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A Model for Forecasting and Policy Analysis in Pakistan: The Role of Government and External Sectors

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Abstract

This paper contributes to the development of the next generation of Forecasting and Policy Analysis System (FPAS) by formulating, estimating and conducting a forecast evaluation of a New Keynesian Dynamic Stochastic General Equilibrium (DSGE) model customized for Pakistan. The DSGE model in this paper contributes through addition of the detailed fiscal and external sectors of the economy. The fiscal block models the behavior of government expenditures, tax revenues and government debt, and allows for government borrowing from the central bank that affects monetary growth. Key additions to a conventional external sector block include the introduction of transaction costs in international borrowing and lending, which weaken the link between returns on domestic assets and the exchange rate adjusted returns on foreign assets. Further, to analyze the dynamics of major components of inflation, CPI is disaggregated in three components: core, food and oil inflation. Application of the model for alternative macroeconomic scenario assessment and forecasting is conducted likewise, especially regarding monetary policy formulation in Pakistan. Forecasts of major macroeconomic variables from this DSGE model are compared with those obtained using a comparable Bayesian VAR model and a DSGE-VAR model. New FPAS outperforms both Bayesian VAR and older generation FPAS model, as shown by the in-sample projection comparison regarding GDP growth, nominal interest rate and CPI, in a rolling window forecasting setup. In addition, the forecast efficacy of both DSGE VAR model and new FPAS model is quite comparable as they yield similar results.

JEL Classification: C53, D5, E5, E17.
Key Words: Monetary Policy, DSGE Model, Pakistan, Forecasting.

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Non-technical Summary

Dynamic Stochastic General Equilibrium (DSGE) framework is now explicitly recognized as a useful tool of monetary policy analysis at the State Bank of Pakistan (SBP). A basic DSGE model called the Forecasting and Policy Analysis System (FPAS) has been approved for use in the deliberations of SBP monetary policy committee. This model is a small-scale linear DSGE model of a small open economy and is based on a basic framework utilized by many central banks and IMF. The present project has developed the next generation of FPAS models to meet the needs of SBP.

One key objective of the project was to extend the current FPAS model to add a fiscal block and revise the external sector block. These extensions are important because fiscal policy exerts an important influence on the formulation of monetary policy in Pakistan and the external sector needs to be developed further to account for a lack of integration of financial markets in Pakistan with global markets. We developed a model with these extensions with strong microeconomic foundations. Although the current FPAS model is motivated by DSGE models with microeconomic foundations, its equations are not explicitly derived from an optimization framework. Our starting point was to develop a micro-founded basic model, which was comparable to the current FPAS model. We then derived a general model by extending the basic model to include a new fiscal block and a revised external block. The new fiscal block models the behavior of government expenditures, tax revenues and government debt, and allows for government borrowing from SBP which affects money growth. The external sector is revised to introduce transaction costs in international borrowing and lending, which weaken the link between the return on domestic assets and the exchange rate adjusted return on foreign assets. To examine the behavior of the major components of CPI, the general model also distinguished three sectors: core products, food and oil.

Behavioral parameters of the current FPAS model have been calibrated using judgment and results from various studies. To improve the fit of the model to data and its forecast performance, we estimated our model parameters employing widely-used Bayesian techniques. We estimated both the basic and the general models using their linearized versions and quarterly data from 2001Q3 to 2015Q4. As data for real GDP are available only on an annual basis, statistical interpolation methods (using information on related indicators at higher frequency) were utilized to construct a quarterly GDP series from annual data. Estimation of the basic model used data for 4 home (real GDP, CPI inflation rate, Treasury Bill rate and exchange rate depreciation) and 3 foreign variables (world real GDP, US CPI inflation rate and US Treasury Bill rate). The general model was estimated using additional data for 3 fiscal block variables (real government expenditures, real tax revenues and money growth rate) and 5 multi-sector block variables (core, food and oil inflation rates, and relative world prices of food and oil).

In Bayesian estimation of the models, the prior values of model parameters were chosen based on values suggested by the current FPAS model and other sources. Posterior estimation of parameters for the two models produced a number of interesting results. Estimates of the habit parameter suggest a significant role for both the forward- and backward-looking components in aggregate demand. Estimated values for indexation parameter imply that current inflation responds more to the expected value of future inflation than to past inflation. Estimation of the monetary policy parameters indicates significant interest rate smoothing and a moderately strong interest rate reaction to inflation. Estimate of the Calvo parameter in the basic model suggests greater flexibility of prices in Pakistan than developed economies, but there are significant differences in the estimates of the Calvo parameter across sectors in the general model.

The next generation FPAS models developed in this project are expected to be used for forecasting macroeconomic variables for monetary policy deliberations. Thus it is important to examine if they improve the forecast performance of the current FPAS model or other potential models that could be used for forecasting. For forecast comparisons, we considered Bayesian vector autoregressions (BVARs), which are empirical models that are widely used for forecasting. We also considered a hybrid model (DSGE-VAR) that combines the forecasts of theoretical DSGE and empirical VAR models. One standard test of forecast accuracy is based on root mean square error (RMSE) of the forecast. For each model, we calculated RMSE of forecast for a horizon of upto 8 quarters using a 20-quarter rolling window (the 20-quarter period is moved ahead, one quarter at a time) for estimation.

Inflation projections have been derived for the current FPAS model and were found to be superior to the best combination of projections available from econometric models, especially for normal or moderate inflation periods. We compared these forecasts of the current FPAS model with those of our basic model (which is comparable to the FPAS model) and found that based on the RMSE test, the basic model performs better at all forecast horizons. We also made forecasts comparisons with BVAR and DSGE-VAR models for real GDP, CPI inflation and the nominal interest rate. Our model performed better (had lower RMSE) than the Bayesian VAR for each variable at all forecast horizons. The hybrid DSGE-VAR model attempts to improve the forecast performance by combining the forecasts of our DSGE model and the VAR model. The performance of the hybrid model is close to our model: it performed marginally better in forecasting interest rates and inflation but slightly worse in predicting output growth. Thus it did not contribute much to improving the overall forecasting ability of our model. We also undertook additional tests of relative forecast accuracy of our model and these tests also produced similar results.

1. Introduction

Dynamic Stochastic General Equilibrium (DSGE) framework is now explicitly recognized as a useful tool of monetary policy analysis at the State Bank of Pakistan[6] (SBP). A basic DSGE model called the Forecasting and Policy Analysis System (FPAS) has been approved for use in the deliberation of SBP monetary policy committee. This model is a small-scale linear DSGE model of a small open economy (Ahmad and Pasha, 2015), and is based on a basic framework utilized by many central banks and IMF (e.g., see Berg et al., 2006). The present project has developed the next generation of FPAS models to meet the needs of SBP. The work on this project was divided into three phases.

The first phase developed the theoeretical framework required to improve and extend the model further into meaningful directions so that additional relevant policy issues and more macroeconomic variables could be addressed and forecasted respectively. The key extensions are revising the external sector block, adding a government sector block, and expanding aggregate supply block. These extensions are important because the external sector needs to be developed further to account for a lack of integration of financial markets in Pakistan with global markets, fiscal policy exerts an important influence on the formulation of monetary policy in Pakistan, and there is interest in understanding the behavior of different components of CPI inflation. DSGE models typically have strong microeconomic foundations. The current FPAS model is based on micro-founded models, but its equations are not explicitly derived from micro foundations. To develop the next generation of FPAS models, the project first developed a micro-founded counterpart to the current model, and then used this counterpart as a starting point for revising existing blocks and adding new ones. In addition to incorporating the needed revisions and extensions, the first phase also derived linear versions of the model, which are suitable for estimation.

