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Time-Frequency Analysis of Determinants of Inflation Rate in Pakistan

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Abstract

By using wavelet decomposition method, this study provides time-frequency analysis of headline inflation and its determinants for the case of Pakistan by using monthly data covering July 1992 to June 2021. Frequency scale-wise correlation and causation analysis indicates that in the short run (at median frequency of 4 to 8 months) policy rate, exchange rate changes, changes in government borrowing and growth in import payments are associated with inflation rate in Pakistan. During medium run (at median frequency of 16 to 64 months), growth in broad money supply and global commodity prices also affect inflation. In the long run (at low frequency scale of above 64 months), growth in government borrowing, global commodity prices and broad money supply determine the inflation rate in Pakistan. The study shows that money supply matters more for inflation than the policy rate in the long run.

JEL Classification: E32, E51.
Key Words: Time Frequency, Wavelet, Inflation Determinants.

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Non-technical Summary

Price stability has been set as primary objective by many central banks around the globe. For example, primary objective of the European Central Bank (ECB) is to achieve price stability with yearly inflation near to 2 percent over the medium term. Similarly, Bank of England monetary policy primary objective is to maintain price stability by achieving government's inflation target of 2 percent. Through a recent amendment, State Bank of Pakistan Act, 2022 also mandates the country's central bank price stability as its primary objective. Price stability has been defined in the Act as 'maintenance of low and stable inflation guided by the government's medium-term inflation target'.

How important are episodes of high or low inflation at a certain time horizon for a central bank with price stability as its primary objective? How phase dependent linkages between inflation and its possible determinants may change over time? This study attempts to address these issues through empirical analysis of inflation and its determinants, under time frequency domain, using data from Pakistan.

The study finds that within high frequency cycle, policy rate, exchange rate growth, government borrowing and imports growth are determining factors for inflation in Pakistan. With reference to causal relationship and correlation coefficient, cyclical movement of inflation are determined by same frequency movements in growth of external commodity prices, government borrowing growth, imports growth and growth in global oil prices. Whereas, during a median frequency, the policy rate, change in exchange rate, growth in government borrowing, fluctuations in global commodity prices (including oil prices), imports, and money supply growth do have causal relationships with inflation in Pakistan. In the long-run, growth in government borrowing, growth in global commodity prices (including oil prices), imports growth, and money supply growth cause inflation in the country.

1. Introduction

Many central banks consider price stability as their primary goal. For instance, primary objective of the European Central Bank (ECB) is to achieve price stability with yearly inflation near to 2 percent over the medium term. Similarly, Bank of England monetary policy primary objective is to maintain price stability by achieving government's inflation target of 2 percent. Through a recent amendment, State Bank of Pakistan Act, 2022 also mandates the country's central bank price stability as its primary objective. Price stability has been defined in the Act as 'maintenance of low and stable inflation guided by the government's medium-term inflation target'.

International experience reveals that instability in prices results in low and volatile output and employment in a country. High inflation or deflation causes systematic distortions in resource allocation. A positive output gap over time results in inflation above the target level while a negative output gap will result in too low level of inflation, or deflation in certain cases, that disincentives the producers. In other words, when an economy grows more (less) than its potential, inflationary (disinflationary or deflationary) pressures build up and tend to make growth path uncertain. Consequently, periods of high inflation (too low inflation of deflation) are followed by periods of undesirable contraction (unsustainable expansion). Such situations can be corrected using a combination of different macroeconomic policies with monetary policy focusing on price stability. Setting price stability as the primary objective of the monetary policy contributes to achieve sustainable economic growth. However, situation in developing countries can be a bit difficult to address where inflation levels are higher and volatile compared to developed countries.

Better we know the sources of inflation and effectiveness of monetary policy tools; better we can address it. Moreover, the knowledge of frequency scale in which the determinants of inflations impact it can further equip policy makers to address inflation. It is possible that different variables (monetary policy tools) causing (or taming) inflation may be linked with it differently at different frequencies scales. Accordingly, this necessitates to explore inflation rate and its determinants using time frequency analysis. Thus, tracking and linking fluctuations in inflation from/with the fluctuations in its main determinants helps devise more appropriate monetary policy stance by the central bank to achieve price stability in the country.

The premise that the determinants of inflation have different frequencies scales is an empirical question that needs assessing this relationship across different frequency bands. For instance, although there is some evidence that money growth causes inflation, yet such relationship can also vary over time (Christiano and Fitzgerald, 2003; Sargent and Surico, 2008; Benati, 2009, Jiang, Chang, and Li, 2015). A positive and one-to-one relationship between money growth and inflation exists in the medium to long run, whereas such relationship may not exist in the short-run due to transitory nature of shock and lag in effects (Jiang, Chang, and Li, 2015). Ryczkowsk (2021), using wavelet analysis, demonstrates that money growth causes inflation, and find strong positive relationship between the two variables around the Great Recession at a typical cycle/median frequency.

How important are episodes of high or low inflation at a certain time horizon for a central bank with price stability as its primary objective? How phase dependent linkages between inflation and its possible determinants may change over time? This study attempts to address these issues through empirical analysis of inflation and its determinants, under time frequency domain, using data from Pakistan. Instead of analyzing the original series, we first decompose the series using wavelet decomposition technique at various scales of resolution. We study the relationship between components of the series of inflation and its determinants across different scales. Then we consider the relationship between inflation and its determinants across different time frequencies. Our study contributes to the growing literature in that it decomposes the headline inflation series of Pakistan and its potential determinants using wavelet decomposition technique at various scales of resolution. Further we obtain the relationship between components of the decomposed series of inflation and its determinants matched to its scale and also across the scale.

We study headline inflation and its determinants in time frequency domain. This study offers guidance for policy makers for both their short and long-term decisions. A comprehensive list of studies on determinants of inflation is furnished below. The empirical literature on subject matter has two streams; one is based on the traditional approach on the determinants of inflation (i.e. monetary, fiscal, supply side, and demand side, structural, external and country specific factors), which is focused on the time series dimension of variables; and other is time-frequency dimension of inflation determinants that may provide much useful information about the behavior of inflation in a country over time. The second approach has gained greater attention of researcher and policy circles at central banks aiming to target inflation or already doing so, in order to gain greater economic stability.

The study finds that within high frequency cycle, i.e. 4-8 months, policy rate, exchange rate growth, government borrowing and imports growth are determining factors for inflation in Pakistan. With reference to causal relationship and correlation coefficient, cyclical movement (4-64 months) of inflation are determined by same frequency movements in growth of external commodity prices, government borrowing growth, imports growth and growth in global oil prices. Whereas, during a median frequency (16-64 months), the policy rate, change in exchange rate, growth in government borrowing, fluctuations in global commodity prices (including oil prices), imports, and money supply growth do have causal relationships with inflation in Pakistan. In the long-run, growth in government borrowing, growth in global commodity prices (including oil prices), imports growth, and money supply growth cause inflation in the country.

Rest of the paper is organized as follows. Next section reviews the main literature on the subject. Section 3 explains methodology and section 4 presents the results. Finally, section 5 concludes the study.

2. Literature Review

There is growing literature on time-frequency based view on determinants of inflation. Assenmacher-Wesche & Gerlach (2006) examine the Euro area inflation at high and low frequencies and find that variations in inflation are well explained by low-frequency movements of money and real income growth and high-frequency fluctuations of the output gap. Rua (2012) employs time-frequency analysis of Money Growth and Inflation in the Euro Area using monthly data for 1970:01- 2007:12. The author finds that there is a stronger link between inflation and money growth at low frequencies. At the typical median frequency range, the link is only present until the beginning of the 1980s. Dar et al. (2014) finds that industrial growth and inflation share an anti-cyclical relationship corresponding to the frequencies of 8 - 16 months in India. Inflation leads the industrial growth at lower frequencies. Hanus & Vacha (2018) work on time-frequency response analysis of monetary policy transmission. They find that both the domestic and foreign output gap are significant drivers of inflation whereas, the former has a bigger influence in the right tail of the conditional distribution of inflation. There is an increase in the response of inflation to the domestic gap in the last decade but only at the lower quantiles, which means time varying response exists. Using data for developed countries, Gallegati et al. (2019) find that there is a close and stable relationship between excess money growth and inflation only over longer time horizons, i.e. periods greater than 16 and 24 years, with money growth mostly leading it. In addition, inflationary upsurges affect regression coefficients but not the closeness of the long-run relationship.

In Pakistan, the studies on determinants of inflation used the original time series of the variables incorporated in the model. For example, Khan et al. (2007) suggest the in Pakistan important determinants of inflation in 2005-06 were adaptive expectations, private sector credit and rising import prices. Bashir et al. (2011) used the annual data for 1972 – 2010 from Pakistan on CPI, M2, GDP, imports, exports, government revenue and expenditure. Using Johansen Co-integration and VEC model the authors find that money supply, gross domestic product, imports and government expenditures cause inflation.

In another study on Pakistan's data from 1970-2007 with co-integration technique, Khan & Gill (2010) find that procurement prices and administered prices along with imported inflation have contributed to higher inflation. Budget deficit, depreciation of exchange rate and increase in the value of imports have contributed shooting up of CPI, WPI, SPI and GDP deflator. The support prices of sugarcane, rice, wheat, and cotton also affect inflation. Bashir et al. (2016) in a study on Pakistan find that the demand side factors of inflation are population, roads and government expenditure while supply side factors are imports, government revenue, electricity generation and external debt. In the long run, inflation is caused by roads, government expenditure, imports, government revenue and external debt. There is decline in price level due to foreign direct investment, electricity generation and population in long run. Ahmed et al. (2014) find through co-integration that in Pakistan exchange rate is the most significant factor of inflation followed by indirect taxes and money growth. In another study on Pakistan, Khan & Qasim (1996) indicate increase in money supply and exchange rate depreciation cause inflation. The supply side variables are important to put downward pressure on price level. Food inflation is co-integrating with money supply, value added in agriculture, and support price of wheat. Whereas, money supply, GDP, electricity price and import price cause non-food inflation. Asghar et al. (2013) find that in Pakistan long-run money supply growth, lagged inflation, foreign inflation and global financial crises have positive and significant impact on inflation whereas; money supply becomes insignificant in the short-run.

Empirical literature on inflation determinants employing econometric models generally use original time series data (for more details see Box 1). Altissimo et al. (2005) attempt to find long-run determinants of inflation in EU through Dynamic Stochastic General Equilibrium. They find that relative variations in productivity in the non-traded sector are the cause of price and inflation differentials, with shocks to productivity in the traded sector being largely absorbed by movements in the terms of trade in the regional economies. In another study on EU, Andersson et al. (2009) find that inflation differentials are primarily determined by cyclical positions and inflation persistence that appears to be partly explained by administered prices and to some extent by product market regulations. Price level in each EU country is governed by GDP per capita. Kandil & Morsy (2009), applied the Co-integration and VEC model on GCC data for CPI, trading partners' CPI, exchange rate, government spending, and broad money from 1970-2007. Their findings are that inflation in major trading partners, oil revenues, and exchange rate depreciation cause inflation whereas, higher government spending and credit growth slow down inflation.

Using panel data of 54 developing countries and GMM, Narayan et al. (2011) find that remittances, trade openness, debt, current account deficits, the agricultural sector, and USA interest rate have a positive effect on inflation; whereas, improvements in democracy reduce inflation. Cottarelli et al. (1998), using panel data of 47 industrial and transition economies discover that relative price changes, central bank independence, the exchange rate regime, and the degree of price liberalization cause inflation. Structural factors, such as those influencing the natural rate of unemployment, have a limited effect on inflation. Morley et al. (2015) using data of G7 countries and Bayesian techniques find that trend inflation and the inflation gap have been consistent and substantial determinants of inflation; whereas, real-activity gap explains a large fraction of the variation in the inflation gap for each country.

In a country specific study on India, Dua & Goel (2021) finds that expected inflation, exchange rate, rate of growth of money supply, output gap and interest rates are the major determinants of inflation. On the supply side global factors like international oil and food prices play a key role in determining both overall and food inflation. Mohanty and John (2015) study the determinants of inflation in India using annual data for 1996 – 2014 applying SVAR on CPI, crude oil prices, output gap, fiscal policy and monetary policy. They find that influence of monetary policy on inflation remained almost steady during the study period. Output gap had an asymmetric impact on inflation with its influence having weakened in the recent period. Crude oil price was the predominant driver of inflation during 2009 - 2011. Fiscal deficit was a key determinant of inflation in 2011 – 2012.

