

## 2.2. Linking to Risk Aversion

The GMM procedure produces consistent estimates of  $\beta$  and  $\gamma$ . By definition, for CRRA preferences:

$$\text{CRRA} = -c \frac{U''(c)}{U'(c)} = \gamma \quad (8)$$

where  $U'(c) = \frac{\partial U}{\partial c}$  and  $U''(c) = \frac{\partial^2 U}{\partial c^2}$  are the first and second derivatives of the utility function with respect to  $C$  i.e. consumption. Hence, the estimated  $\gamma$  directly corresponds to the CRRA.

## 2.3. Getting to Final Result

To come up with a plausible range for the parameters, we estimated the desired parameters using different data frequencies, tried out several utility function specifications, and looked across different time horizons. This helped us narrow down the parameters to a reasonable range.

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<sup>3</sup> The source of equation is Singleton (1982).

### 2.3.1. Data

In the estimation, we used per capita real private consumption, the real interest rate, and the natural real interest rate. To construct these variables, we relied on private consumption, CPI inflation, population, and interest rate data obtained from PBS and SBP. The natural real interest rate estimates are taken from Ahmed et al. (forthcoming). The analysis employs two data frequencies: annual (FY1978–FY2024) and quarterly (FY1978–FY2024).

As a prerequisite for GMM estimation, variables need to be stationary. Per capita real private consumption is non-stationary. However, the structure of the Euler equation allows us to use this variable without transforming it into a stationary series. This is because the Euler equation employs the consumption growth ratio  $\frac{c_{t+1}}{c_t}$ , which is stationary even if the consumption levels themselves, are non-stationary. The real interest rate and natural real interest rate, on the other hand are both stationary. Hence, the prerequisite of GMM is satisfied.

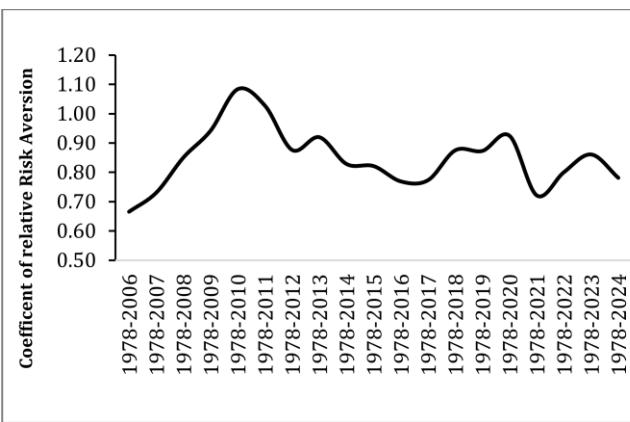
## 3. Results

First, we estimated the Euler equation described in equation (1) using the GMM methodology on annual data from FY78 to FY24. To check the validity of the model, we applied several tests, including the Hansen J-test and the Chow stability test, all of which the model passed. This analysis indicates that the discount factor ( $\beta$ ) is 0.99, while the coefficient of relative risk aversion ( $\gamma$ ) is 0.8.

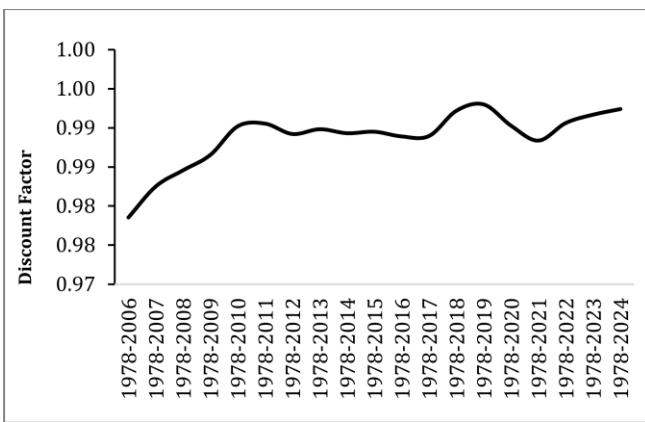
### 3.1. Estimating the range of parameters