In the second phase, the next generation versions of FPAS were estimated using Bayesian techniques (e.g., see An and Schorfheide, 2007), which is the method that is now generally preferred for estimating DSGE models. One challenge for estimation is that time series data for Pakistan for key macro variables such as GDP is not available and, only available from 2003 onwards for government expenditures and tax revenues at quarterly frequencty (the typical time unit used in DSGE models). To address this data problem, quarterly series of these variables were estimated by interpolating annual data and utilizing certain proxies for the variables available at higher frequency. Parameters of the current version of the FPAS model have not been estimated but calibrated using information from various studies and judgement. In the Bayesian estimation process, we utilized these values and other available information in choosing priors, and then combined prior information with time series data to identify model parameters. This process provides a closer fit of the model to data and is expected to improve the forecasting performance of the model.

The third and final phase of the project was concerned with evaluating the performance of the estimated next generation models in fitting the data and forecasting important macro variables of interest. Several tests were used to assess model performance.

The work for each phase is discussed in the next three sections. Section 2 discusses the the theoretical structure of the next generation FPAS models, which we have deloped in module form. We first discuss (in sub-section 2.1) the basic model, which is designed to be a micro-founded counterpart to the current FPAS model. In this model we also introduce frictions to allow for a lack of integration between domestic and

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[6] State Bank of Pakistan is the central bank of Pakistan.

global financial markets. We then add the government block to the basic model in sub-section 2.2. This extended model is useful in examining the implications of fiscal behavior for monetary policy and forecasting the movements of fiscal variables. The model is further extended (in subsection 2.3) to expand the aggrgate supply block to meet the need for forecasting and analyzing not only the headline inflation, but also core, food and oil inflation.

Section 3 discusses the estimation of next generation FPAS models. In this section, we focus on the estimation of the basic model and a general version that includes both government and multiple sectors. We first discuss (in sub-section 3.1) the data used for estimating the model. Then we describe (in sub-section 3.2) our calibration and selection of priors for Bayesian estimation. The results for both the basic and the general model are reported in sub-section 3.3. Section 4 evaluates the properties and forecasting performance of estimated models. Section 4.1 analyzes impulse response functions of basic version and multi-sector models. Section 4.2 focuses upon forecasting performance of the new FPAS model in comparison with the current FPAS (Ahmad and Pasha, 2015) and the widely used Bayesian VAR and DSGE-VAR models.

2. Theoretical Structure

2.1. Basic Model

The basic model assumes a micro-founded framework that yields linear relations which are similar to those for FPAS. It is based on a simple DSGE model for a small open economy developed by Gali and Monacelli (2005). As in this model, there is one good (consisting of differentiated home and foreign varieties) and one factor (labor). Wages are flexible, but prices are set as in the Calvo model. Money and government are not explicitly modeled. Foreign economy is large. We introduce a number of variations to the Gali-Monacelli model. Instead of complete asset markets, we assume that household's international financial transactions consist of buying and selling of foreign bonds subject to transaction costs. We also allow for departures from the law of one price for imports along the lines of Monacelli (2005).We also incorporate habit formation in consumption and partial indexation to inflation in the Calvo price setting as in Justiniano and Preston (2010) and Rudolf and Zurlinden (2014).

2.1.1. Households

The Utility function for the representative household is

$$U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} X_{H,t} \left(\frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma} - \frac{\chi_N N_t^{1+\nu}}{1+\nu} \right),$$

(1)

where, C_t and N_t are the household's aggregate consumption and labor supply, and $X_{H,t}$ is a shock to household preferences. Parameters β, h, σ and ν represent, respectively, the discount factor, the habit persistence index, the inverse elasticity of intertemporal substitution in consumption and, inverse elasticity labor supply.

The consumption aggregate is given by

$$C_t = \left[(1-\alpha)^{1/\eta} (C_{H,t})^{(\eta-1)/\eta} + \alpha^{1/\eta} (C_{F,t})^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)}, \tag{2}$$

where, $C_{H,t} = \left(\int_0^1 C_{H,t}(i)^{(\varepsilon-1)/\varepsilon} di \right)^{\varepsilon/(\varepsilon-1)}$ and $C_{F,t} = \left(\int_0^1 C_{F,t}(i)^{(\varepsilon-1)/\varepsilon} di \right)^{\varepsilon/(\varepsilon-1)}$ are the bundles of home and

foreign varieties (indexed by i). The elasticity of substitution between the two bundles, η, is assumed to be different than the elasticity between varieties, ε. The demand functions for the domestic and imported bundles are given by

$$C_{H,t} = (1-\alpha) \left(\frac{P_{H,t}}{P_t} \right)^{-\eta} C_t, \quad C_{F,t} = \alpha \left(\frac{P_{F,t}}{P_t} \right)^{-\eta} C_t, \tag{3}$$

$$P_t = \left[(1-\alpha) P_{H,t}^{1-\eta} + \alpha P_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}, \tag{4}$$

where, P_t is the price of aggregate consumption, and $P_{H,t}$ and $P_{F,t}$ are the price indexes for domestic and

foreign bundles given by $P_{H,t} = \left(\int_0^1 P_{H,t}(i)^{1-\varepsilon} di \right)^{1/(1-\varepsilon)}$ and, $P_{F,t} = \left(\int_0^1 P_{F,t}(i)^{1-\varepsilon} di \right)^{1/(1-\varepsilon)}$. Using a bar over

a variable to denote steady state values, we normalize these prices as $\bar{P}_{H,t} = \bar{P}_{F,t} = \bar{P}_t = 1$, so that α represents the share of foreign goods in consumption. Analogous relations hold in the foreign large economy (treated as closed) with an asterisk used to denote foreign variables and parameters.

The household budget constraint is

$$P_t C_t + P_t B_t + S_t P_t^* B_t^* = R_{t-1} P_t B_{t-1} + R_{t-1}^* TC_{t-1} S_t P_t^* B_{t-1}^* + W_t N_t + PR_t, \tag{5}$$

Where, B_t and B_t^* are household's holdings of real domestic and foreign bonds (in terms of each country's

aggregate price level); R_t and R_t^* are the gross home and foreign interest rates; S_t is the nominal exchange rate; W_t is the nominal wage rate; PR_t represents nominal profits distributed to households; and TC_t denotes the cost for transactions in foreign bonds. We assume that the transaction cost is a function of real value of foreign bonds as follows:

$$TC_t = e^{-\psi_1 B_t^*} X_{TC,t}, \tag{6}$$

Where, $\psi_1 > 0$ and $X_{TC,t}$ is a transaction cost shock. Note that for $X_{TC,t} = 1$, $TC_t = 1$ for $B_t^* = 0$,

$TC_t < 1$ for $B_t^* > 0$ (international lending), and $TC_t > 1$ for $B_t^* < 0$ (international borrowing). The transaction costs for foreign bonds can be viewed broadly as also including risk premium. It has been suggested that risk premium is negatively correlated with the expected change in the exchange rate (forward premium puzzle). To address this issue, Adolfson et al. (2008) let the risk premium depend on the expected

change in the exchange rate between $t+1$ and $t-1$. We can incorporate this effect by modifying the above transaction cost function as

$$TC_t = e^{-\psi_1 B_t^* - \psi_2 (\frac{E_t S_{t+1}}{S_{t-1}} - 1)} X_{TC,t},$$ (7)

with $0 < \psi_2 < 1$.