Ubide (1997) estimates determinants of inflation in Mozambique with Monthly data for 1989:01-1996:12 on CPI, Exchange rate, M2, Budget deficit, GDP growth, weather and political shocks. Using OLS and VAR the author finds that Monetary expansion, exchange rate and unpredictable events in the agricultural and politics, are responsible for inflation. Gottschalk et al. (2008) a named Determinants of Inflation in Sierra Leone used VAR on monthly data from 2001:01 to 2007:12 on CPI, international oil prices, exchange rate and reserve money. The find that higher oil prices, higher money supply, and nominal exchange rate depreciation, are contributes to inflation. In another country specific study on Albania, Domac & Elbirt (2013) employed univariate analysis, Co-integration and VEC model using CPI, CPI components, power consumption, base money, M1, M2, M3, government credit and bilateral exchange rate of major trading economies. The results suggest that inflation exhibits strong seasonal patterns owing to agriculture seasonality. Credit to government, M2, cause inflation. Bi-directional causality exists between inflation and credit to government and rent, water, fuel, and power component.

In another study on Kuwait, Al-Mutairi et al. (2020) employ multiple linear regression analysis and find that change in CPI is positively influenced by change in interest rate spreads, imports of goods and services and money supply whereas it is negatively influenced by the changes in tax revenue and current account balance. Basher& Elsamadisy (2012) suggest that inflation is determined by money supply, non-hydrocarbon output, foreign prices and the nominal effective exchange rate in the Gulf Arab states. Oppong et al. (2015) finds that in Ghana crude oil price at the world market and exchange rate are key determinants of inflation. In another study on industrial economies, Deniz et al. (2016) find that real effective exchange rate has a more negative impact on inflation in emerging than industrialized economies. Money growth rate contributes to inflation in emerging economies only. Real wage has a positive impact in emerging economies, whereas the impact is negative for industrialized economies. Budget balance has a negative impact on inflation however; it has a positive impact in non-inflation targeting countries.

3. Methodology

In order to explore scale dependent linkages between inflation and its potential determinants using time frequency domain, we first decompose all the relevant variables into various scales, based on frequency (or resolution), and estimate relationships of time frequency scales of inflation with as well as across the time frequency scales of its potential determinants. There are different ways to decompose a series into different frequency scale components including Fourier transformation and Wavelet transformation, depending upon if the underlying series in stationary or otherwise.

After decomposing the time series into various frequency scales, we start estimating the correlation coefficients of scales of inflation with the scales of its potential determinants to understand the direction and strength of an underlying relationship between different pairs of (components at various) scales. In order to understand the direction of causation, we apply Granger causality test upon each pair of scales of inflation with the scales of its potential determinants. We consider a variable to be a determinant of inflation if there exists a significant correlation as well as well causation towards inflation. On the basis of these results we interpret the scale-wise determinants of inflation in Pakistan.

3.1. Wavelet Transformation

A Fourier transformation is well-known and frequently used approach to decompose a time series into a spectrum of cycles of varying length and to identify trend (low frequency component), cycles (median frequency components), and noise (high frequency component) (Granger and Hatanaka, 1964 and Benhmad, 2013). This spectral method (Fourier transformation) requires underlying time series to be stationary which is usually not the case in economics time series. On the other hand, Wavelet transformation has an advantage (over Fourier transformation) as its window size adjusts itself optimally to longer basis functions at low frequencies and to shorter basis function at high frequencies (Benhmad, 2013). Similarly, it has good frequency resolution for low frequency movements and good time resolution for high frequency movements. Thus, due to local analysis ability of wavelet transform, it is capable to capture sudden regime changes, unexpected jumps, ARCH effects, outliers, shocks, long memory in an economic time series. Moreover, Wavelet transformation extracts information from both time-domain and frequency-domain irrespective of data generating process of underlying time series. Thus, in this study, we use Wavelet transformation to decompose time series of inflation and its potential determinants to explore scale wise determinants of inflation rate in Pakistan. We brief the Wavelet transformation based decomposition methodology in the following subsection and we provide relevant technical details in Appendix A.

3.1.1. Wavelet Analysis based Decomposition

Wavelets are mathematical expansions that transform data from the time domain into different layers of frequency levels (Iqbal and Hanif, 2017 and Hanif et. al., 2020). Conventionally economists consider only two scales in an economic time series: the short run and the long run. There are actually more time scales in between the short run and the long run horizon of a time series (Dalkir, 2004). Through decomposition based upon Multiresolution Wavelet analysis we can analyze a time series into different frequency zones, very high frequency zone (noise part) and very low frequency zone (smooth part or trend part) and a cyclical part (median frequency component) which is sum of the frequency zones in between the high and very low frequency zones. Thus, using Multiresolution Wavelet analysis we can decompose a (seasonally adjusted) economic time series into different noise, cycle, and trend components (Yogo, 2008). A time series X_{t} can be decomposed as

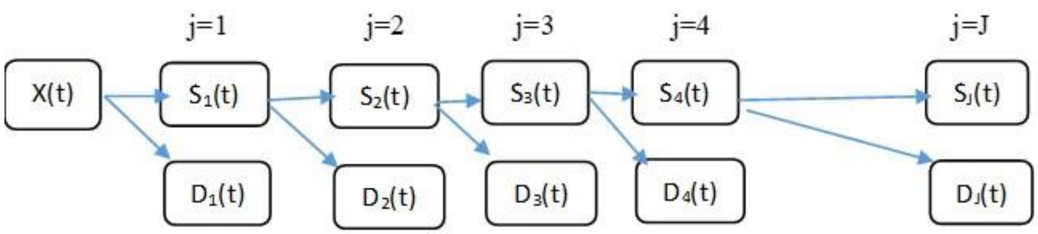
$$X_t = S_t^J + \sum_{j=1}^{J} D_t^j \qquad (1)$$

Where S_t^J is a cycle with periodicity greater than2^{J+1} andD_t^j denotes cycles with periodicity between 2^j and 2^{j+1}

j=1, 2, 3, . . . J. And selection of "J" depends on the frequency of data used. Researchers use J = 5, 4 and 2 for monthly, quarterly and annual data series respectively (Yogo, 2008; Crowley, 2010 and Hanif et. al., 2020). Since, in this study, we are working with monthly data series, we select J=5. For monthly data series, S_t^5 is the trend component with periodicity greater than 64 months and D_t^2, D_t^3,$D_t^4$and D_t^5 are the business-cycle components with periodicity of 4-8, 8-16, 16-32 and 32-64 months respectively and D_t^1 is a high frequency noise with periodicity less than 4 months. Hence for monthly seasonally adjusted data series

$X_t = S_t^5 + D_t^2 + D_t^3 + D_t^4 + D_t^5 + D_t^1 = S_t^5 + \sum_{j=2}^{5} D_t^j + D_t^1$= trend component+ cycle components+ noise

Pyramid Algorithm



Depending on their application, different types of wavelet have been introduced like Haar wavelet and Daubechies wavelet (Daubechies, 1992 and Misiti et al. 2000). Before applying wavelet technique for decomposition of data series, we first have to select a suitable type of wavelet filter according to our data series. The Haar wavelet is the simplest possible wavelet. The technical disadvantage of the Haar wavelet is that it is not continues and therefore not differentiable. Daubechies wavelet family is also very simple in use like Haar wavelet. Advantage of Daubechies wavelet over Haar wavelet is that it is orthogonal (Hanif et al. 2020) and the filter length is more than two. So it is more smooth and localized (Sharif and Khare, 2014). In this study we will use Daubechies (1992)'s Daub4 wavelet which is orthogonal symmetric wavelet filter.

As discussed in Javed and Hanif (2017), most of the filtering methods (like HP, MHP, band pass filters, wavelet etc) having endpoint biasness due to their structure and unavailability of one sided values at boundaries. To overcome this problem, Javed and Hanif (2017) purposed ARIMA(p,q) model to extend data to both ends for estimations. After estimation of cyclical components, extended data length is removed from both ends and choose the same time period of decomposed component as in the original data series. This study also adopted this method to resolve the end point problem. Technical detail of wavelet method used in this study can be found in Javed and Hanif (2017).

4. Empirical Findings

4.1 Data description

Considering the primary objective of State Bank of Pakistan – price stability – our main variable of interest to study in this paper is inflation rate. Inflation rate is measured by year on year (YoY) change in consumer price index as compiled by Pakistan Bureau of Statistics. For the potential determinants of inflation rate in Pakistan, we have selected a list of variables from relevant literature - including Gottschalk et al. (2008), Bashir et al (2011), Ahmed et al. (2014), Bashir et al. (2016), Al-Mutairi et al. (2020)) - on Pakistan having explanatory power for inflation rate in the country. These variables are: Policy rate (DISR),[2] Broad Money supply (M2), PKR/ US$ Exchange Rate (EXR), Large Scale of Manufacturing Index (LSM), Government Borrowing (GB), Global Commodity Prices (GCP) excluding oil,[3] Global Oil Price (OILP) and Import Payments[4]. We use seasonally adjusted monthly data of all these variable from July 1992 to June 2021.[5] For the analysis we take Year on Year (YoY) growth of all these variable except policy rate. Global commodity prices and oil prices data are obtained from IMF; LSM and CPI are taken from Pakistan Bureau of Statistics (PBS); and data of all other variables are taken from State Bank of Pakistan (SBP).

Figure 1 represents the growth rate of all variables in this study while policy rate is at level. From graph we can observe different frequency cycles across the series as well as across the time period. Policy rate, Exchange rate and Global commodity prices consists of cycles with low frequencies while LSM, Inflation rate showing high frequency cyclical movements. We will discuss in the following section, how cycles, based on their frequencies, shift along the time period.

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[2] Discount rate was replaced by SBP policy rate on 23rd May, 2015.

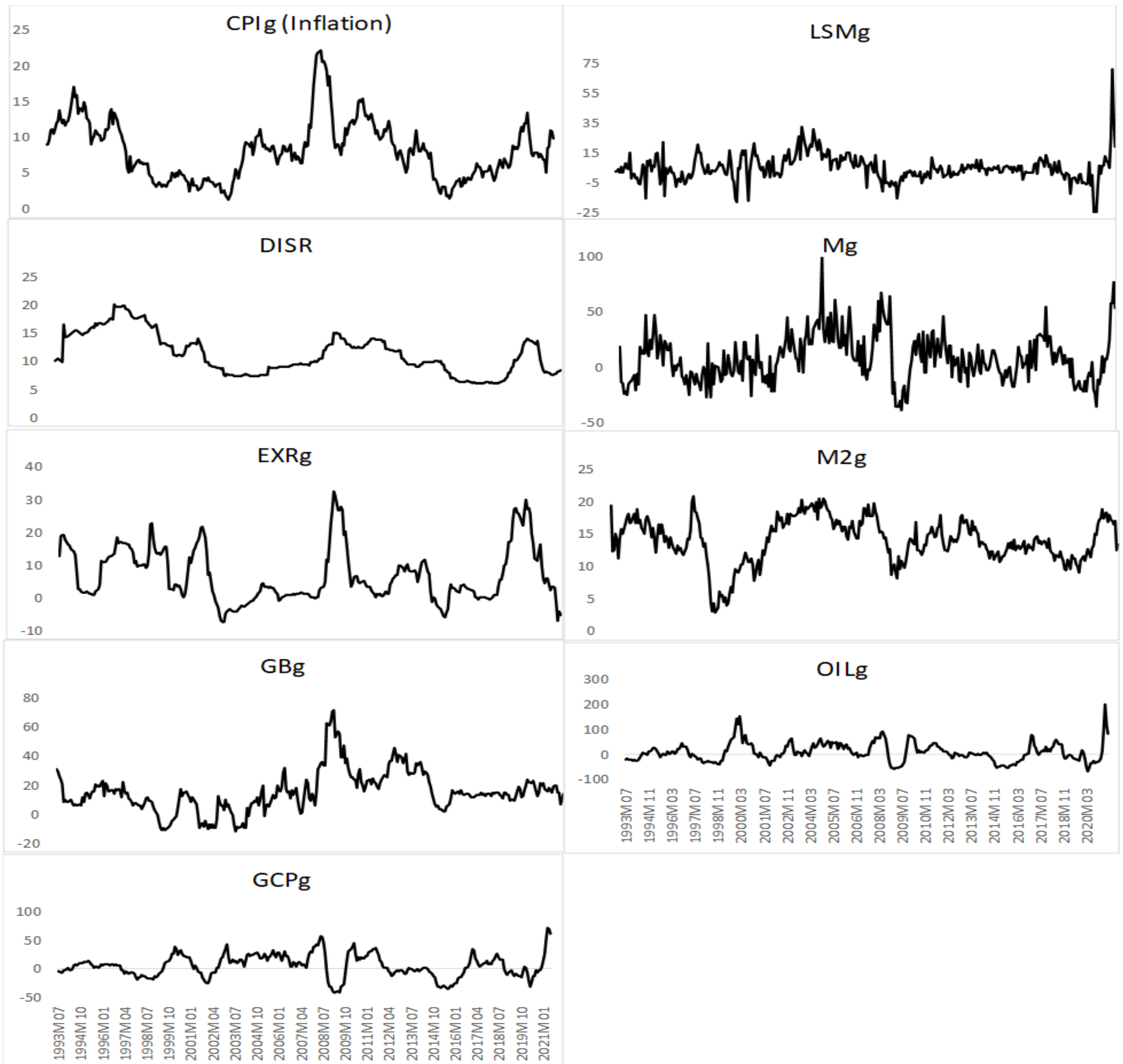
[3] Global commodity prices index is prepared by IMF. It comprises of 68 commodities prices. It is weighted average of respective prices where their shares in international trade have been used as weights. Pakistan is a net importer of most of the commodities in the IMF's global commodity price index (see Hanif et al (2017) for detail). For example, in Financial Year 2021, the major imports are agriculture and other chemicals (16.34%), machinery (15.9%) etc. Crude oil is separated being the largest imported product group is 'petroleum group' as it was almost one fifth of Pakistan's imports in FY2021. Other than textiles, major exported products included rice (8.83%), fruits (1.88%), fish and fish Preparations (1.67%) and meat and meat Preparations (1.41%).