#### 3.1.1. Evidence from recursive estimates

To find the plausible range of CRRA, we estimated it across expanding sample windows starting with the period 1978–2006 and then gradually extending the sample by adding one additional year at each step, up to the final window covering 1978–2024. These estimates, referred to as recursive parameter estimates, are reported in **Figure 1**. The coefficient stays stable over time, without evidence of structural breaks. This stability confirms the robustness of our estimates. However, the CRRA coefficient gradually fluctuates between 0.7 and 1.1, reflecting households' adjustments in consumption behavior in response to changing economic conditions. From this analysis, we can



**Figure 2: Coefficient of Relative Risk Aversion (Recursive Estimates)**



**Figure 2: Discount Factor (Recursive Estimates)**

safely conclude that CRRA lies between 0.7 and 1.1. In similar fashion, the estimates of the discount factor lie between 0.98 and 0.99 (**Figure 2**).

### 3.1.2. Evidence from different data frequency

To further explore the plausible values of the desired parameters, we estimated the Euler equation with quarterly data. We then converted the results into annual values to see how they differ only by changing the frequency. The quarterly estimates give a risk aversion coefficient of 0.36. For equivalent annual estimate, we used following relation:

$$\gamma_{\text{annual}} = \gamma_{\text{quarter}} \frac{\text{Var}(\text{Consumption Growth})_{\text{quarter}}}{\text{Var}(\text{Consumption Growth})_{\text{annual}}}$$

The implied annual value is about 0.73, which is close to 0.8 (annual data-based estimate). The results confirm the robustness of the model across different data frequencies. Discount factor values also came very close to each other.

### 3.1.3. Evidence from different set of methodologies

To probe further into the plausible range of the parameters, we estimated the parameters using two different processes:

First, the discount factor value set at the steady state of the Euler equation. Since, ( $c_{t+1} = c_t = c$ ) and ( $r_{t+1} = r_t = r$ ) at the steady state, hence the equation (1) will be reduced to:

$$\beta(1 + r) - 1 = 0$$

By rearranging, we obtained,  $\beta = \frac{1}{1+r}$ , where  $r$  is average real interest rate<sup>4</sup>, The result shows  $\beta = 0.98$  which is consistent with our prior estimates.

Second, we estimate  $\beta$  with natural rate, while natural interest rate is estimated using quarterly data from Q3-FY91 to Q4-FY24, within a modified Holston, Laubach, & Williams (2023) framework, which jointly estimates the IS and Phillips curves together along with the Euler equation<sup>5</sup>. This approach yields a  $\beta$  value of 0.95, somewhat lower than model-based estimates, largely because of comparatively shorter time span.

### 3.1.4. Evidence from different utility function specifications

The Euler equation may vary with the utility function's specification and, consequently, may alter the CRRA estimates. To examine how different specifications affect the CRRA estimates, we consider a version of the utility function with external habit persistence, where households compare their consumption to the average consumption in the economy.

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<sup>4</sup> Average is taken for the time span (Q1-FY78 to Q4FY24)

<sup>5</sup> Ahmed et al. forthcoming SBP Working Paper

**3.1.4.1. External Habit Formation** – where utility depends on own consumption relative to average consumption in the economy.

$$U_i \left[ c_{t+i}, \frac{M_{t+i}}{P_{t+i}}, (1 - n_{t+i}) \right] = \frac{(c_{t+i} - h\bar{c}_{t+i})^{1-\gamma}}{1-\gamma} + \frac{\zeta_{m,t+i} \left( \frac{M_{t+i}}{P_{t+i}} \right)^{1-\eta}}{1-\eta} + \frac{(1 - n_{t+i})^{1-\nu}}{1-\nu}$$

Since we do not have direct values of persistent parameter  $h$ . Therefore, we fixed  $h$  at 0.5 and 0.25, to see how different persistence effect the CRRA estimates.