Optimization of (1) subject to the budget constraint (5) yields

$$\chi_N (C_t - hC_{t-1})^\sigma N_t^\nu = \frac{W_t}{P_t},$$ (8)

$$\frac{\beta E_t X_{H,t+1} P_t}{X_{H,t} E_t (P_{t+1})} \left(\frac{(E_t C_{t+1} - hC_t)}{(C_t - hC_{t-1})} \right)^{-\sigma} = \frac{1}{R_t},$$ (9)

$$R_t = \frac{E_t S_{t+1} R_t^* TC_t}{S_t}.$$ (10)

Similar conditions hold for the foreign economy. We assume that $\beta = \beta^*$. Under this assumption, $\bar{R} = \bar{R}^*$ and $\overline{TC} = 1$ according to (9), its foreign counterpart and (10). Then (6) or (7) imply that $\bar{B}^* = 0$. Thus in steady state, there are zero foreign assets and trade is balanced.

2.1.2. Firms

There are two types of monopolistically-competitive firms: a continuum of producers of varieties of the home good, and a continuum of retailers of imports who convert import bundles into varieties of the foreign good. The production function of a producer i is $Y_t(i) = X_{Y,t} N_t(i)$, where $X_{Y,t}$ represents a common productivity shock. The nominal marginal costs for the producers is

$$MC_{H,t} = \frac{W_t}{X_{Y,t}}.$$ (11)

The retail activity is assumed to be costless for simplicity. The nominal marginal cost for retailers is thus the cost of the import bundle:

$$MC_{F,t} = S_t P_{F,t}^*.$$ (12)

The prices for both the home good and imports in the domestic market are set according to the Calvo mechanism, modified to allow for partial indexation to inflation. For $j = H, F$, let $1 - \theta$ be the probability that a firms sets a new optimal price, $\bar{P}_{j,t}(i)$, in period t. The firms that do not reoptimize,

simply index their price to past inflation as $P_{j,t}(i) = P_{j,t-1}(i)\left(\dfrac{P_{j,t-1}}{P_{j,t-2}}\right)^{\kappa}$, where $0 \le \kappa \le 1$ is the indexation parameter. The new optimal price is set to maximize expected profits

$$\sum_{k=0}^{\infty} \theta^k E_t \left[DR_{t,t+k} D_{j,t+k}(i) \left(\bar{P}_{j,t}(i) \left(\frac{P_{j,t+k-1}}{P_{j,t-1}} \right)^{\kappa} - MC_{t+k} \right) \right], j = H, F, \text{ where}$$

$DR_{t,t+k} = \beta^k (X_{H,t} C_t / X_{H,t+k} C_{t+k})^{\sigma} (P_t / P_{t+k})$ is the stochastic discount rate, and

$$D_{H,t+k}(i) = \left[\frac{\bar{P}_{H,t}(i)}{P_{H,t+k}} \left(\frac{P_{H,t+k-1}}{P_{H,t-1}} \right)^{\kappa} \right]^{-\varepsilon} (C_{H,t+k} + C^*_{H,t+k}) \text{ is the demand for a producer while}$$

$$D_{F,t+k}(i) = \left[\frac{\bar{P}_{F,t}(i)}{P_{F,t+k}} \left(\frac{P_{F,t+k-1}}{P_{F,t-1}} \right)^{\kappa} \right]^{-\varepsilon} C_{F,t+k} \text{ is the demand for a retailer. The optimal condition for setting}$$

the new price is

$$\sum_{k=0}^{\infty} \theta^k E_t \left[DR_{t,t+k} D_{j,t+k}(i) \left(\bar{P}_{j,t}(i) \left(\frac{P_{j,t+k-1}}{P_{j,t-1}} \right)^{\kappa} - \frac{\varepsilon}{\varepsilon - 1} MC_{j,t+k} \right) \right] = 0, j = H, F. \tag{13}$$

Since prices are symmetric across reoptimizers and other firms in each period, we also have

$$P_{j,t} = \left[\theta \left(P_{j,t-1} \left(P_{j,t-1} / P_{j,t-2} \right)^{\kappa} \right)^{1-\varepsilon} + (1-\theta)(\bar{P}_{j,t})^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}, j = H, F, \tag{14}$$

where, $\bar{P}_{j,t}$ is the common new optimal price.

2.1.3. Equilibrium

Define aggregate indexes of output and employment as $Y_t \equiv \left(\int_0^1 Y_t(i)^{(\varepsilon-1)/\varepsilon} di \right)^{\varepsilon/(\varepsilon-1)}$ and $N_t \equiv \int_0^1 N_t(i)\, di$. Using the firm production function, aggregate employment can be related to aggregate output as

$$N_t = \frac{Y_t}{X_{Y,t}} \xi_t, \tag{15}$$

where, $\xi_t \equiv \int_0^1 \dfrac{Y_t(i)}{Y_t}$. Since output of a firm producing a variety of the home good equals home and foreign demand for its variety, we have $Y_t(i) = C_{H,t}(i) + C^*_{H,t}(i) = \left(\dfrac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} \left(C_{H,t} + C^*_{H,t} \right)$. Define the real

exchange rate as $Z_t \equiv SP_t^* / P_t$, Using (3), its foreign counterpart, and the definition of aggregate output, and noting that $\frac{P_{H,t}}{S_t P_t^*} = \frac{P_{H,t}}{P_t Z_t}$, we obtain

$$Y_t = C_{H,t} + C_{H,t}^* = \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} \left[(1-\alpha)C_t + \alpha^* Z_t^\eta C_t^*\right]. \tag{16}$$

Since the large foreign economy is treated as closed, $C^* = Y^*$.

Noting that $B_t = B_{t-1} = 0$ in symmetric household equilibrium, we can rearrange the budget constraint (5) to derive the following relation determining the evolution of foreign bonds:

$$B_t^* = TC_{t-1} R_{t-1}^* B_{t-1}^* + (HI_t - C_t)/Z_t, \tag{17}$$

where, $HI_t = (W_t N_t + PR_t)/P_t$ is the real household income. Letting $PR_{H,t}$ and $PR_{F,t}$ denote profits for producing and retailing firms, we have $PR_t = PR_{H,t} + PR_{F,t}$. Since $Y_t = (W_t N_t + PR_{H,t})/P_{H,t}$, $PR_{F,t} = (P_{F,t} - S_t P_{F,t}^*)C_{F,t}$ and $P_F^* = P_t^*$ for a large foreign economy, we can express

$$HI_t = \frac{P_{H,t}}{P_t} Y_t + (\frac{P_{F,t}}{P_t} - Z_t)C_{F,t}. \tag{18}$$

The model is completed by adding a monetary policy rule. We assume a simple rule which targets inflation and smoothes interest rate movements:

$$R_t = \frac{1}{\beta}\left(R_{t-1}\right)^{\delta_R} \left(E_t \Pi_{t+1}\right)^{(1-\delta_R)\delta_\pi} X_{R,t}, \tag{19}$$

where, $\Pi_{t+1} \equiv \frac{P_{t+1}}{P_t}$, $0 \leq \delta_R < 1$, $\delta_\pi > 1$, and $X_{R,t}$ is a monetary policy shock.