[4] Almost a quarter CPI basket of Pakistan consists of traded goods (Hanif and Shah, 2019). Share of imported inflation in inflation in a country depends upon a) share of imported goods in CPI basket, b) country's exchange rate, and c) global commodity prices of goods imported by the country. The share of imported goods in CPI basket of Pakistan is around one-sixth.

[5] We have used X12 ARIMA technique to adjust the seasonality.

Descriptive statistics of inflation and its potential determinants are reported in Table 1. Sample means of all variables are positive with government borrowing growth having highest mean value while large scale manufacturing growth mean is the lowest one. Standard deviation figures indicate that global oil prices

Figure 1: Spectrum of inflation rate (in Pakistan) and its potential determinants (from July 1992 to May 2021). All time series are in year-on-year growth except policy rate which is at level.



growth is highly volatile. Skewness and Kurtosis measure the shape of the distribution of data series. The shape of distribution helps to understand where the most information is lying and analyze the outliers in a given data. M2 growth is the only variable in our study which is negatively skewed. Large scale manufacturing growth having highest kurtosis value indicate that most of its values lies around the mean, which is low as stated above. The Jarque-Bera results for all the data series shows that the nulls, data is normally distributed, are rejected.

Table1. Descriptive Statistics

	CPIg	DISR	EXRg	GBg	LSMg	M2g	OILg	GCPg
Mean	8.18	11.28	7.02	15.57	4.55	14.03	9.87	5.29
Median	7.95	10.33	4.08	14.53	4.00	14.14	4.31	5.08
Maximum	22.17	20.08	32.52	71.35	70.72	20.85	200.71	72.86
Minimum	1.34	6.13	-7.40	-11.57	-40.00	2.77	-69.05	-41.85
Std. Dev.	4.07	3.58	8.29	13.89	9.17	3.48	37.55	19.95
Skewness	0.77	0.52	0.86	0.93	0.97	-0.62	0.97	0.15
Kurtosis	3.64	2.39	3.14	5.25	12.66	3.54	5.37	3.35
Jarque-Bera	38.51	20.58	41.53	119.85	1358.94	25.97	130.76	35.25
Probability	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

4.2. Wavelet depcompositions of inflation and its potential determinants

In this study, we use Daubechies (1992)'s (Daub4) wavelet filter to decompose the inflation rate and all its potential determinants. Following Benhmad (2013), Hanif and Ejaz (2018), Tahir et al. (2018), we are taking sixty-four months' length of overall business cycle of macroeconomic series. Through wavelet analysis, we decompose all these time series into different components of frequency lengths between $(2^j, 2^{j+1})$ months. As disused above, all these time series are decomposed into a set of five orthogonal frequency components D1, D2, D3, D4 and D5 which stand for 'details' at different frequency bands of respective time series representing the deviation from the own trend, and a frequency component S5 representing secular trend component of the given series. Decomposed components along with their frequency and category are given in Table 2. The detail D1 accounts for highest frequency noise (2–4 months). The level 2 through level 5 details (D2, D3, D4 and D5) account for the cyclical components of respective frequency lengths which in aggregate (summing over from D2 to D5 for 4–64 months' length) represents cycle (Tiwari et.al., 2019 and Benhmad, 2013) component of underlying time series. Excluding noise (D1), and cycle component (sum of D2 to D5) from the seasonally adjusted time series gives the secular trend for the underlying time series which is denoted by S5 (in this study) and pertains to frequency length of more than 64 months. Cyclical component can be further divided into different frequencies, high frequency cycle (D2, 4-8 months), medium frequency cycle (D3 or D4, 8-32 months) and median frequency cyclical components D5, 32-64 months).

This study explores the relationship of inflation with other variables at each frequency level. The smooth (S5) and detail (D5 to D2) components of each series are estimated using Matlab[6].

Table 2: Frequency scale levels in multiresolution analysis

Scale level	Monthly Frequency	Category	Description
D1	2-4 months	High Frequency	Noise
D2	4-8 months	Highest median Frequency	Cycle
D3	8-16 months	Higher median Frequency	
D4	16-32 months	Lower median Frequency	
D5	32-64 months	Lowest median Frequency	
S5	Above than 64 months	Low Frequency	Trend

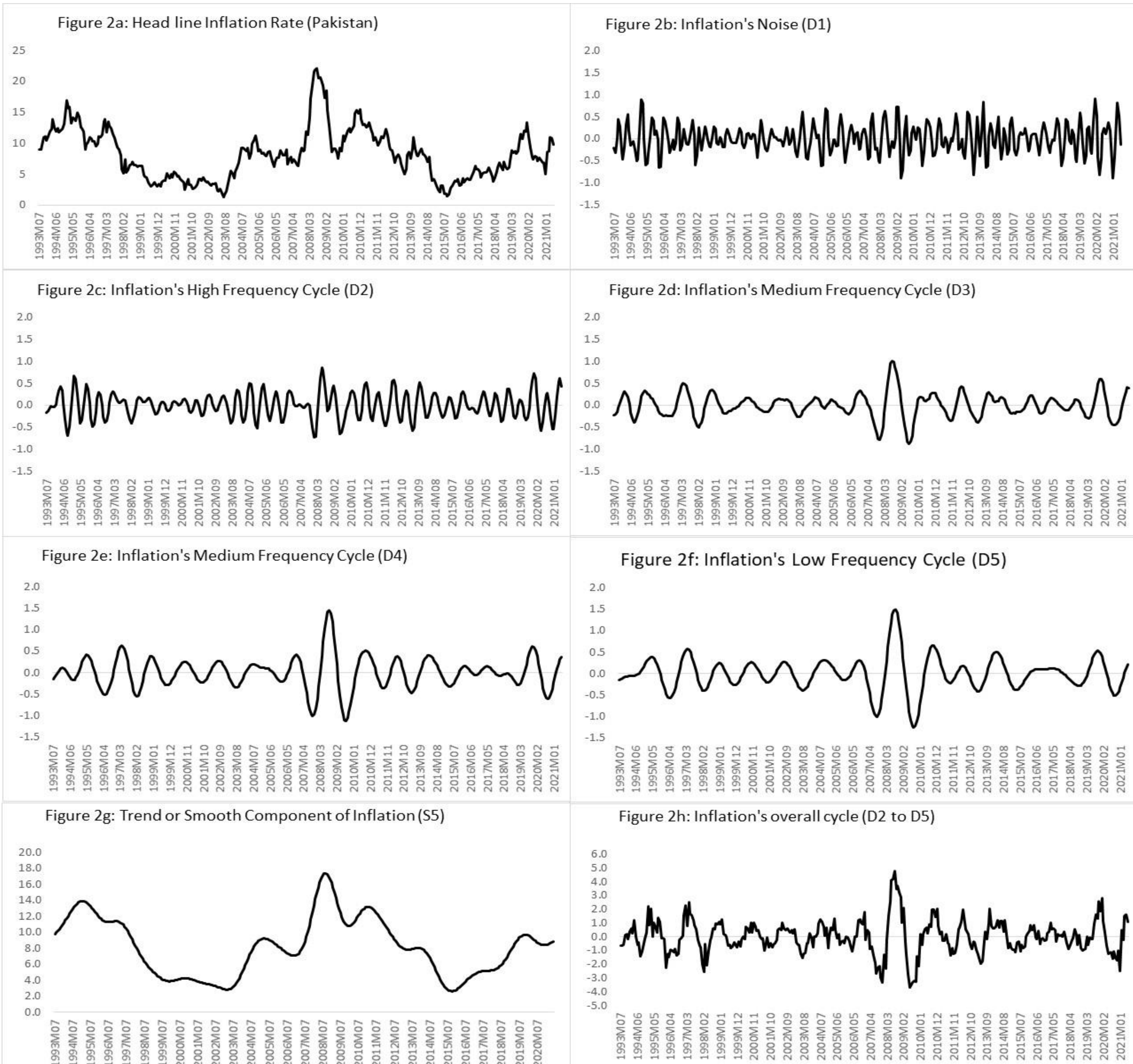
Figure 2 plots inflation rate in Pakistan (during FY 1993 to FY 2021) and all its different frequency scale levels as described in Table 2 and estimated using Wavelet analysis. Figure 2a shows observed headline inflation in Pakistan over the period of study. Figure 2b represents noise part (i.e. with 2-4 months' frequency length). This part is described as noise and hence is not part of the overall cyclical component of underlying series (Benhmad, 2013). Combining the exhibits 2a and 2b visually one can observe "higher the inflation - higher the noise" as has already be documented by Iqbal and Hanif (2012). The same phenomenon can also be recalled if we look at relatively low inflation regimes in Pakistan 1999-2003 and 2105-2018 (as identified in Iqbal and Hanif, 2012). In further analysis, we will explore the main sources of noise part of inflation in Pakistan.

Figures 2c to 2f exhibit cyclical components of inflation rate in Pakistan. We can see that the wavelet reveals interesting dynamics of the volatility in cyclical components of inflation at different frequency scales. As it should be, the most volatility in cyclical components occurred during 2007 to 2011 at the frequency scales corresponding to 16-32 and 32-64 months. This was the period when world went through global financial crisis and subsequent international commodity prices shock also impacted Pakistan inflation landscape.

We can see that how cyclical fluctuations vary across the different time frequency scales (D2 and D5, for example) over low, middle and high inflation regimes. Which, if ignored, as in the case of figure 2 where we sum up cycles over D2 to D5 simply disappears. This has implications for analysis of cyclical fluctuations in inflation with those of its determinants across the different time frequency scales (including D2 and D5). That is the main benefit of using time frequency scale analysis for studies like nexus between inflation and its determinants.

[6] Matlab code has been provided in the Appendix.

Figure 2: Head Line Inflation and It's Frequency components.



4.3. Results

4.4.1. Correlation Analysis

Correlation analysis helps exploring the strength and direction of the relationship between two (or more) variables before making any inference about the underlying relationship. In this study, we start from studying Pearson's correlation coefficients between components of inflation and those of its potential determinants. It may be possible that high frequency component of inflation rate series is correlated with median or low frequency component of any of its determinants. Thus, we explore the correlation coefficients across different frequency scales in addition to those between actual/composed inflation rate and its determinants.