It is imperative to note that for the standard CRRA case, the estimated  $\gamma$  is directly interpreted as relative risk aversion. For the external habit models, however,  $\gamma$  must be adjusted by definition to obtain the effective risk aversion:

$$\gamma_{\text{External habit}}^{\text{eff}} = \text{CRRA} = -c \frac{U''(c)}{U'(c)} = \gamma \frac{1}{T} \sum \frac{c_t}{c_t - h\bar{c}}$$

So estimated gamma from GMM methodology is adjusted with consumption multiplier  $(\frac{1}{T} \sum \frac{c_t}{c_t - h\bar{c}})$  to get effective or comparative CRRA.

**Table 1: Effective Relative Risk Aversion Across Utility Specifications**

Specification	Structural $\gamma$	Consumption Multiplier (m)	Effective CRRA = $\gamma \times m$
Baseline (no habit)	0.8	1.0	0.8
External habit: $h=0.25$	0.6	1.4	0.8
External habit: $h=0.50$	0.4	2.3	0.9

**Table 1** shows that the estimates of CRRA from external-habit specifications close to baseline and within the range already identified through recursive estimates.

### 3.2. Final Results

Based on the above discussion and robustness checks, the estimates suggest that  $\beta$  ranges from 0.95 to 0.99, while  $\gamma$  varies between 0.7 and 1.1.

## 4. Sensitivity of Macroeconomic Dynamics to Estimated Ranges of $\beta$ and $\gamma$ in a DSGE Framework

In our analysis, we estimated the range values of the discount factor and CRRA. However, DSGE models typically use a single value for each parameter. This raises following questions:

1. Which value should we choose from the range?
2. How sensitive is the outcome of DSGE to the values in the range?
3. Will it influence the direction of IRFs in DSGE models?

To explore these questions, we simulated the standard DSGE model using three different combinations of  $\beta$  and  $\gamma$ . We then compared the impulse response functions (IRFs) for output, inflation, interest rates, and the exchange rate after a productivity shock.

For the analysis, we used the DSGE model of Ahmed et al. (forthcoming SBP Working Paper). All parameters retained at their calibrated values from Ahmed et al. (*Annexure-Table 3*), except for  $\beta$  and  $\gamma$ , which were set as follows:

- **Scenario 1 (Upper bound):**  $\beta = 0.99$ ;  $\gamma = 0.57$
- **Scenario 2 (Average bound):**  $\beta = 0.97$ ;  $\gamma = 0.47$
- **Scenario 3 (Lower bound):**  $\beta = 0.95$ ;  $\gamma = 0.36$

The results are illustrated in **Figure 3** and **Table 2**. All three parameter scenarios produce theoretically consistent responses; output rises, inflation and interest rates fall, and the exchange rate first depreciates before appreciating, consistent with Gali & Monacelli (2005) and Ahmed et al. (forthcoming SBP Working Paper). Despite the consistent IRFs paths, the magnitude of the responses differ significantly for output, and marginally for the other variables (**Table 2**). Therefore, we recommend using the average values of  $\beta$  and  $\gamma$  for baseline calibration. However, in Bayesian estimation, one can play around the boundaries to construct reliable posterior distribution.