2.1.4. Linearized Model

To obtain a linearized version of the model, we derive first-order approximations of log deviations around steady state values, and use lower case letters to denote the log deviations. Linearizing (8)-(10), we have

$$rw_t = \frac{\sigma}{1-h}c_t - \frac{\sigma h}{1-h}c_{t-1} + \nu n_t, \tag{20}$$

$$c_t = \frac{1}{1+h}E_t(c_{t+1}) + \frac{h}{1+h}c_{t-1} - \frac{1-h}{\sigma(1+h)}\left[r_t - E_t(\pi_{t+1}) + x_{H,t} - E_t(x_{H,t+1})\right], \tag{21}$$

$$r_t = E_t \Delta s_{t+1} + r_t^* + tc_t \tag{22}$$

Where, $rw_t \equiv w_t - p_t$, $\pi_t \equiv p_t - p_{t-1}$, $\Delta s_t \equiv s_t - s_{t-1} = z_t - z_{t-1} + \pi_t - \pi_t^*$.

Next, letting $\dot{B}_t^*$ denote change in foreign assets per unit of time, and noting that $\bar{B}^* = 0$, we linearize (7) as

$$tc_t = -\psi_1 \dot{B}_t^* - \psi_2 (\Delta s_{t+1} + \Delta s_t) + x_{TC,t} , \tag{23}$$

Setting $\psi_2 = 0$ in (23), we can obtain the linear form of the simpler transaction cost function (6).

Letting $\bar{P}_{j,t}(i)$ in (13) equal to the common price $\bar{P}_{j,t}$, and using linearized versions of (13) and (14), we can derive the following relations:

$$\pi_{j,t} = \frac{\beta}{1 + \beta\kappa} E_t(\pi_{j,t+1}) + \frac{\kappa}{1 + \beta\kappa} \pi_{j,t-1} + \frac{(1-\theta_j)(1-\beta\theta_j)}{\theta_j(1+\beta\kappa)} rmc_{j,t}, \quad j = H, F, \tag{24}$$

Where $\pi_{j,t} = p_{j,t} - p_{j,t-1}$ and $rmc_{j,t} = mc_{j,t} - p_{j,t}$ are the inflation rates and real marginal costs for home and foreign goods. Define $rp_{F,t} \equiv p_{F,t} - p_{H,t}$. Since (4) linearizes as $p_t = (1-\alpha)p_{H,t} + \alpha p_{F,t}$ under our normalization $(\bar{P}_{H,t} = \bar{P}_{F,t} = \bar{P}_t = 1)$, $p_t - p_{H,t} = \alpha rp_{F,t}$ and $p_t - p_{F,t} = -(1-\alpha)rp_{F,t}$. Using these conditions, noting that $p_{F,t}^* = p_t^*$ under the assumption of a large foreign economy, the linearized relations for (11) and (12) can be expressed as

$$rmc_{H,t} = rw_t + \alpha rp_{F,t} - x_{Y,t} , \tag{25}$$

$$rmc_{F,t} = z_t - (1-\alpha)rp_{F,t} . \tag{26}$$

The log relative price of foreign to home goods is related to the inflation rates for the two goods as

$$rp_{F,t} = rp_{F,t-1} + \pi_{F,t} - \pi_{H,t} . \tag{27}$$

Also, the linearized form of (4) implies that

$$\pi_t = \alpha\pi_{F,t} + (1-\alpha)\pi_{H,t} . \tag{28}$$

Noting that $\xi_t \equiv \int_0^1 \frac{Y_t(i)}{Y_t} = \int_0^1 \frac{P_{H,t}(i)}{P_{H,t}}$ and using our normalization of prices, we linearize (15) as

$$n_t = y_t - x_{Y,t} . \tag{29}$$

We also normalize $\bar{S} = \bar{P}^* = 1$, which (given our normalization that $\bar{P} = 1$) implies that $\bar{Z} = 1$. Also, since trade is balanced in steady state, $\alpha\bar{C} = \alpha^*\bar{C}^*$. Then, noting that $p_{H,t} - p_t = -\alpha rp_{F,t}$ and $c_t^* = y_t^*$, we can linearize (16) as:

$$y_t = -\eta rp_{H,t} + (1-\alpha)c_t + \alpha y_t^* + \alpha\eta z_t. \tag{30}$$

Linearization of (17) and (18) yields

$$\dot{B}_t^* = \dot{B}_{t-1}^* + hi_t - c_t, \tag{31}$$

$$hi_t = -\alpha rp_{F,t} + y_t + \alpha((1-\alpha)rp_{F,t} - z_t), \tag{32}$$

where, in deriving (31) and (32) we have assumed that $\bar{C} = 1$ (so that $\bar{C}_F = \alpha$). Finally, the monetary policy rule (19) is linearized as

$$r_t = \delta_R r_{t-1} + (1-\delta_R)\delta_\pi E_t(\pi_{t+1}) + x_{R,t}. \tag{33}$$

2.2. Model with Government

In this section, we extend the model to include government. In specifying the fiscal behavior of the government, we allow for the fact that the government borrows from SBP to finance part of its expenditures, and thus fiscal policy influences growth of money supply. Money demand is modeled by introducing real money balances in the utility function. In discussing the model below, we focus on the relations that are new or modified.

2.2.1. Revisions

To add a role for money, the utility function (1) is revised as:

$$U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} X_{H,t} \left(\frac{(C_t - hC_{t-1})^{1-\sigma}}{1-\sigma} - \frac{\chi_N N_t^{1+\nu}}{1+\nu} + \frac{\chi_M (M_t / P_t)^{1-\mu}}{1-\mu} \right), \tag{34}$$

where, M_t the money stock held by households, and the parameter μ determines the elasticity of money demand with respect to the interest rate and household expenditure (as shown below). Household budget constraint is modified as:

$$P_t C_t + M_t + P_t B_t + S_t P_t^* B_t^* = M_{t-1} + R_{t-1} P_t B_{t-1} + R_{t-1}^* TC_{t-1} S_t P_t^* B_{t-1}^* + W_t N_t + PR_t - P_t TR_t, \tag{35}$$

where, B_t now represents real stock of government bonds held by households (for simplicity, we ignore private domestic bonds, which equal zero in symmetric household equilibrium), and TR_t is tax revenue in real terms. Optimization of (34) subject to the budget constraint (35) still yields (8)-(10), but also implies that

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$$\left(\frac{M_t}{P_t}\right)^{-\mu} = \frac{C_t^{-\sigma}}{\chi_M}\left(\frac{R_t-1}{R_t}\right).\tag{36}$$

Let G_t denote government's consumption of the composite good. The government's flow budget constraint in real terms is given by

$$B_t = G_t - TR_t - (M_t - M_{t-1})/P_t + R_{t-1}B_{t-1}.\tag{37}$$

We assume that the government has a long-run debt target, $\bar{B}$ and adjusts tax revenue to move the debt towards the target. The fiscal variables are assumed to be determined as

$$G_t = X_{G,t}Y_t P_{H,t}/P_t,\tag{38}$$

$$TR_t = X_{TR,t}(B_{t-1}/\bar{B})^{\delta_{TR}}Y_t P_{H,t}/P_t,\tag{39}$$

Where, $X_{G,t}$ and $X_{TR,t}$ are fiscal shocks representing stochastic shares of government expenditures and tax revenue in GDP. Given $\bar{P}_H = \bar{P}$ by normalization, the steady state constraint for the government can be obtained from (37)-(39) and expressed as $\frac{\Delta \bar{M}}{\bar{M}}\frac{\bar{M}}{\bar{P}} = \bar{X}_G\bar{Y} - \bar{X}_{TR}\bar{Y} + (\bar{R}-1)\bar{B}$. Thus, given the level of government debt in the long run, the long-run expenditure and tax revenue shares determine the rate of long-run money growth and inflation.