4.4.1.1. Correlation Analysis within same scale

i) Correlation between Original data series

Table 3 (in Annexure), presents the Pearson correlation coefficients of inflation and with its determinants at same frequency scales. There is a possibility that same frequency components of two series are correlated with each other with time lags, so it is useful to estimate and analyze the correlation coefficients between these series at different lag length for original data series. We observe that all the variables, except large-scale manufacturing growth, are positively significantly correlated with inflation rate contemporaneously as well as at different lag lengths. However, after two years lags, the positive correlation between inflation and policy rate is turned to negative but economically insignificant.

ii) Correlation between high frequency (2-4 months) components

Table 3 also presents scale-wise correlation coefficients. Within high frequency component (2-4 months), global oil prices and global commodity prices are significantly positively correlated with inflation at level and one-month lag. However, this relationship dies out after two months lag for both commodities. This implies that high frequency changes in external commodity prices create immediate positive high frequency changes in inflation in Pakistan. Within same high frequency scale, we observe opposite finding in the case of exchange rate and large scale manufacturing growth as compared to original data series. Change in exchange rate, which was positively correlated with inflation in the case of original data series, is negatively correlated within high frequency scale. Being insignificant and counter intuitive, this correlation between inflation and change in exchange rate at high frequency may be ignored.[7] Large-scale manufacturing growth, which was negatively correlated with inflation for original data series, significantly positively correlated with inflation under high frequency movement. There is no correlation between noise part of all other variables and high frequency component of inflation and that is intuitive.

[7] The pass-through of exchange rate changes on inflation depends on a lot of things in addition to exchange rate regimes. During the period of study, Pakistan has followed different exchange rate regimes including managed exchanged rate and market-determined exchange rate. Pass-through may be different for different regimes which may need to be assessed in another study.

iii) Correlation between cycle/median frequency and its components (4-64 months)

For cycle or median frequency components (D2-D5), policy rate showing almost same pattern, highly correlated at level to lags 2 and after lag 2 it moves to opposite direction till twelve months' lag where it is negatively correlated with inflation for all the frequency scales. Exchange rate changes are positively significantly correlated with inflation within scales D3, D4, D5 and cycle till 2 months lags and after two months lags we observe opposite (negative) correlation. Within these frequency components, we observe co-movement up till two month lags and after this we have opposite movement between inflation and exchange rate components. For government borrowing, we observed no correlation during high frequency components D2 and D3, while for all other scales, from 16 to 64 months, there exist significant positive correlation between inflation and government borrowing up till lags 3. Which implies that short run government borrowing does not have any short run impact in inflation. External sector like global commodity prices and global oil prices are significantly positively correlated to inflation within (D3, D4 and D5) and overall median scale up to six month lags. In case of large scale manufacturing growth, we have no evidence of correlations of cycle and its components with relevant scales of inflation rate, however its high frequency components are positively correlated with high frequency component of inflation which opposite to original data series results. Imports growth, which was positively correlated to inflation in case of actual data series, we observe the same pattern within median frequency scales (D3, D4 and D5) and cycle. For short run (D1 and D2), there is no association between inflation and imports. Growth in money supply showing different stories as compared to all other variables, for median frequency cyclical components (D3, D4 and D5) / cycle, it is negatively correlated with inflation up to three months lags and shifted to positive side after six months.

IV) Correlation between trend component (above than 64 months' frequency)

For long run trend, we observe that policy rate, exchange rate, Government borrowing, Global commodity prices, money supply and global oil prices are all significantly positively correlated with long run movement in inflation as in the case of original data series. Imports growth, which was positively correlated with inflation rate in the case of original data series, have no long run correlation with inflation. Its positive correlation with inflation is mainly due to positive correlations in median frequency.

4.4.1.2. Correlation Analysis across scales

In the above section, we discussed correlations between inflation and its potential determinants within same frequency scale. However, if we have a scenario where positive change in high frequency of one variable (say global oil prices) during depression creates positive change in medium frequency of other variable (say inflation), it is not possible to have accurate picture of association between the two series within same scale analysis. Correlation analysis across the frequency scales is a better way to handle such situations. In the following paragraph, we will see how correlations differs across short, medium and large frequency scales.

Table 4 (in Annexure) presents the values of correlations coefficient across the frequency components with two months' lags. There is no correlation between high frequency component of inflation and any frequency component of all variables. We observe frequency component (4-8 months) of inflation significantly positively correlated with median frequency cyclical components (D2, D3, D4 and d5) and cycle of external variables (global commodity prices and oil prices). It is also correlated with medium frequency (D3 and D4) component of Government borrowing. Median frequency components (D3, D4 and D5) of inflation showing almost similar results. All these components are positively significantly correlated with the same frequency components and cycle. There is no correlation between median to low frequency components of inflation and trend components of other variables. Long run trend in inflation is positively significantly correlated with trend of policy rate, exchange rate, government borrowing, global commodity prices and money supply. However, it is negatively correlated with trend in large-scale manufacturing index. There is no correlation between long run trend in inflation and cyclical components of other variables, which implies that short run change in a variable does not have any impact on long run trend of inflation.

4.4.2. Causality Analysis

In previous section we explored association between frequency components of inflation and other variables through correlation estimations. Correlation describes the size of a relationship between two or more variables. A correlation between variables, however, does not automatically mean that the change in a variable causes variation in the other variables. If two variables are correlated, then the next step is to explore if one variable have any impact on other variable. For this purpose, we estimate and discuss causal relationship between cyclical components of inflation and other economic variables in this section. We apply Granger (1969) causality test for determining whether a macroeconomic time series is useful in predicting inflation or not. Granger causality testing provides a much more stringent criterion for information flow than simply observing high correlation with some lead-lag relationship (Tiwari et al., 2019). In Table 6 (in Annexure), we present Granger causality test results within a scale as well as across the scales.

4.4.2.1. Causal relationship within Scale

In this section, we discuss the causality results within same scales. From Table 6, with in scale D1 (2-4 months' frequency), we observe no causal relationship between CPI inflation and any of the economic variables in this study. Within scale D2 (4-8 months), policy rate, exchange rate growth, government borrowing and imports growth Granger cause inflation rate while all other variables do not have any causal relationship with inflation (see Table 6, row 20). This implies that these variables have a relatively brisk pass-through to the domestic prices in the wake of faster transmission of such shocks to the inflation. Within the same scale, we observe bi-directional causal relationship in the case of exchange rate changes, government borrowing and imports growth. Under the same frequency scale, we observe unidirectional causal relation from inflation to large scale manufacturing growth as inflation compromises the economic growth. Under scale D3 (8-16 months' cycle), only Government borrowing (as in D2) and money supply does cause inflation rate with feedback. No other variable having causal relationship with inflation. Within frequency scale D4 (16-32 months), exchange rate growth (as in D2) and large scale manufacturing growth does Granger cause inflation showing that growth also decreases inflationary pressures but after some lag. However, out of these two variables only exchange rate growth is significantly correlated with inflation which implies that inflation rate can be determined by exchange rate under this frequency movement. Under median frequency scale after sufficient time passes, D5 (32-64 moths), all the variables, except large scale manufacturing growth, Granger causes inflation owing to established theoretical relationship between inflation and these determinants in such frequency domain. This causality is bi-direction in all the cases except for global commodity prices where we observed only uni-directional causality. As all these variables under this frequency scale are significantly correlated (Table 3-5) with inflation along with causal relationships. We can infer that cyclical movement of inflation can be determined by same frequency movements in policy rate, exchange rate, external commodity prices, government borrowing, money supply and imports growth. Long run trend causal analysis indicates that government borrowing, global commodity, oil prices, imports and money supply does Granger cause inflation rate. Although this outcome is statistically significant, but not all variables have economically meaningful potency. This is because out of these five variables only, government borrowing, global commodity prices and money supply are significantly positively impacting with inflation. Which implies that mainly these three variables determine inflation in the long run. Thus, our results support the evidence that money supply growth determines inflation, even in time frequency domain (Christiano and Fitzgerald, 2003; Sargent and Surico, 2008; Benati, 2009, Jiang, Chang, and Li, 2015).

For causality between cycle (4- to 64-month length) of inflation and other variables, the government borrowing, global commodity prices, imports and global oil prices do have some causal relationship with inflation. Since all these variables are also significantly positively correlated with inflation rate at level as well as along some lags, we deduce that changes in business cyclical movement in any one of these variable create some changes in cycle of inflation rate in same direction. Consequently, we have evidence that not only degree of causal relationship varies across scales, but also the direction of causality differs by timescales which is in the line of Ramsey and Lampart (1998) study.

4.4.2.2. Causal relationship across the scales

i) Inflation rate and Policy rate

From Table 6, we observe scale D2 (4-8 months) component of policy rate does Granger cause inflation rate of scale D2, D3 and D4. Other Median scales (D3, D4 and D5) components of policy rate do cause almost every component of inflation rate. Cyclical component Granger causes long run trend component of inflation while there is no evidence of Granger causality from trend component as well as original data series of policy rate to inflation rate and its components. We also observed that all these combinations are significantly negatively correlated with each other after six months' lag (Table 5), which implies that cycle and all of its components of policy rate playing role in explaining inflation rate in line with the conventional theory. We find that price puzzle phenomenon might have frequency dependent effect as discussed in Lobus and Lokus (2018). Hence, timescale analysis resolves the issues of price puzzle which has been discussed in many studies like (Javid and Munir, 2010).

ii) Inflation rate and Exchange rate growth

We observe that D2 component of exchange rate Granger causes original data series as well as high frequency cyclical (D1 and D2) components of inflation. There is no evidence of Granger causality from this component to any other component of inflation rate. Medium to high frequency (D3, D4 and D5) components of exchange rate Granger cause almost all the frequency components of inflation rate.

Table 5: Correlation coefficient between inflation and policy rate at Six Months' Lags

Policy Rate (-6) \ Inflation	D1	D2	D3	D4	D5	S5	BC
D1	-0.01	0.01	-0.04	-0.03	-0.02	0.00	-0.02
D2	0.01	-0.08	-0.20	-0.16	-0.10	-0.01	-0.14
D3	-0.04	-0.18	-0.36	-0.31	-0.23	-0.02	-0.29
D4	-0.04	-0.16	-0.35	-0.34	-0.27	-0.01	-0.31
D5	-0.03	-0.12	-0.24	-0.24	-0.20	0.01	-0.22
S5	0.00	-0.01	-0.01	0.00	0.01	0.39	0.00
BC (D2-D5)	-0.02	-0.14	-0.30	-0.28	-0.22	-0.01	-0.24

Also we observe that all these components Granger cause long run trend and cycle of inflation rate. There is no evidence of Granger causality from original data series and long run trend component of exchange rate to any frequency component of inflation. However, cyclical component of exchange rate does Granger cause inflation at sub components levels of cyclical part of inflation (D5 and S5, above than 32 months' cycles). In the correlation section, we observed positive significant correlation between these sub components (D3, D4, D5 and cycle itself) of cyclical components of inflation and exchange rate. We observe that cycle (and its sub components) of exchange rate playing significant role in explaining cyclical movements of inflation.

iii) Inflation rate and Government borrowing growth

From causality results, we observe that frequency scales D3, D4 and D5 of government borrowing Granger causes all the components of inflation rate. However, cycle of government borrowing does Granger cause long run trend, cycle and different median frequency components of inflation rate. While, long run trend component of government borrowing does Granger cause long run trend and low frequency component of inflation. Cycle and its sub components are all significantly correlated with same group of components. Keeping in mind both causality and correlation results it is inferred that cycle and its sub components can explain inflation and its cycle components.

IV) Inflation rate and Global commodity prices growth

Low frequency components (D3, D4 and D5) of GCPg Granger cause low frequency components, long run trend and cycle of inflation. We also observe that cycle of GCPg does Granger cause cycle, long run trend and actual inflation rate. Due to week correlation between D3, D4 and D5 of GCPg and trend component of inflation, no business component has any explanatory power to long run trend of inflation. However, by considering both correlations and causal relationship, we conclude that cycle and all its components played significant role in explaining overall inflation rate individually.

V) Inflation rate and Large scale manufacturing growth

From Table 6, median frequency components (D4 and D5) of large-scale manufacturing index Granger cause median frequency components (D4, D5 and S5) of inflation. There is no long run causal link between inflation and LSM growth. These results are in line of Tiwari et al (2019) results. Although these median frequency components of LSM growth Granger cause of median frequency components of inflation but we observed zero correlation between them, which implies that large scale manufacturing growth itself and its component have nothing to explain inflation or any of its component.