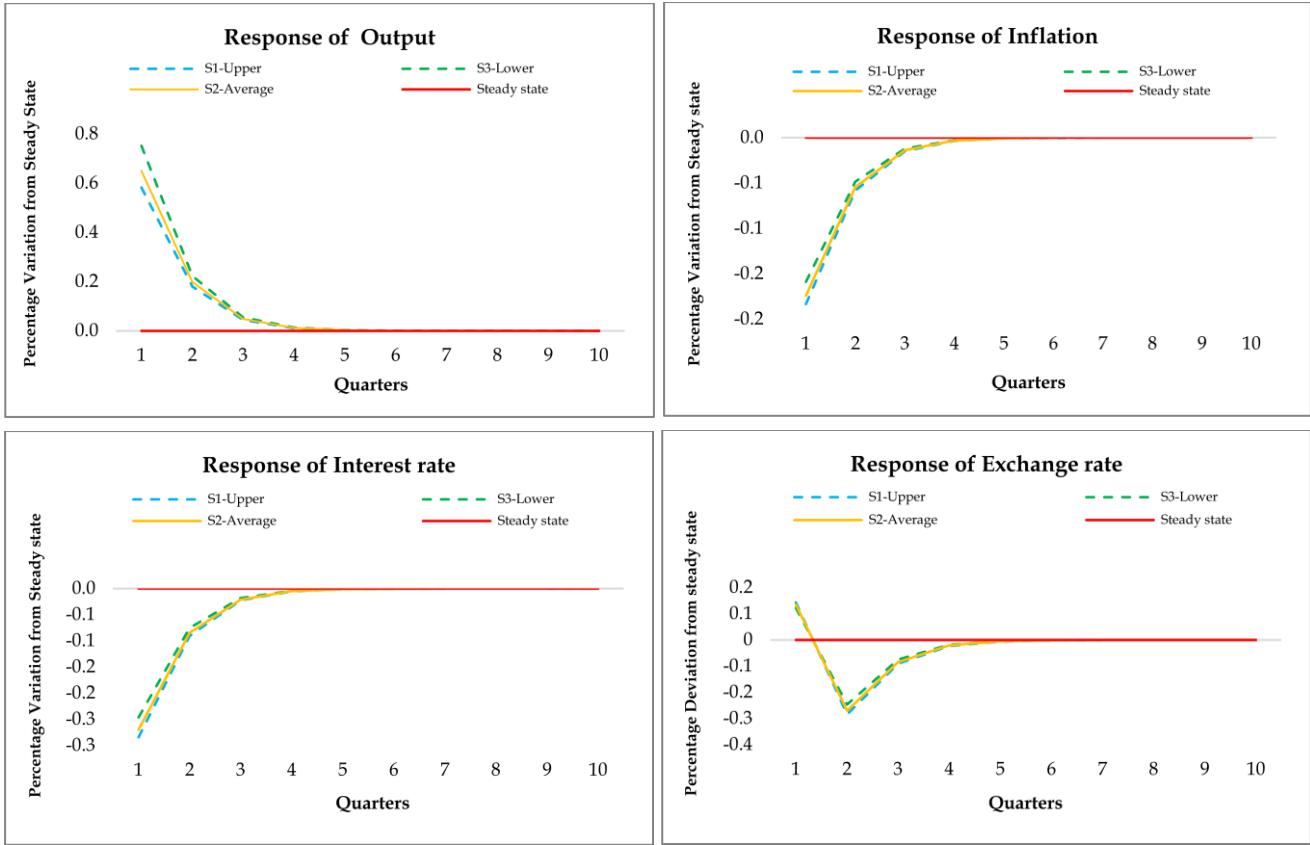
**Table 2: Cumulative Response of Key Macroeconomic Variables (10 Quarters)**

Variables	S1: Upper Bound <sup>1</sup>	S2: Average Bound <sup>2</sup>	S3: Lower Bound <sup>3</sup>
Output	0.82	0.91	1.05
Inflation	-0.26	-0.25	-0.22
Interest rate	-0.40	-0.38	-0.35
Exchange rate	-0.26	-0.25	-0.22

<sup>1</sup>  $\beta = 0.99$ ;  $\gamma = 0.57$

<sup>2</sup>  $\beta = 0.97$ ;  $\gamma = 0.47$

<sup>3</sup>  $\beta = 0.95$ ;  $\gamma = 0.36$



**Figure 3:** Response of Key Macroeconomic Variables to Domestic Productivity Shock

## 5. Conclusion

This research estimates the  $\gamma$  (coefficient of relative risk aversion) and  $\beta$  (discount factor) through the Euler equation. The baseline GMM estimates, using annual data from FY78 to FY24, show values of  $\gamma = 0.8$  and  $\beta = 0.99$ . To ensure robustness, we refine the estimates by using multiple approaches such as recursive estimates, applying different frequencies and tracking changes over time, cross-methodological check, and testing alternative utility functions specifications.