The presence of government also modifies relations (16)-(18) as follows. First, since output now equals $Y_t = C_{H,t} + C_{H,t}^* + G_t$, use this equality along with(3) and its foreign counterpart to revise (16) as

$$Y_t = G_t + \left(\frac{P_{H,t}}{P_t}\right)^{-\eta}\left[(1-\alpha)C_t + \alpha^* Z^{-\eta}C_t^*\right].\tag{40}$$

Next, as the the government's budget constarint (37) implies that $G_t = TR_t + (M_t - M_{t-1})/P_t + B_t - R_{t-1}B_{t-1}$, we can make use of this expression in the household budget constraint (35) to revise (17) as

$$B_t^* = TC_{t-1}R_{t-1}^*B_{t-1}^* + (HI_t - C_t - G_t)/Z_t.\tag{41}$$

Finally, letting $G_{F,t} = \alpha_G G_t$ represent the government's consumption of the imported good, profits for importers now equal $PR_{F,t} = (P_{F,t} - S_t P_{F,t}^*)(C_{F,t} + G_{F,t})$, and (18) is revised as

$$HI_t = \frac{P_{H,t}}{P_t}Y_t + (\frac{P_{F,t}}{P_t} - Z_t)(C_{F,t} + G_{F,t}).\tag{42}$$

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2.2.2. Linearized Relations

The new equations (36)-(39) are linearized as

$$rm_t = \frac{\sigma}{\mu}c_t - \frac{\beta}{\mu}r_t + \frac{1}{\mu}x_{M,t}, \tag{43}$$

$$b_t = \frac{1}{\beta}(b_{t-1} + r_{t-1}) + \varphi_{GB}g_t - \varphi_{TRB}tr_t - \varphi_{RMB}mg_t, \tag{44}$$

$$g_t = y_t + x_{G,t}, \tag{45}$$

$$tr_t = y_t + \delta_{TR}b_{t-1} + x_{TR,t}, \tag{46}$$

where, $rm_t \equiv m_t - p_t$, $mg_t \equiv rm_t - (rm_{t-1} + \pi_t)/\bar{\Pi}$, $\varphi_{GB} = \bar{G}/\bar{B}$, $\varphi_{TRB} = \overline{TR}/\bar{B}$ and $\varphi_{RMB} = \bar{M}/(\bar{P}\bar{B})$. The linearized forms of the modified equations (40)-(42) are

$$y_t = \varphi_{GY}g + (1-\varphi_{GY})[\alpha\eta rp_{F,t} + (1-\alpha)c_t + \alpha y_t^* + \alpha\eta z_t], \tag{47}$$

$$\dot{B}_t^* = \dot{B}_{t-1}^* + hi_t - c_t - \varphi_{GC}g_t, \tag{48}$$

$$hi_t = -\alpha rp_{F,t} + y_t + (\alpha + \alpha_G\varphi_{GY})((1-\alpha)rp_{F,t} - z_t), \tag{49}$$

where $\varphi_{GY} = \bar{G}/\bar{Y} = \bar{G}/\bar{C}$. Equations (47)-(49) replace (30)-(32).

2.3. Multi-Sector Model

There is interest in examining and forecasting the movements of different components of inflation. FPAS model includes disaggregated Phillips curve relations for core, food and oil inflation. In this section we develop a multi-sector model that provides theoretical underpinnings for these relations. We distinguish three types of goods: food, core products and oil. Both home and foreign firms produce varieties of food and core goods while oil is not produced at home. In the discussion below, we consider the multi-sector model without government. However, it would be straightforward to combine the outcomes of sections 2.2 and 2.3 to build a model with both government and multiple sectors.

2.3.1. Revisions

Instead of (2), assume a two tier consumption function, as follows:

$$C_t = \left[(1-\gamma_f-\gamma_o)^{1/\eta}(C_{c,t})^{(\eta-1)/\eta} + \gamma_f^{1/\eta}(C_{f,t})^{(\eta-1)/\eta} + \gamma_o^{1/\eta}(C_{o,t})^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)}, \tag{50}$$

$$C_{c,t} = \left[(1-\alpha_c)^{1/\eta}(C_{Hc,t})^{(\eta-1)/\eta} + \alpha_c^{1/\eta}(C_{Fc,t})^{(\eta-1)/\eta}\right]^{\eta/(\eta-1)}, \tag{51}$$

$$C_{f,t} = \left[(1-\alpha_f)^{1/\eta} (C_{Hf,t})^{(\eta-1)/\eta} + \alpha_f^{1/\eta} (C_{Ff,t})^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)}, \qquad (52)$$

where, $C_{c,t}, C_{f,t}$ and $C_{o,t}$ are aggregate indexes for core, food and oil products;

$C_{Hc,t} = \left(\int_0^1 C_{Hc,t}(i)^{(\varepsilon-1)/\varepsilon} di \right)^{\varepsilon/(\varepsilon-1)}$ and $C_{Hf,t} = \left(\int_0^1 C_{Hf,t}(i)^{(\varepsilon-1)/\varepsilon} di \right)^{\varepsilon/(\varepsilon-1)}$ represent bundles of home

varieties of core and food products; $C_{Fc,t} = \left(\int_0^1 C_{Fc,t}(i)^{(\varepsilon-1)/\varepsilon} di \right)^{\varepsilon/(\varepsilon-1)}$ and

$C_{Ff,t} = \left(\int_0^1 C_{Ff,t}(i)^{(\varepsilon-1)/\varepsilon} di \right)^{\varepsilon/(\varepsilon-1)}$ represent the corresponding bundles of foreign varieties, and

$C_{o,t} = \left(\int_0^1 C_{o,t}(i)^{(\varepsilon-1)/\varepsilon} di \right)^{\varepsilon/(\varepsilon-1)}$ is the bundle of imported varieties for oil. The substitution elasticities

between varieties are assumed to be the same at each tier for simplicity.