VI) Inflation rate and Imports growth

In causality analysis, we observed that overall imports growth does Granger cause inflation rate, with significant positive correlation, which implies that imports are playing significant role in explaining inflation. For components analysis, we observe that cycle of imports growth Granger causes cycle and almost all of its components of inflation. S5 (long run trend) of imports growth Granger causes median frequency components (greater than 16 months' frequency including trend) of inflation rate. However other frequency components are showing mixed causality response. As we observed in previous section that only cycle of imports is significantly positively correlated to cycle and its sub components of inflation which implies that cycle of imports can be a determinant of inflation itself and all its sub components.

VII) Inflation rate and Money Supply growth

From causality results, higher timescale components (D3, D4 and D5) of money supply does Granger cause of almost same group of components (D3, D4, D5 and cycle) of inflation rate. Cycle of money supply does only cause median frequency components of inflation. There is no evidence of causality from BC of money supply to BC and long run trend component of inflation. While long run trend of money supply does Granger cause inflation rate itself. Also in the case of original data, money supply does Granger cause of inflation with significant positive correlation. Considering correlations between these components, we observe counter cyclical behavior between cycles and its components. Only trend components of both series are positively correlated to each other, which implies that long run trend of money supply can be a determinant of long run inflation and inflation itself.

VIII) Inflation rate and Global Oil Price growth

For overall data series, global oil prices Granger cause inflation rate with significant positive correlation. Global oil price can be powerful tool to explain inflation rate. While frequency analysis, long run trend component of inflation is caused by all frequency components (D2-D5, S5 and BC) of global oil prices. This causality is meaning less as there is no correlations between all these pairs. However, cycle of inflation rate is caused by cycle and its sub component of global oil prices with significant positive correlation. We concluded here that cycle and its components of inflation rate can be explained by cycle and its components of oil prices.

5. Conclusion

Low and stable inflation rate is the primary, if not only, objective of many central banks. To achieve this goal, it is important for the policy makers to know what determines inflation. In addition to general theoretical literature, there exists huge empirical evidence to explain what determines inflation in a small open economy like Pakistan. A simple survey of available empirical research studies can help distinguish short run and long run determinants of inflation. However, the methodologies used in different studies exploring inflation determinant in Pakistan do not guide anything about how short is the short run and how long is the long run for any of the determinants to impact inflation in Pakistan. There could be different determinants of inflation in Pakistan in different time scales. In this study, we have used time frequency analysis to explore scale wise determinants of inflation in Pakistan.

By decomposing the time series data into different time-frequency scales through multi-scale Wavelet approach, we analyzed the nexus between inflation and its determinants through correlation and causation analysis using monthly data for July 1992 to June 2021. First we decompose time series into three frequency scales, noise part (less than 4 months' frequency component), cyclical part (4-64 months' frequency) and long run trend component (above than 64 months' frequency). Cyclical component can be further divided into different frequencies subcomponents (D2, 4-8 months, D3 or D4, 8-32 months, and D5, 32-64 months).

Pearson's correlation coefficient based results indicate that in case of actual data series, all variables, except large scale manufacturing, are positively significantly correlated with inflation up to six month lags for actual data series. Large scale manufacturing growth is negatively correlated with inflation for up-to 6 month lags. Mix of intuitive and counter-intuitive results for shorter lags are observed up-to six months' lags. Some variables are correlated with inflation as per economic theory and other showed theorized impact on inflation after some longer lags. In particular, we found that after two years, the correlation between policy rate and inflation rate becomes negative, making policy rate an effective monetary policy tool for the case of Pakistan.

Once all series are decomposed, we observe that only a) global commodity prices and b) international oil prices are positively significantly correlated with inflation under high frequency scale (at noise level). It implies that any change in global commodity prices passes through to domestic prices in Pakistan relatively quickly. Global commodity prices are strongly correlated with inflation during this scale as compared to original data series. Such relationships fade over the period of time. For example, over median frequency scale, global commodity prices and imports growth remain positively correlated with inflation while both becomes uncorrelated with inflation in the long run (at low frequency scale).
We observe that at median frequency scale, policy rate, which is positively correlated with inflation in the short run at 'composed' level, turns to negative relationship with inflation after just two lags – a theoretically consistent result that interest rate affects inflation with lag.

Granger causality results show that at median frequency scale and its components (D3, D4 and D5) of policy rates is associated with all the frequency components of inflation. We also observe that all these components are significantly negatively correlated with inflation and its components after six months' lags, which implies that cycle and all its components of policy rate playing role in taming inflation and its frequency components.

Cycle of exchange rate does Granger cause median frequency (D5) and low frequency (S5) components of inflation. Thus, we conclude that cyclical movement of inflation can be explained by cycle and its subcomponent of exchange rate.

Cycle of government borrowing does Granger cause cycle of inflation. We also observed positive significant correlation between these series, which implies that cyclical movement of inflation can be explained by cyclical movement of government borrowing growth. Similarly, cyclical components (D3, D4 and D5) of government borrowing can explain inflation and its cycle components. Cyclical part and its sub components (D2-D5) of global commodity prices does Granger cause cycle, its sub components and trend part of inflation. By considering both correlations and causal relationship, we concluded that cycle and its sub components of global commodity prices playing significant role in explaining overall inflation rate individually.

Due to causal relationship and significant positive correlation, overall imports growth playing significant role in explaining inflation. For components analysis, with the causal relationship and significantly positively correlation, only cycle part of imports growth can be a determinant of inflation itself and all its sub components.

Frequency analysis suggests, cycle and its components of inflation rate can be explained by cycle and its components of oil prices. Cycle and its sub components of money supply growth does Granger cause some sub components cyclical part of inflation. Trend component (S5) of money supply does Granger cause long run trend of inflation and inflation itself. With strong positive correlation we can concluded that trend part of money supply can be a determinant of long run inflation.

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Appendix

Table 3: Correlation coefficients between frequency components of inflation and macro-economic variables (in growth) with in same Scale.

	Lag Length	DISR	EXRg	GBg	GCPg	LSMg	Mg	M2g	OILg
Original Series	Level	0.52	0.34	0.64	0.21	-0.23	0.13	0.17	0.11
	Lag 1	0.49	0.32	0.63	0.26	-0.22	0.17	0.20	0.15
	Lag 2	0.46	0.30	0.62	0.30	-0.23	0.20	0.23	0.17
	Lag 3	0.42	0.28	0.60	0.33	-0.24	0.22	0.26	0.20
	Lag 6	0.32	0.20	0.50	0.37	-0.17	0.22	0.33	0.24
	Lag 12	0.15	0.11	0.31	0.24	-0.11	0.08	0.38	0.10
	Lag 24	-0.10	-0.04	0.32	0.21	-0.02	0.23	0.27	0.05
D1 (2-4 months)	Level	-0.03	-0.17	-0.06	0.36	0.23	-0.01	-0.05	0.36
	Lag 1	-0.03	-0.17	-0.06	0.29	0.09	-0.01	0.03	0.25
	Lag 2	0.05	-0.03	-0.06	-0.03	-0.10	-0.03	0.10	-0.08
	Lag 3	0.04	0.11	-0.05	-0.25	-0.15	-0.02	0.07	-0.29
	Lag 6	-0.01	-0.02	0.13	0.04	0.03	0.00	-0.04	0.10
	Lag 12	0.08	0.10	0.07	-0.19	-0.13	0.03	0.04	-0.18
	Lag 24	-0.11	-0.11	-0.03	-0.10	0.07	0.05	-0.05	-0.14
D2 (4-8 months)	Level	0.24	-0.06	-0.10	0.44	0.25	0.03	0.02	0.37
	Lag 1	0.11	-0.15	-0.13	0.40	0.20	0.00	0.05	0.33
	Lag 2	-0.04	-0.15	-0.05	0.21	0.05	0.00	0.04	0.16
	Lag 3	-0.16	-0.07	0.08	-0.05	-0.10	0.04	0.00	-0.06
	Lag 6	-0.08	0.04	0.18	-0.17	-0.03	0.10	-0.12	-0.13
	Lag 12	-0.09	0.03	0.00	-0.23	-0.11	-0.14	0.12	-0.20
	Lag 24	-0.14	-0.05	0.11	-0.16	-0.01	0.20	-0.14	-0.17
D3 (8-16 months)	Level	0.40	0.34	0.49	0.36	0.01	0.26	-0.27	0.27
	Lag 1	0.29	0.22	0.50	0.44	0.08	0.34	-0.24	0.37
	Lag 2	0.16	0.07	0.46	0.47	0.13	0.40	-0.19	0.41
	Lag 3	0.00	-0.07	0.38	0.45	0.16	0.41	-0.12	0.42
	Lag 6	-0.36	-0.38	-0.04	0.19	0.14	0.26	0.19	0.23
	Lag 12	-0.11	-0.06	-0.37	-0.20	-0.07	-0.29	0.27	-0.16
	Lag 24	-0.11	0.08	0.10	-0.04	-0.05	0.10	-0.07	-0.04

	Lag Length	DISR	EXRg	GBg	GCPg	LSMg	Mg	M2g	OILg
D4 (16-32 months)	Level	0.41	0.36	0.57	0.26	-0.06	0.10	-0.39	0.14
	Lag 1	0.30	0.26	0.58	0.40	-0.03	0.14	-0.30	0.29
	Lag 2	0.17	0.13	0.54	0.50	0.01	0.17	-0.19	0.41
	Lag 3	0.03	-0.01	0.46	0.56	0.05	0.19	-0.06	0.50
	Lag 6	-0.34	-0.36	0.05	0.48	0.09	0.14	0.33	0.52
	Lag 12	-0.23	-0.22	-0.53	-0.20	0.00	-0.13	0.32	-0.13
	Lag 24	-0.04	0.12	0.39	0.09	0.03	0.13	-0.17	0.07
D5 (32-64 months)	Level	0.44	0.29	0.56	0.28	-0.08	0.05	-0.29	0.14
	Lag 1	0.35	0.22	0.57	0.43	-0.06	0.09	-0.22	0.30
	Lag 2	0.24	0.13	0.53	0.56	-0.04	0.13	-0.12	0.44
	Lag 3	0.13	0.03	0.46	0.65	-0.02	0.15	-0.01	0.56
	Lag 6	-0.20	-0.24	0.08	0.68	0.06	0.16	0.29	0.67
	Lag 12	-0.35	-0.22	-0.62	-0.12	0.18	0.04	0.30	-0.03
	Lag 24	0.09	0.11	0.51	0.00	-0.15	-0.09	-0.24	-0.03
S5 (Trend Part) (Above than 64 months)	Level	0.55	0.38	0.72	0.19	-0.42	-0.04	0.28	0.06
	Lag 1	0.52	0.37	0.71	0.22	-0.42	-0.04	0.30	0.09
	Lag 2	0.50	0.36	0.70	0.26	-0.41	-0.03	0.32	0.11
	Lag 3	0.48	0.35	0.68	0.29	-0.40	-0.03	0.33	0.13
	Lag 6	0.39	0.31	0.63	0.35	-0.38	-0.04	0.38	0.17
	Lag12	0.21	0.18	0.51	0.38	-0.31	-0.07	0.43	0.18
	Lag24	-0.11	-0.06	0.35	0.36	-0.16	0.02	0.42	0.13
Cycle (BC) (4-64 months)	Level	0.35	0.28	0.43	0.29	0.00	0.14	-0.26	0.18
	Lag 1	0.26	0.18	0.44	0.40	0.05	0.19	-0.19	0.30
	Lag 2	0.13	0.09	0.41	0.44	0.03	0.24	-0.14	0.34
	Lag 3	0.04	-0.01	0.37	0.47	0.04	0.25	-0.03	0.40
	Lag 6	-0.24	-0.28	0.07	0.38	0.06	0.16	0.22	0.40
	Lag 12	-0.16	-0.16	-0.40	-0.19	0.01	-0.12	0.27	-0.13
	Lag 24	-0.06	0.09	0.29	0.03	-0.02	0.07	-0.17	0.01

Note: Absolute value of correlation, |r|>0.11 is significant at 5% level. Lag length is of macro-economic variables.

Table 4: Scale Wise Cross Correlation coefficients between inflation and macro-economic variables.