Recursive estimates indicate that  $\gamma$  remains steady over time, ranging from 0.7 to 1.1, whereas  $\beta$  consistently falls between 0.98 and 0.99. When the frequency of data was changed from annual to quarterly, the annualized estimates remained quite consistent with the quarterly estimates. Furthermore, cross-methodological tests using steady-state conditions and the natural interest rate confirm that  $\beta$  falls between 0.95 and 0.99. Alternative specifications based on the utility function and external habit persistence produce effective CRRA estimations similar to the baseline.

Altogether, the consolidated results show that  $\gamma$  ranges from 0.7 to 1.1, whereas  $\beta$  ranges from 0.95 to 0.99. These estimated ranges are theoretically consistent and in line with the estimates for developing countries. Importantly, we extend the analysis by embedding these ranges into a DSGE framework. Simulations under different scenarios confirm that the direction of IRFs of key macroeconomic variables remains theoretically consistent, except the magnitude of the output.

Therefore, for baseline DSGE calibration, we recommend to use average values of the estimated parameters of discount factor and CRRA.

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## Annexure

**Table 3: List of Calibrated Parameters<sup>6</sup>**

Parameter	Definition	Values
Beta ( $\beta$ )	Time preferences (Discount Factor)	0.98
Sigma ( $\sigma$ )	Coefficient of Relative risk aversion (CRRA)	0.31
Alpha ( $\alpha$ )	Share of import in domestic consumption (Degree of Openness)	0.15
Gamma ( $\gamma$ )	Elasticity of substitution between goods produced in foreign economy	1.00
Eta ( $\eta$ )	Substitutability between domestic and foreign goods	0.78
Phi ( $\varphi$ )	Disutility from supplying labor	0.70
Theta ( $\theta$ )	Price stickiness parameter	0.49
Tau ( $\tau$ )	Employment subsidy	0.70
Mu ( $\mu$ )	Optimal markup on MC	1.16
Rho_a ( $\rho_a$ )	Persistent of domestic labor productivity shock (parameter of AR(1) for productivity)	0.23
Rho_y ( $\rho_y$ )	Persistent of world output shock (parameter of AR(1) for $y^*$ )	0.86
Phi_pi ( $\phi_\pi$ )	Response of monetary policy to inflation under Taylor rule	1.55
Epsilon ( $\varepsilon$ )	Elasticity of substitution between Differentiated goods	7.20
<b>Derived Model Parameters<sup>7</sup></b>		
Sigma_a ( $\sigma_a$ )	Sensitivity of domestic output to terms of trade	0.44
Rho ( $\rho$ )	Discount rate (Log of discount factor)	0.02
Tau ( $\Gamma$ )	Sensitivity of natural level of domestic output to productivity	1.49

<sup>6</sup> Taken from Ahmed et al. (forthcoming SBP Working Paper)

<sup>7</sup> These parameters are the functional transformations of the baseline parameters provided above in the Table 3.

Theta ( $\theta$ )	Part of sensitivity of natural interest rate to expected change in world output	-1.25
Xi ( $\psi$ )	Sensitivity of natural level of domestic output to world output	1.79
Kappa ( $\kappa_a$ )	Sensitivity of inflation to output gap in Philips curve	0.62
Omega ( $\Omega$ )	Constant representing the natural level of output	-0.17

**Table 4: CRRA – Developing Countries<sup>8</sup>**

Country	CRRA
Bangladesh	1.30
Bosnia & Herzegovina	0.72
Botswana	0.94
Brazil	0.63
El Salvador	0.54
Georgia	0.88
India	0.92
Indonesia	1.24
Madagascar	0.72
Myanmar	1.01
Mexico	0.78
Russia	0.65
Sri Lanka	0.68
Uruguay	0.90
Vietnam	1.15

**Source:** Gadelman & Hernández-Murillo (2015)

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<sup>8</sup> The discount factor ( $\beta$ ) for developing countries is typically around 0.99, with very limited variation. Therefore, we did not include a separate column for  $\beta$  and instead focused on reporting variations in CRRA values.