The demand functions for the aggregates and for the domestic and imported bundles of food and core are given by

$$C_{c,t} = \gamma_c \left(\frac{P_{c,t}}{P_t} \right)^{-\eta} C_t, \quad C_{f,t} = \gamma_f \left(\frac{P_{f,t}}{P_t} \right)^{-\eta} C_t, \quad C_{o,t} = \gamma_o \left(\frac{P_{o,t}}{P_t} \right)^{-\eta} C_t, \qquad (53)$$

$$C_{Hc,t} = (1-\alpha_c) \left(\frac{P_{Hc,t}}{P_{c,t}} \right)^{-\eta} C_{c,t}, \quad C_{Hf,t} = (1-\alpha_f) \left(\frac{P_{Hf,t}}{P_{f,t}} \right)^{-\eta} C_{f,t}, \qquad (54)$$

$$C_{Fc,t} = \alpha_c \left(\frac{P_{Fc,t}}{P_{c,t}} \right)^{-\eta} C_{c,t}, \quad C_{Ff,t} = \alpha_f \left(\frac{P_{Ff,t}}{P_{f,t}} \right)^{-\eta} C_{f,t}, \qquad (55)$$

where,

$$P_t = \left[(1-\gamma_f - \gamma_o) P_{c,t}^{1-\eta} + \gamma_f P_{f,t}^{1-\eta} + \gamma_o P_{o,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}, \qquad (56)$$

$$P_{c,t} = \left[(1-\alpha_c) P_{Hc,t}^{1-\eta} + \alpha_c P_{Fc,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}, \quad P_{f,t} = \left[(1-\alpha_f) P_{Hf,t}^{1-\eta} + \alpha_f P_{Ff,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}. \qquad (57)$$

The production function of a producer i of a core or food variety is $Y_{j,t}(i) = X_{Yj,t} N_{j,t}(i)$, $j = c, f$, where $Y_{c,t}(i)$ and $N_{c,t}(i)$ are the output and employment for a producer of core products, $Y_{f,t}(i)$ and $N_{f,t}(i)$ are the corresponding variables for a food producer, while $X_{Yc,t}$ and $X_{Yf,t}$ represent productivity shocks for core and food sectors (assumed to be the same for all firms in each sector). The nominal marginal costs for producers and retailers in different sectors are

$$MC_{Hj,t} = \frac{W_t}{X_{Yj,t}}, \quad j = c, f \ . \tag{58}$$

$$MC_{Fj,t} = S_t P_{Fj,t}^*, \quad j = c, f \ , \tag{59}$$

$$MC_{o,t} = S_t P_{o,t}^* \ . \tag{60}$$

Prices for producers of the home varieties of core and food products as well as retailers of imported varieties of the core, food and oil products are set according to the Calvo mechanism with partial indexation to inflation. For $j = c, f$, let $1 - \theta_j$ be the probability that a producer or a retailer sets a new optimal price, $\bar{P}_{lj,t}(i), \ l = H, F$, in period t . The firms that do not reoptimize simply index their price to past inflations as $P_{lj,t}(i) = P_{lj,t-1}(i) \left(\frac{P_{lj,t-1}}{P_{lj,t-2}} \right)^{\kappa_j}, l = H, F \ , \ j = c, f$, where $0 \le \kappa_j \le 1$ is the indexation parameter for each sector. The optimal condition for setting the new price is

$$\sum_{k=0}^{\infty} \theta_j^k E_t \left[DR_{t,t+k} D_{lj,t+k}(i) \left(\bar{P}_{lj,t}(i) \left(\frac{P_{lj,t+k-1}}{P_{lj,t-1}} \right)^{\kappa_j} - \frac{\varepsilon}{\varepsilon - 1} MC_{lj,t+k} \right) \right] = 0, \quad l = H, F, \quad j = c, f \ , \tag{61}$$

Where, $D_{Hj,t+k}(i) = \left[\frac{\bar{P}_{Hj,t}(i)}{P_{Hj,t+k}} \left(\frac{P_{Hj,t+k-1}}{P_{Hj,t-1}} \right)^{\kappa_j} \right]^{-\varepsilon} (C_{Hj,t+k} + C_{Hj,t+k}^*)$ is the demand for a producer while

$D_{Fj,t+k}(i) = \left[\frac{\bar{P}_{Fj,t}(i)}{P_{Fj,t+k}} \left(\frac{P_{Fj,t+k-1}}{P_{Fj,t-1}} \right)^{\kappa_j} \right]^{-\varepsilon} C_{Fj,t+k}$ is the demand for a retailer. Moreover, prices indexes for home and foreign goods in each sector are given by

$$P_{lj,t} = \left[\theta_j \left(P_{lj,t-1} \left(P_{lj,t-1} / P_{lj,t-2} \right)^{\kappa_j} \right)^{1-\varepsilon} + (1 - \theta_j)(\bar{P}_{lj,t})^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}, \quad l = H, F, \quad j = c, f \ , \tag{62}$$

Where, $\bar{P}_{lj,t}$ are the common new optimal prices for producers and retailers in core and food sectors.

Similar relations can be derived for oil importers as follows:

$$\sum_{k=0}^{\infty} \theta_o^k E_t \left[DR_{t,t+k} D_{o,t+k}(i) \left(\bar{P}_{o,t}(i) \left(\frac{P_{o,t+k-1}}{P_{o,t-1}} \right)^{\kappa_o} - \frac{\varepsilon}{\varepsilon - 1} MC_{o,t+k} \right) \right] = 0, \tag{63}$$

$$P_{o,t} = \left[\theta_o \left(P_{o,t-1} \left(P_{o,t-1} / P_{o,t-2} \right)^{\kappa_j} \right)^{1-\varepsilon} + (1-\theta_o)(\bar{P}_{o,t})^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}, \tag{64}$$

Where, $1-\theta_o$ is the probability that an oil retailer would set a new optimal price, $\bar{P}_{o,t}(i)$, in period t;

κ_o is the indexation parameter; and $D_{o,t+k}(i) = \left[\dfrac{\bar{P}_{o,t}(i)}{P_{o,t+k}} \left(\dfrac{P_{o,t+k-1}}{P_{o,t-1}} \right)^{\kappa_o} \right]^{-\varepsilon} C_{o,t+k}$.

Define sector-level indexes for output andemployment as $Y_{j,t} \equiv \left(\int_0^1 Y_{j,t}(i)^{(\varepsilon-1)/\varepsilon} di \right)^{\varepsilon/(\varepsilon-1)}$ and

$N_{j,t} \equiv \int_0^1 N_{j,t}(i)\, di$. Using the firm-level production functions for core and food sectors, we obtain

$$N_{j,t} = \frac{Y_{j,t}}{X_{Yj,t}} \xi_{j,t}, \quad j = c, f, \tag{65}$$

Where, $\xi_{j,t} \equiv \int_0^1 \dfrac{Y_{j,t}(i)}{Y_{j,t}}$. Noting that $Y_{j,t}(i) = C_{Hj,t}(i) + C^*_{Hj,t}(i) = \left(\dfrac{P_{Hj,t}(i)}{P_{Hj,t}} \right)^{-\varepsilon} \left(C_{Hj,t} + C^*_{Hj,t} \right)$ for

$j = c, f$, using, (53), (54) and its foreign counterpart, the definition of sectoral output, and letting $C^* = Y^*$, we obtain

$$Y_{j,t} = C_{Hj,t} + C^*_{Hj,t} = \left(\frac{P_{Hj,t}}{P_t} \right)^{-\eta} \left[(1-\alpha_j)\gamma_j C_t + \alpha_j^* \gamma_j^* Z^\eta Y_t^* \right], \quad j = c, f. \tag{66}$$