Inflation Vs Macro Variables	DISR	EXRg	GBg	GCPg	LSMg	Mg	M2g	OILg
D1 Vs D1	0.05	-0.03	-0.06	-0.03	-0.10	-0.03	0.10	-0.08
D1 Vs D2	-0.04	-0.10	-0.10	0.10	0.01	0.00	0.06	0.08
D1 Vs D3	0.01	-0.02	0.03	0.07	0.07	0.07	-0.01	0.08
D1 Vs D4	0.01	0.00	0.04	0.06	0.04	0.04	-0.02	0.06
D1 Vs D5	0.00	0.01	0.04	0.04	0.04	0.05	-0.02	0.04
D1 Vs S5	0.00	0.00	0.01	0.02	-0.02	-0.03	0.00	0.01
D1 Vs BC	0.00	-0.03	0.01	0.06	0.02	0.06	0.00	0.05
D2 Vs D1	-0.04	-0.08	-0.07	0.09	-0.02	-0.03	0.06	0.04
D2 Vs D2	-0.04	-0.15	-0.05	0.21	0.05	0.00	0.04	0.16
D2 Vs D3	0.06	-0.01	0.18	0.25	0.13	0.18	-0.09	0.24
D2 Vs D4	0.07	0.03	0.21	0.23	0.03	0.07	-0.10	0.22
D2 Vs D5	0.27	0.06	0.04	0.19	0.19	0.11	0.17	-0.07
D2 Vs S5	0.00	0.00	0.02	0.06	-0.04	-0.03	0.00	0.04
D2 Vs BC	0.04	-0.02	0.14	0.25	0.09	0.15	-0.05	0.22
D3 Vs D1	0.00	-0.01	0.03	0.06	0.01	0.01	0.01	0.05
D3 Vs D2	0.07	0.00	0.18	0.24	0.05	0.15	-0.06	0.18
D3 Vs D3	0.16	0.07	0.46	0.47	0.13	0.40	-0.19	0.41
D3 Vs D4	0.16	0.10	0.50	0.50	0.05	0.17	-0.20	0.44
D3Vs D5	0.14	0.11	0.46	0.45	0.11	0.24	-0.15	0.38
D3 Vs S5	0.02	0.03	0.06	0.11	-0.08	-0.02	0.00	0.09
D3 Vs BC	0.14	0.08	0.43	0.47	0.10	0.28	-0.15	0.39
D4 Vs D1	0.01	0.01	0.03	0.05	0.01	0.02	0.00	0.04
D4 Vs D2	0.07	0.04	0.19	0.20	0.02	0.16	-0.06	0.14
D4 Vs D3	0.15	0.11	0.47	0.42	0.06	0.40	-0.16	0.35
D4 Vs D4	0.17	0.13	0.54	0.50	0.01	0.17	-0.19	0.41
D4 Vs D5	0.18	0.13	0.53	0.52	0.02	0.18	-0.15	0.42
D4 Vs S5	0.04	0.05	0.07	0.14	-0.07	0.04	0.00	0.12
D4 Vs BC	0.16	0.12	0.47	0.47	0.03	0.25	-0.14	0.37
D5 Vs D1	0.01	0.01	0.02	0.04	0.01	0.02	0.00	0.03
D5 Vs D2	0.06	0.04	0.15	0.15	0.01	0.13	-0.04	0.11
D5 Vs D3	0.13	0.11	0.42	0.34	-0.01	0.33	-0.11	0.27
D5 Vs D4	0.19	0.13	0.51	0.46	-0.05	0.13	-0.15	0.36
D5 Vs D5	0.24	0.13	0.53	0.56	-0.04	0.13	-0.12	0.44

Inflation Vs Macro Variables	DISR	EXRg	GBg	GCPg	LSMg	Mg	M2g	OILg
D5 Vs S5	0.06	0.07	0.09	0.18	-0.06	0.11	0.01	0.16
D5 Vs BC	0.17	0.12	0.44	0.43	-0.03	0.19	-0.11	0.32
S5 Vs D1	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.01
S5 Vs D2	0.01	0.01	0.02	0.02	0.00	0.02	-0.01	0.01
S5 Vs D3	0.04	0.03	0.05	0.05	0.02	0.09	-0.01	0.04
S5 Vs D4	0.07	0.04	0.07	0.08	0.07	0.11	0.00	0.06
S5 Vs D5	0.11	0.05	0.09	0.12	0.11	0.17	0.01	0.10
S5 Vs S5	0.50	0.36	0.70	0.26	-0.41	-0.03	0.32	0.11
S5 Vs BC	0.07	0.04	0.06	0.08	0.07	0.16	0.00	0.06
BC Vs D1	0.00	-0.02	0.01	0.05	-0.01	0.01	0.02	0.03
BC Vs D2	0.04	-0.01	0.13	0.22	0.03	0.13	-0.03	0.16
BC Vs D3	0.14	0.08	0.43	0.41	0.07	0.36	-0.15	0.34
BC Vs D4	0.17	0.11	0.49	0.47	0.01	0.15	-0.17	0.39
BC Vs D5	0.18	0.11	0.48	0.48	0.04	0.19	-0.14	0.40
BC Vs S5	0.04	0.05	0.07	0.14	-0.07	0.04	0.00	0.12
BC Vs BC	0.13	0.09	0.41	0.44	0.03	0.24	-0.14	0.34

Note: Absolute value of correlation, $|r|>0.11$ is significant at 5% level.

Table 6. Multiscale Granger causality test (Probability) between inflation and growth of economic variables.

		Mode 1		Model 2		Mode 3		Model 4		Mode 5		Model 6		Mode 7		Model 8	
Causal Scale	**Caused (CPIg) Scale**	**DISR to CPIg**	**CPIg to DISC**	**EXRg to CPIg**	**CPIg to EXRg**	**GBg to CPIg**	**CPIg to GBg**	**GCPg to CPIg**	**CPIg to GCPg**	**LSMg to CPIg**	**CPIg to LSMg**	**Mg to CPIg**	**CPIg to Mg**	**M2g to CPIg**	**CPIg to M2g**	**OILg to CPIg**	**CPIg to OILg**
D0	D0	0.10	0.00	0.79	0.12	0.20	0.01	0.00	0.11	0.23	0.01	0.01	0.12	0.05	0.65	0.00	0.13
D1	D0	0.69	0.76	0.30	0.30	0.97	0.83	0.00	0.88	0.08	0.85	1.00	0.89	0.74	0.96	0.00	0.90
D2	D0	0.05	0.11	0.31	0.03	0.59	0.02	0.01	0.10	0.21	0.31	0.75	0.00	0.81	0.46	0.03	0.14
D3	D0	0.00	0.16	0.02	0.40	0.04	0.31	0.00	0.50	0.20	0.34	0.00	0.11	0.22	0.40	0.00	0.18
D4	D0	0.00	0.19	0.01	0.72	0.00	0.12	0.00	0.02	0.17	0.41	0.04	0.10	0.01	0.73	0.00	0.05
D5	D0	0.00	0.61	0.01	0.33	0.00	0.15	0.00	0.86	0.23	0.85	0.06	0.50	0.01	0.06	0.00	0.81
S5	D0	0.00	0.00	0.27	0.05	0.00	0.24	0.00	0.00	0.20	0.01	0.95	0.58	0.08	0.00	0.03	0.00
BC	D0	0.02	0.04	0.09	0.27	0.65	0.32	0.00	0.17	0.18	0.06	0.04	0.22	0.22	0.44	0.00	0.09
D0	D1	0.96	0.56	0.88	0.16	0.81	0.61	0.49	0.00	0.82	0.10	0.91	1.00	0.57	0.73	0.49	0.00
D1	D1	0.10	0.06	0.09	0.48	0.26	0.46	0.13	0.68	0.61	0.21	0.46	0.79	0.35	0.66	0.11	0.81
D2	D1	0.03	1.00	0.13	0.00	0.13	0.66	0.00	0.05	0.01	0.06	0.91	0.14	0.60	0.83	0.00	0.10
D3	D1	0.21	0.64	0.22	0.40	0.61	0.36	0.18	0.01	0.44	0.16	0.29	0.55	0.90	0.63	0.23	0.05
D4	D1	0.45	0.91	0.40	0.97	0.39	0.95	0.34	0.79	0.88	0.14	0.50	0.10	0.53	0.90	0.33	0.68
D5	D1	0.54	0.95	0.56	0.73	0.51	0.91	0.36	0.91	0.87	0.85	0.71	0.77	0.56	0.74	0.31	0.93
S5	D1	0.95	0.97	0.98	0.97	0.99	0.89	0.89	0.98	0.65	0.89	0.63	0.73	0.94	0.96	0.90	0.96
BC	D1	0.30	0.30	0.72	0.10	0.80	0.52	0.13	0.00	0.62	0.10	0.65	0.91	0.58	0.64	0.23	0.00
D0	D2	0.12	0.20	0.01	0.07	0.01	0.68	0.19	0.00	0.23	0.04	0.00	0.97	0.42	0.60	0.02	0.00
D1	D2	0.93	0.02	0.02	0.00	0.27	0.10	0.19	0.00	0.29	0.01	0.36	0.43	0.66	0.81	0.31	0.00
D2	D2	0.03	0.06	0.00	0.00	0.00	0.00	0.34	0.58	0.17	0.00	0.01	0.00	0.35	0.53	0.71	0.46
D3	D2	0.00	0.23	0.00	0.01	0.00	0.02	0.00	0.00	0.55	0.02	0.00	0.14	0.00	0.10	0.00	0.00
D4	D2	0.00	0.59	0.00	0.26	0.00	0.85	0.00	0.07	0.00	0.12	0.08	0.05	0.00	0.79	0.00	0.04
D5	D2	0.00	0.64	0.00	0.03	0.00	0.07	0.00	0.26	0.13	0.51	0.00	0.20	0.00	0.20	0.00	0.33
S5	D2	0.41	0.31	0.44	0.45	0.61	0.14	0.00	0.51	0.59	0.76	0.99	0.51	0.52	0.41	0.02	0.00
BC	D2	0.00	0.02	0.00	0.00	0.00	0.17	0.00	0.00	0.50	0.01	0.01	0.70	0.02	0.34	0.02	0.00
D0	D3	0.94	0.00	0.74	0.04	0.28	0.18	0.59	0.00	0.20	0.19	0.09	0.01	0.21	0.14	0.89	0.00
D1	D3	0.51	0.13	0.55	0.20	0.41	0.77	0.02	0.03	0.31	0.67	0.86	0.65	0.64	0.93	0.04	0.11
D2	D3	0.01	0.00	0.28	0.00	0.01	0.00	0.00	0.00	0.07	0.00	0.32	0.00	0.14	0.00	0.00	0.00
D3	D3	0.53	0.21	0.96	0.07	0.02	0.05	0.66	0.00	0.36	0.01	0.14	0.00	0.00	0.00	0.53	0.00
D4	D3	0.05	0.73	0.15	0.00	0.00	0.70	0.10	0.00	0.83	0.22	0.33	0.05	0.00	0.35	0.20	0.00
D5	D3	0.01	0.03	0.04	0.00	0.00	0.00	0.00	0.00	0.18	0.15	0.52	0.00	0.00	0.00	0.00	0.00
S5	D3	0.54	0.00	0.64	0.00	0.42	0.00	0.09	0.00	0.51	0.30	0.40	0.16	0.65	0.00	0.11	0.00
BC	D3	0.77	0.00	0.65	0.00	0.01	0.00	0.72	0.00	0.17	0.05	0.67	0.00	0.08	0.02	0.28	0.00
D0	D4	0.55	0.00	0.74	0.04	0.29	0.01	0.23	0.00	0.12	0.34	0.16	0.00	0.88	0.03	0.58	0.00