Letting Y_t denote aggregate output, and $\tilde{P}_t$ the price index for aggregate output, we have

$$\tilde{P}_t Y_t = P_{Hc,t} Y_{c,t} + P_{Hf,t} Y_{f,t}. \tag{67}$$

Finally, as the present model with multiple sectors implies that $Y_t = (W_t N_t + PR_{H,t}) / \tilde{P}_{H,t}$, and $PR_{F,t} = (P_{Fc,t} - S_t P^*_{c,t}) C_{Fc,t} + (P_{Ff,t} - S_t P^*_{f,t}) C_{Ff,t} + (P_{o,t} - S_t P^*_{o,t}) C_{o,t}$, (18) is revised as

$$HI_t = \frac{P_{H,t}}{\tilde{P}_t} Y_t + (\frac{P_{Fc,t}}{P_t} - \frac{P^*_{c,t}}{P^*_t} Z_t) C_{Fc,t} + (\frac{P_{Ff,t}}{P_t} - \frac{P^*_{f,t}}{P^*_t} Z_t) C_{Ff,t} + (\frac{P_{o,t}}{P_t} - \frac{P^*_{o,t}}{P^*_t} Z_t) C_{o,t} \tag{68}$$

2.3.2. Linearized Relations

Using linearized versions of (61)-(64), we can derive the following relations:

$$\pi_{lj,t} = \frac{\beta}{1+\beta\kappa_j} E_t(\pi_{lj,t+1}) + \frac{\kappa_j}{1+\beta\kappa_j} \pi_{lj,t-1} + \frac{(1-\theta_j)(1-\beta\theta_j)}{\theta_j(1+\beta\kappa_j)} rmc_{lj,t}, \quad l = H, F, \quad j = c, f, \tag{69}$$

$$\pi_{o,t} = \frac{\beta}{1+\beta\kappa_o} E_t(\pi_{o,t+1}) + \frac{\kappa_o}{1+\beta\kappa_0} \pi_{o,t-1} + \frac{(1-\theta_o)(1-\beta\theta_o)}{\theta_o(1+\beta\kappa_o)} rmc_{o,t}, \tag{70}$$

Where, $\pi_{lj,t} = p_{lj,t} - p_{lj,t-1}, \pi_{o,t} = p_{o,t} - p_{o,t-1}$, $rmc_{lj,t} = mc_{lj,t} - p_{lj,t}$ and $rmc_{o,t} = mc_{o,t} - p_{o,t}$.

Normalize all price indexes to equal unity under steady state. Under this normalization, linearization of (56) and (57) implies that $p_t = (1-\gamma_f - \gamma_o)p_{c,t} + \gamma_f p_{f,t} + \gamma_o p_{o,t}$, and for $j = c, f$, $p_{j,t} = (1-\alpha_j)p_{Hj,t} + \alpha_j p_{Fj,t}$. These relations imply that

$$\pi_t = (1-\gamma_f - \gamma_o)\pi_{c,t} + \gamma_f \pi_{f,t} + \gamma_o \pi_{o,t}, \tag{71}$$

$$\pi_{c,t} = \alpha_c \pi_{Fc,t} + (1-\alpha_c)\pi_{Hc,t}, \tag{72}$$

$$\pi_{f,t} = \alpha_f \pi_{Ff,t} + (1-\alpha_f)\pi_{Hf,t}. \tag{73}$$

Defining $pr_{fc,t} \equiv p_{f,t} - p_{c,t}$ and $pr_{oc,t} \equiv p_{o,t} - p_{c,t}$, we can express $p_t - p_{c,t} = \gamma_f pr_{fc,t} + \gamma_0 pr_{oc,t}$, $p_t - p_{f,t} = -(1-\gamma_f)pr_{fc,t} + \gamma_0 pr_{oc,t}$ and $p_t - p_{o,t} = \gamma_f pr_{fc,t} - (1-\gamma_0)pr_{oc,t}$. Moreover, for $j = c, f$, defining $rp_{Fj,t} \equiv p_{Fj,t} - p_{Hj,t}$, we have $p_{j,t} - p_{Hj,t} = \alpha_j rp_{Fj,t}$ and $p_{j,t} - p_{Fj,t} = -(1-\alpha)rp_{Fj,t}$. Using these relations and the large foreign economy assumption, and noting that $rmc_{Hj,t} = mc_{Hj,t} - p_{Hj,t}$, $rmc_{Fj,t} = mc_{Fj,t} - p_{Fj,t}$ for $j = c, f$, and $rmc_{o,t} = mc_{o,t} - p_{o,t}$ the linearized versions of (58)-(60) can be stated as:

$$rmc_{Hc,t} = rw_t + \alpha_c rp_{Fc,t} + \gamma_f pr_{fc,t} + \gamma_o pr_{oc,t} - x_{Yc,t}, \tag{74}$$

$$rmc_{Fc,t} = z_t + pr_{c,t}^* - (1-\alpha_c)rp_{Fc,t} + \gamma_f pr_{fc,t} + \gamma_o pr_{oc,t}, \tag{75}$$

$$rmc_{Hf,t} = rw_t + \alpha_f rp_{Ff,t} + (\gamma_f -1)pr_{fc,t} + \gamma_o pr_{oc,t} - x_{Yf,t}, \tag{76}$$

$$rmc_{Ff,t} = z_t + pr_{f,t}^* - (1-\alpha_f)rp_{Ff,t} + (\gamma_f -1)pr_{fc,t} + \gamma_o pr_{oc,t}, \tag{77}$$

$$rmc_{o,t} = z_t + pr_{o,t}^* + (\gamma_o -1)pr_{oc,t} + \gamma_f pr_{fc,t} - x_{o,t}, \tag{78}$$

where, $pr_{c,t}^* \equiv p_{c,t}^* - p_t^*$, $pr_{f,t}^* \equiv p_{f,t}^* - p_t^*$, and $pr_{c,t}^* \equiv p_{o,t}^* - p_t^*$.

Real marginal costs are thus related to relative prices of foreign to home goods in core and food sectors, and food-core and oil-core price ratios. These variables can be linked to corresponding inflation rates as follows:

$$rp_{Fc,t} = rp_{Fc,t-1} + \pi_{Fc,t} - \pi_{Hc,t}, \tag{79}$$

$$rp_{Ff,t} = rp_{Ff,t-1} + \pi_{Ff,t} - \pi_{Hf,t}, \tag{80}$$

$$pr_{f,t} = pr_{f,t-1} + \pi_{f,t} - \pi_{c,t}, \tag{81}$$

$$pr_{o,t} = pr_{o,t-1} + \pi_{o,t} - \pi_{c,t}. \tag{82}$$

Log-linearization of (67) yields $\tilde{p}_t + y_t = (1-\gamma_f')(p_{Hc,t} + y_{c,t}) + \gamma_f'(p_{Hf,t} + y_{f,t})$, where $\gamma_f' = \bar{Y}_{f,t}/\bar{Y}_t$ given our normalization of price indexes. Defining $\tilde{p}_t \equiv (1-\gamma_f')p_{Hc,t} + \gamma_f' p_{Hf,t}$ [which can be interpreted as the GDP deflator], we have

$$y_t = (1-\gamma_f')y_{c,t} + \gamma_f' y_{f,t}. \tag{83}$$

Now linearize (66) to get $y_{j,t} = -\eta(p_{Hj,t} - p_t) + (1-\alpha_j')c_t + \alpha_j'(c_t^* + \eta z_t)$, $j = c, f$, where $\alpha_j' = \bar{C}_{Hj}^* / \bar{Y}_j$ is the steady state share of exports in output for sector j. Note that