		Mode 1		Model 2		Mode 3		Model 4		Mode 5		Model 6		Mode 7		Model 8	
Causal Scale	**Caused (CPIg) Scale**	**DISR to CPIg**	**CPIg to DISC**	**EXRg to CPIg**	**CPIg to EXRg**	**GBg to CPIg**	**CPIg to GBg**	**GCPg to CPIg**	**CPIg to GCPg**	**LSMg to CPIg**	**CPIg to LSMg**	**Mg to CPIg**	**CPIg to Mg**	**M2g to CPIg**	**CPIg to M2g**	**OILg to CPIg**	**CPIg to OILg**
D1	D4	0.93	0.44	0.97	0.56	0.96	0.58	0.60	0.29	0.88	0.98	0.88	0.57	0.98	0.81	0.64	0.50
D2	D4	0.06	0.00	0.59	0.00	0.28	0.00	0.02	0.00	0.78	0.03	0.27	0.00	0.79	0.00	0.07	0.00
D3	D4	0.01	0.00	0.00	0.00	0.04	0.00	0.00	0.00	0.03	0.05	0.06	0.00	0.11	0.00	0.00	0.00
D4	D4	0.99	0.31	0.03	0.02	0.95	0.17	0.27	0.00	0.03	0.64	0.39	0.30	0.09	0.53	0.08	0.00
D5	D4	0.00	0.00	0.88	0.00	0.02	0.00	0.00	0.00	0.00	0.07	0.01	0.00	0.19	0.00	0.05	0.00
S5	D4	0.14	0.00	0.35	0.00	0.00	0.00	0.00	0.00	0.86	0.05	0.01	0.12	0.38	0.00	0.00	0.00
BC	D4	0.98	0.00	0.72	0.00	0.84	0.00	0.16	0.00	0.03	0.26	0.05	0.00	0.73	0.00	0.11	0.00
D0	D5	0.50	0.00	0.68	0.06	0.32	0.01	0.39	0.00	0.66	0.36	0.35	0.00	0.16	0.03	0.20	0.00
D1	D5	0.99	0.59	0.98	0.73	0.96	0.70	0.96	0.39	0.99	0.99	0.91	0.67	0.83	0.81	0.86	0.57
D2	D5	0.60	0.00	0.47	0.00	0.12	0.00	0.44	0.00	0.95	0.17	0.57	0.00	0.46	0.00	0.64	0.00
D3	D5	0.06	0.00	0.00	0.00	0.00	0.00	0.03	0.00	0.25	0.18	0.02	0.00	0.00	0.01	0.03	0.00
D4	D5	0.05	0.00	0.00	0.47	0.00	0.00	0.01	0.00	0.07	0.92	0.09	0.10	0.00	0.47	0.00	0.00
D5	D5	0.01	0.00	0.00	0.00	0.00	0.00	0.00	0.15	0.10	0.12	0.10	0.00	0.00	0.00	0.00	0.05
S5	D5	0.11	0.00	0.38	0.00	0.00	0.00	0.00	0.00	0.88	0.02	0.01	0.10	0.31	0.00	0.01	0.00
BC	D5	0.64	0.00	0.03	0.01	0.02	0.00	0.46	0.00	0.38	0.37	0.08	0.01	0.00	0.01	0.23	0.00
D0	S5	0.71	0.00	0.04	0.14	0.26	0.00	0.00	0.02	0.63	0.06	0.00	0.01	0.03	0.78	0.00	0.05
D1	S5	0.92	0.98	0.98	0.99	0.95	0.99	0.95	0.87	0.99	1.00	0.97	0.95	0.98	0.99	0.98	0.93
D2	S5	0.14	0.52	0.72	0.64	0.18	0.77	0.32	0.03	0.96	0.77	0.39	0.02	0.82	0.71	0.62	0.10
D3	S5	0.00	0.40	0.01	0.74	0.00	0.82	0.00	0.00	0.31	0.72	0.02	0.11	0.03	0.78	0.00	0.03
D4	S5	0.00	0.20	0.00	0.58	0.00	0.01	0.00	0.00	0.01	0.23	0.91	0.05	0.00	0.29	0.00	0.00
D5	S5	0.00	0.42	0.00	0.66	0.00	0.00	0.00	0.00	0.00	0.86	0.01	0.97	0.00	0.72	0.00	0.00
S5	S5	0.53	0.00	0.11	0.00	0.00	0.00	0.00	0.00	0.18	0.00	0.00	0.16	0.02	0.03	0.00	0.00
BC	S5	0.00	0.56	0.02	0.91	0.00	0.70	0.00	0.05	0.08	0.80	0.76	0.58	0.17	0.93	0.03	0.14
D0	BC	0.24	0.04	0.99	0.13	0.29	0.90	0.00	0.00	0.14	0.04	0.01	0.19	0.87	0.30	0.00	0.00
D1	BC	0.72	0.12	0.26	0.22	0.98	0.70	0.00	0.47	0.10	0.56	0.99	0.85	0.65	0.72	0.00	0.63
D2	BC	0.11	0.00	0.43	0.00	0.40	0.00	0.00	0.15	0.14	0.08	0.47	0.00	0.95	0.01	0.01	0.29
D3	BC	0.00	0.09	0.01	0.04	0.00	0.01	0.00	0.09	0.29	0.01	0.00	0.00	0.20	0.00	0.00	0.04
D4	BC	0.00	0.63	0.00	0.58	0.00	0.31	0.00	0.25	0.50	0.39	0.06	0.33	0.00	0.53	0.00	0.05
D5	BC	0.00	0.76	0.00	0.00	0.00	0.00	0.00	0.40	0.69	0.13	0.17	0.01	0.00	0.00	0.00	0.43
S5	BC	0.34	0.00	0.65	0.01	0.31	0.00	0.25	0.01	0.15	0.45	0.57	0.54	0.45	0.02	0.26	0.02
BC	BC	0.35	0.00	0.66	0.00	0.03	0.01	0.00	0.00	0.11	0.03	0.04	0.18	0.95	0.01	0.00	0.01

Significance level of 5%; p-values <0.05, Ho rejected.

Appendix: A

Matlab Code for estimation of time-frequency components of data series by using Wavelet Technique.

```
% Matlab Code for decomposition of a time series (matrix) by wavelet method%
 % 1)   we obtained business cycle(from 4-64 months frequency for monthly data)
 % 2)   Cyclical components (D2(4-8 months), D3(8-16 months, D4 (16-32 months) and D5 (32-64 months;
 % 3)   D1 is Noise part of the series
 % 4)   Trend component (above than 64 months frequency
 % 5)   we extend data from both ends with ARIMA model to handle end points biasness%
 % 6)   This code is for multiple time series
load data
siz=size(data);
n=siz(1,1);
nr=siz(1,2);
Final_Results=zeros(n,7*nr); % first seven series are the components related to series 1st in data;
% Next seven series for 2nd series in data and so on%
for ji=1:nr
y=data(:,ji); %origional data series
siz=size(y);
r=siz(1,1);
nn=siz(1,2);
cycles=zeros(r,5);
%J=2 for cycle of length 1-4 months, J=3 for cycle lengh 1-8 months.%
% J=6 for cycle length of 1-64 months. for monthly data J=5 finally.
for J=2:6
z=y;
siz=size(z);
t=siz(1,1);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%For wavelet%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%%%%%%%%%%%Following Portion for Lead lags values by AR model
yy=z(2:t);

%ttt=round(0.2*t);
ttt=20;
xxx= z(1:t-1);
[betahat,Ibeta,res,Ires,stats]=regress(yy,xxx,0.05);
se=(res'*res)/(t-1);
zza=zeros(t+2*ttt,1);
zza(ttt+1:end-ttt,:)=z;
for i=1:ttt
   zza(ttt-i+1,1)=(1/betahat)*zza(ttt-i+2,1);
end
for i=1:ttt
   zza(end-ttt+i,1)=betahat*zza(end-ttt+i-1,1);
end
z=zza;
siz=size(z);
t=siz(1,1);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%wavelet estimation%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%%Following is cyclical and trend part using wavelet Filter%%%%%%%%%%%%%%%%%%%%%%
```

```
  %%%%%%%%Daubechies Filter%%%%%%%%%%%%%%%
 a=[0.482962913
0.836516304
0.224143868
-0.129409523
];
  n=t;
  tt=t;
AA=zeros(n,n*J);
for j=1:J
A=zeros(n,n);
for i=1:n-3*j
    for jjt=1:3
        A(i,i)=a(1,1);
        A(jjt*j+i,i)=a(jjt+1,1);
    end
end
A(n-3*j+1:n, n-3*j+1:n)=A(n-3*j*2+1:n-3*j,n-3*j*2+1:n-3*j);
A(1:3*j, n-3*j+1:n)=A(n-3*j+1:n,n-3*j*2+1:n-3*j);
AA(:,j*n-n+1:j*n)=A;
end
za=z;

wt=zeros(tt,J);
for ij=1:J
wt(1:tt,ij)=AA(:,ij*n-n+1:ij*n)*za/sqrt(2);
%wc(1:tt,ij)=za-wt(1:tt,ij);
za=wt(1:tt,ij);
end
wtt1=transpose(AA(:,1:n))*wt(1:tt,1)/sqrt(2);
wcc1=z-wtt1;
wttt=wt(1:tt,J);
for ij=1:J
   wttend=transpose(AA(:,(J-ij+1)*n-n+1:(J-ij+1)*n))*wttt/sqrt(2);
   wttt=wttend;
end
wt=wttend;
wc=z-wt;
wccendminus=wc-wcc1;
wc=wccendminus;
wcc1=wcc1(ttt+1:end-ttt,1);
wt=wt(ttt+1:end-ttt,1);
wc=wc(ttt+1:end-ttt,1);
result=[y wt wc wcc1];
trend=wt;
cycles(:,J-1)=wc;
end
results1=[cycles(:,1) cycles(:,2)-cycles(:,1) cycles(:,3)-cycles(:,2)];
results2=[cycles(:,4)-cycles(:,3) cycles(:,5)-cycles(:,4)];
Results_single_series=[results1 results2 y-trend-cycles(:,1) trend];
% Results_single_series=[D1 D2 D3 D4 D5 {Business Cycle} Trend]
Final_Results(:,7*ji-6:7*ji)=Results_single_series;
end
```

Box 1: Summary of recent literature on determinants of inflation

No.	Author(s)	Theme	Data and Frequency	Methodology	Variables	Results /Remarks
1	Ubide (1997)	Determinants of Inflation in Mozambique	Monthly data for 1989:01-1996:12	OLS and VAR	CPI, Exchange rate, M2, Budget deficit, GDP growth, weather and political shocks	Monetary expansion, exchange rate and unpredictable events in the agricultural and politics, are responsible for inflation.
2	Gottschalk et al. (2008)	Determinants of Inflation in Sierra Leone	Monthly data for 2001:01 - 2007:12	VAR	CPI, international oil prices, exchange rate and reserve money	Higher oil prices, higher money supply, and nominal exchange rate depreciation, are responsible for inflation
3	Bashir et al. (2011)	Determinants of Inflation in Pakistan	Annual data for 1972 - 2010	Johansen Co-integration and VEC model	CPI, M2, GDP, imports, exports, government revenue and expenditures	Money supply, gross domestic product, imports and government expenditures cause inflation.
4	Kandil & Morsy (2009)	Determinants of Inflation in GCC	Annual data for 1970 - 2007	Co-integration and VEC model	CPI, trading partners CPI, exchange rate, government spending, and broad money	Inflation in major trading partners, oil revenues, and exchange rate depreciation cause inflation whereas, higher government spending and credit growth slow down inflation.
5	Domac & Elbirt (2013)	Determinants of inflation in Albania	Monthly data for 1993:01 - 1997:09	Univariate analysis, Co-integration and VEC model	CPI, CPI components, power consumption, base money, M1, M2, M3, government credit and bilateral exchange rate of major trading economies	Inflation exhibits strong seasonal patterns owing to agriculture seasonality. Credit to government, M2, cause inflation. Bi-directional causality exists between inflation and credit to government and rent, water, fuel, and power component.
6	Khan & Gill (2010)	Determinants of inflation in Pakistan	Annual data for 1970 - 2007	Co-integration technique	CPI, WPI, SPI, and GDP Deflator, exchange rate, imports, M2, budget deficit and support prices of sugarcane, rice, wheat, and cotton	The procurement prices and administered prices along with imported inflation have contributed to higher inflation. . Budget deficit, depreciation of exchange rate and increase in the value of imports has contributed shooting up of CPI, WPI, SPI and GDP deflator. The support prices of sugarcane, rice, wheat, and cotton also affect inflation.
7	Mohanty and John (2015)	Determinants of inflation in India	Annual data for 1996 - 2014	SVAR model	CPI, crude oil prices, output gap, fiscal policy and monetary policy	Influence of monetary policy on inflation remained almost steady during the study period. Output gap had an asymmetric impact on inflation with its influence having weakened in the recent period. Crude oil price was the predominant driver of inflation during 2009 - 2011. Fiscal deficit was a key determinant of inflation in 2011 - 2012.
8	Al-Mutairi et al. (2020)	Determinants of inflation in Kuwait	Annual data for 1979 - 2015	Multiple linear regression analysis	CPI, M2, GDP, exchange rate, interest rate, taxation, imports, current	Change in CPI is positively influenced by change in interest rate spreads, imports of goods and services and money supply

No.	Author(s)	Theme	Data and Frequency	Methodology	Variables	Results /Remarks
					account and unemployment	whereas it is negatively influenced by the changes in tax revenue and current account balance.
9	Dua & Goel (2021)	Determinants of inflation in India	Monthly data for 1996:04 - 2017:03	Co-integration approach	CPI, expected inflation, output gap, rate of growth of money supply, exchange rate, interest rate, fiscal deficit, minimum support prices, rainfall international oil and food prices	Expected inflation, exchange rate, rate of growth of money supply, output gap and interest rates are the major determinants of inflation. On the supply side global factors like international oil and food prices play a key role in determining both overall and food inflation.
10	Basher& El samadisy (2012)	Determinants of inflation in the Gulf Arab states	Annual data for 1980 - 2008	Co-integration approach	CPI, M2, non-hydrocarbon output, foreign prices and the nominal effective exchange rate	Inflation is determined by money supply, non-hydrocarbon output, foreign prices and the nominal effective exchange rate.
11	Oppong et al. (2015)	Determinants of inflation in Ghana	Monthly data for 2000:01 - 2014:12	OLS estimates	CPI, crude oil price at the world market, exchange rate, and electioneering spillover quaternary effects (ESQE)	Crude oil price at the world market, exchange rate, and ESQE are key determinants of inflation.
12	Bashir et al. (2016)	Determinants of inflation in Pakistan	Annual data for 1972 - 2014	ARDL model	Wholesale price index, exports, imports, roads, population, M2, government expenditure, government revenue, electricity generation, FDI, industrial sector output, and external debt	The demand side factors of inflation are population, roads and government expenditure while supply side factors are imports, government revenue, electricity generation and external debt. In the long run, inflation is caused by roads, government expenditure, imports, government revenue and external debt. There is decline in price level due to foreign direct investment, electricity generation and population in long run
13	Bane (2018)	Determinants of inflation in Ethiopia	Annual data for 1975 - 2015	ARDL model	CPI, government expenses, GDP, money supply, bank credit, and agriculture output	Both monetary sector and structural factors determine inflation wherein credit and money printing; government spending and the real interest rate are more potent sources.
14	Narayan et al. (2011)	Determinants of inflation in 54 developing economies	Annual data for 1995 - 2004	System GMM estimator	CPI, Remittances, trade openness, debt, current account deficits, the agricultural sector, and USA interest rate, and democracy	Remittances, trade openness, debt, current account deficits, the agricultural sector, and USA interest rate have a positive effect on inflation; whereas, improvements in democracy reduce inflation.
15	Kabundi (2012)	Determinants of inflation in Uganda	Monthly data for 1999:01 - 2011:10	ECM estimates	CPI, world food prices, domestic food prices, domestic demand and supply and demand,	Over the long run, monetary aggregate, world food prices, and domestic supply and demand effects in agricultural sector are main determinants of

No.	Author(s)	Theme	Data and Frequency	Methodology	Variables	Results /Remarks
					agriculture output, money supply, and energy supply	inflation in Uganda. While money growth, world food prices, and energy prices, combined with domestic food prices have short-term impact on inflation.
16	Ahmed et al. (2014)	Determinants of inflation in Pakistan	Annual data for 1973 - 2013	Co-integration technique	CPI, exchange rate, government borrowing, non-government borrowing, real GNP, indirect taxes, growth rate of money supply, import price index, real demand relative to real supply, wheat support price and money supply	Variable are co0integrating and exchange rate is the most significant factor of inflation.
17	Khan & Qasim (1996)	Determinants of inflation in Pakistan	Annual data for 1972 - 1994	OLS estimates	CPI, CPI food, CPI non-food, money supply, exchange rate, domestic economic activity, interest rate, support prices, agriculture prices, and import price index	Increase in money supply and exchange rate depreciation cause inflation. The supply side variables are important to put downward pressure on price level. Food inflation is co-integrating with money supply, value added in agriculture, and support price of wheat. Whereas, money supply, GDP, electricity price and import price cause non-food inflation.
18	Cottarelli et al. (1998)	Non-monetary determinants of inflation in 47 industrial and transition economies	Annual data for 1993 - 1996	Panel-data econometric techniques (GMM and SURE)	Fiscal balance, the degree of development of the government securities market, and the unemployment rate, private sector share in GDP, price liberalization, trade liberalization, central bank independence and exchange rate pegging	Relative price changes, central bank independence, the exchange rate regime, and the degree of price liberalization cause inflation. Structural factors, such as those influencing the natural rate of unemployment, have a limited effect on inflation.
19	Kim (2001)	Determinants of inflation in Poland	Annual data for 1991 - 1999	Co-integration and VEC model	Inflation, money supply, labor, wages, foreign price, exchange rate, real output, and price of tradeable and non-tradeable goods	Labor and external sector determines the inflation. Exchange rate appreciation and excessive increase in wages sustained inflation, whereas, monetary sector appears indifferent to inflation.
20	Akinboade et al. (2004)	Determinants of inflation in South Africa	Quarterly data for 1970:1 - 2000:2	Co-integration and VEC model	Inflation, money market, labor market and foreign exchange market conditions related variables	Inflation is largely a structural phenomenon. In the short run, labor costs, and broad money supply cause domestic inflation whereas, increase in the nominal effective exchange rate will lower inflation. In the long run, rising labor costs and money

No.	Author(s)	Theme	Data and Frequency	Methodology	Variables	Results /Remarks
						supply contribute significantly to inflation.
21	Altissimo et al. (2005)	Long-run determinants of inflation in EU	Monthly data for 1990:01 - 2004:02	Agent based model (DSGE)	Parameters include discount factor; risk aversion; elasticity of substitution between traded, and non-traded goods, and between domestic and foreign traded goods; substitution across goods within a sector; share of traded goods; inverse of the Frisch elasticity of labor supply; labor share, and curvature of the production function	Relative variations in productivity in the non-traded sector are the cause of price and inflation differentials, with shocks to productivity in the traded sector being largely absorbed by movements in the terms of trade in the regional economies.
22	Khan et al. (2007)	Determinants of inflation in Pakistan	Annual data for 1972 - 2005	OLS estimates	CPI, bank credit, real demand, real supply, price of imports, exchange rate, taxes to manufacturing sector value added, and lagged CPI	Important determinants of inflation in 2005-06 were adaptive expectations, private sector credit and rising import prices.
23	Adigozalov (2009)	Determinants of inflation in Azerbaijan	Annual data for 2006 - 2008	OLS estimates	CPI, money supply, exchange rate, inflation in major trade partners, trade partner’s weight in overall trade turnover, value added by oil sector, value added by non-oil, average interest rate on deposits and average interest rate on credits	Inflation is the most relevant external shock. In addition, oil revenues have reinforced inflationary pressures through growth of credit and aggregate spending. In the short-run, binding capacity constraints also explain higher inflation given increased government spending.
24	Andersson et al. (2009)	Determinants of inflation in EU	Annual data for 1999 - 2006	GMM and OLS estimates	Inflation, inflation persistence, nominal effective exchange rate, output gap, fiscal position, and demand- and supply-shocks	Inflation differentials are primarily determined by cyclical positions and inflation Persistence that appears to be partly explained by administered prices and to some extent by product market regulations. Price level in each EU country is governed by GDP per capita.
25	Jaradat (2011)	Determinants of inflation in Jordan	Quarterly data for 2000:1 - 2010:3	ECM	Inflation, imports, exports, GDP, M2, banking facilities, workers' remittances, and external shocks	Inflation is caused by imported inflation, national exports, credit facilities, staff transfers and external shocks whereas; GDP and money supply don’t have significant impact on inflation.
26	Nguyen et al. (2012)	Determinants of inflation in Vietnam	Annual data for 2001 - 2009	OLS and causality estimates	Inflation, money supply, exchange rate, prices of tradeable and of non-	Oil prices and rice prices have strongest influences on CPI inflation.

No.	Author(s)	Theme	Data and Frequency	Methodology	Variables	Results /Remarks
					tradeable goods, oil prices and rice prices	
27	Asghar et al. (2013)	Determinants of inflation in Pakistan	Annual data for 1972 - 2010	VAR, OLS and ARDL estimates	Inflation, exchange rate, output gap,money supply, lagged inflation, foreign inflation and dummy variable for global financial crises of 2008	The long-run money supply growth, lagged inflation, foreign inflation and global financial crises have positive and significant impact on inflation whereas; money supply becomes insignificant in the short-run.
28	Morley et al. (2015)	Determinants of inflation in G7 countries	Quarterly data for 1957:2 - 2010:2	Bayesian techniques	Inflation, unemployment, inflation gap and output gap	Trend inflation and the inflation gap have been consistent and substantial determinants of inflation; whereas, real-activity gap explains a large fraction of the variation in the inflation gap for each country.
29	Deniz et al. (2016)	Determinants of inflation in emerging and industrial economies	Annual data for 2002-2012	Panel data estimators	Inflation, money growth, real effective exchange rate, budget balance, GDP growth, real wages and output gap	Real effective exchange rate has a more negative impact on inflation in emerging- than industrialized-economies. Money growth rate contributes to inflation in emerging economies only. Real wage has a positive impact in emerging economies, whereas the impact is negative for industrialized economies. Budget balance has a negative impact on inflation however; it has a positive impact in non-inflation targeting countries.
30	Boujelbene & Boujelbene (2010)	Determinants of inflation in Tunisia	Annual data for 1962 - 2003	Co-integration technique	Inflation, expected inflation, wage rate, GDP, M2, and imports prices	Inflation is explained by mixed factors: monetary ones such as money supply, the interest rate and the real effective exchange rate; and structural ones like the nominal average annual wage rate, the import prices and the real output.
Time-frequency based view on determinants of inflation						
No.	**Author(s)**	**Theme**	**Data and Frequency**	**Methodology**	**Variables**	**Results /Remarks**
1	Rua (2012)	Time-frequency analysis of Money Growth and Inflation in the Euro Area	Monthly data for 1970:01 - 2007:12	Wavelet analysis	Inflation and M3 growth	Stronger link between inflation and money growth at low frequencies. At the typical cycle frequency range, the link is only present until the beginning of the 1980s.
2	Dar et Al. (2014)	Time and frequency dependent relationship between industrial	Monthly data for 1992:01 - 2011:06	Wavelet analysis	CPI, whole sale price index (WPI) and index of industrial production (IIP)	Industrial growth and inflation share an anti-cyclical relationship corresponding to the frequencies of 8 - 16 months. Inflation leads the

No.	Author(s)	Theme	Data and Frequency	Methodology	Variables	Results /Remarks
		growth and inflation in India				industrial growth at lower frequencies.
3	Hanus & Vacha (2018)	Time-frequency response analysis of monetary policy transmission	Quarterly data for 1980:1 - 2016:4	Quantile and expectile regression analysis	Domestic and foreign inflation gap, and domestic and foreign output gap	Both the domestic and foreign output gap are significant drivers of inflation whereas, the first has a bigger influence in the right tail of the conditional distribution of inflation. There is an increase in the response of inflation to the domestic gap in the last decade but only at the lower quantiles, which means time varying response exists.
4	Gallegati et al. (2019)	Evidence on high inflation episodes for developed countries	Annual data for 1871 - 2013	Wavelet analysis	Inflation and money supply	There is a close stable relationship between excess money growth and inflation only over longer time horizons, i.e. periods greater than 16 and 24 years, with money growth mostly leading it. In addition, inflationary upsurges affect regression coefficients but not the closeness of the long-run relationship.
5	Assenmacher-Wesche & Gerlach (2006)	Euro area inflation at high and low frequencies	Quarterly data for 1970:2 - 2004:4	Phillips spectral estimator	Inflation, growth rate of money and real output, interest rate, and the rate of change of velocity	Variations in inflation are well explained by low-frequency movements of money and real income growth and high-frequency fluctuations of the output gap.
6	Ryczkowski (2021)	Money and inflation in inflation-targeting regimes	Quarterly data for 1990:1 1 2017:1	Band-pass filtering techniques and wavelet analysis	Inflation, money supply, real output, and velocity of money	According to the band-pass filtering techniques, the link between money growth and inflation was weak and statistically non-significant over the investigated period: from 1990 onwards. The wavelet analysis demonstrated significant causality running from money growth to inflation, and strong significant co-movements between the two variables around the Great Recession at a typical cycle frequency.