$p_{Hc,t} - p_t = p_{Hj,t} - p_{c,t} + p_{c,t} - p_t = -(\alpha_c rp_{Fc,t} + \gamma_f pr_{fc,t} + \gamma_0 pr_{oc,t})$, and

$p_{Hf,t} - p_t = p_{Hf,t} - p_{f,t} + p_{f,t} - p_t = -(\alpha_f rp_{Ff,t} - (1-\gamma_f)pr_{fc,t} + \gamma_0 pr_{oc,t})$. Making use of these relations, letting $c^* = y^*$ and using (83), we obtain

$$\begin{aligned} y_t &= \eta[(1-\gamma_f')\alpha_c rp_{Fc,t} + \gamma_f'\alpha_f rp_{Ff,t} + (\gamma_f - \gamma_f')pr_{fc,t} + \gamma_o pr_{oc,t}] \\ &+ [(1-\alpha_c')(1-\gamma_f') + (1-\alpha_f')\gamma_f']c_t + [\alpha_f'\gamma_f' + \alpha_c'(1-\gamma_f')](y_t^* + \eta z_t). \end{aligned} \tag{84}$$

Linearization of (65) gives $n_{jt} = y_{j,t} - x_{Yj,t}$, $j = c, f$. Normalizing $\bar{X}_{Yc} = \bar{X}_{Yf}$, defining $n_t \equiv n_{c,t} + n_{ft}$ and using (83), we have

$$n_t = y_t - \gamma_f' x_{Yf,t} - (1-\gamma_f')x_{Yc,t} \tag{85}$$

Given our normalizations, (68) is linearized as

$$\begin{aligned} hi_t &= (1-\gamma_f')(p_{Hc,t} - p_t) + \gamma_f'(p_{Hf,t} - p_t) + y_t + (p_{Fc,t} - p_t - pr_{c,t}^* - z_t)\bar{C}_{Fc,t} \\ &+ (p_{Ff,t} - p_t - pr_{f,t}^* - z_t)\bar{C}_{Ff,t} + (p_{o,t} - p_t - pr_{o,t}^* - z_t)\bar{C}_{o,t}. \end{aligned}$$

Using the relations discussed above to substitute for $p_{Hc,t} - p_t$, $p_{Hf,t} - p_t$, $p_{Fc,t} - p_t$, $p_{Ff,t} - p_t$ and $p_{o,t} - p_t$ in the above expression, and letting $\bar{C} = 1$ (so that $\bar{C}_{Fc,t} = \alpha_c(1-\gamma_f - \gamma_o)$, $\bar{C}_{Ff,t} = \alpha_f\gamma_f$, and $\bar{C}_{o,t} = \gamma_o$), we can express

$$hi_t = y_t + \alpha_c[(1-\alpha_c)((1-\gamma_f-\gamma_o)-(1-\gamma_f'))]rp_{Fc,t} + \alpha_f[(1-\alpha_f)\gamma_f-\gamma_f']rp_{Ff,t}$$
$$+[(1-\gamma_f)(\alpha_f\gamma_f+\gamma_f')-\gamma_f(\alpha_c(1-\gamma_f-\gamma_o)+1-\gamma_f'+\gamma_o)]pr_{fc,t}$$
$$+\gamma_o[(1-\gamma_o)-(1-\gamma_f')-\gamma_f'-\alpha_c(1-\gamma_f-\gamma_o)-\alpha_f\gamma_f]pr_{oc,t} \tag{86}$$
$$-\alpha_c(1-\gamma_f-\gamma_o)pr_{c,t}^* - \alpha_f\gamma_f pr_{f,t}^* - \gamma_o pr_{o,t}^* - [\alpha_c(1-\gamma_f-\gamma_o)-\alpha_f\gamma_f-\gamma_o]z_t.$$

With a view to obtain monetary policy response compatible with current FPAS, we add output gap and real exchange rate in Taylor type interest rate rule. According, (30) becomes

$$r_t = \delta_R r_{t-1} + (1-\delta_R)E_t(\delta_\pi \pi_{t+1} + \delta_y y_{t+1} + \delta_z z_{t+1}) + x_{R,t}$$

3. Estimation

We have estimated the basic model and its extended versions. In this section, we will discuss the estimation of the basic model and a general version that includes both government and multiple sectors. Estimated linear versions of both models are summarized in Appendix I. The number of shocks introduced in each model conforms to the number of observed home and foreign variables available from data (discussed below). All shocks are assumed to follow a first-order auto-regressive process.

3.1. Data

The model is estimated using quarterly data from 2001Q3 to 2015Q4. For the basic model, we use data on 4 home and 3 foreign variables. The 4 home variables are real GDP, CPI inflation rate, Treasury Bill rate and the rate of depreciation of the (Pak rupee-US dollar) exchange rate. The 3 foreign variables are US real GDP, US CPI inflation rate and US Treasury Bill rate. The extended model adds 3 variables (real government expenditures, real tax revenues and money growth rate for the government block, and 5 observable variables (core, food and oil inflation rates, and relative world prices of food and oil) for the multi-sector block.

As quarterly series are not available for real GDP, they are estimated from annual series using statistical interpolation methods which make use of the information obtained from related indicators observed at the desired frequency.[7] Time series for real values of GDP, government expenditures and tax revenues as well as for relative world prices of food and oil are non-stationary. To relate these series to stationary model variables, they are detrended using Hodrick-Prescott filter. Moreover, all series are demeaned since the model variables are expressed as deviations from steady-state values. Table 1 provides a list of observed variables used in estimation and relates them to model variables. In relating real GDP to the corresponding model variable, we allow for a measurement error arising from interpolation.[8] Figures 1 shows the behavior of the transformed data series over our sample for observed variables used in the basic model and Figure 2 for the additional variables used in the extended model.

3.2. Calibration and Priors

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[7] The quarterly data for government expenditures and tax revenues are only available from 2003. Therefore, prior to 2003 these series have also been estimated by interpolation techniques. CPI price index is used to estimate real values of the fiscal series.

[8] Since fiscal series are only interpolated for two years, we do not introduce measurement errors for these variables.

Table 2 shows the values of calibrated parameters. We set the quarterly discount factor (β) equal to 0.99. This value is typically used in the literature and is consistent with the evidence for Pakistan (Ahmed et al., 2012). The remaining parameters in the table represent steady state values, which are calibrated to Pakistan's economy using evidence from studies or data for the sample period. The value for Share of imports in consumption (α) is based on Ali (2014). Steady state quarterly gross inflation rate $(\bar{\bar{\Pi}})$ is calibrated to the average value of CPI gross inflation rate over the sample period. In the government block, we do not have information on the share of imports in government expenditures, and assume that this share (α_G) is the same as the import share in consumption. The average values over the sample period are used to calibrate the ratios of tax revenues, government expenditures and real money to domestic debt (φ_{TRB}, φ_{GB} and φ_{RMB}). For government expenditures, we used data on total budgetary spending (that includes current and development expenditures) rather than only current expenditures since our model abstracts from investment and capital flows. Also, since our model does not include a banking sector, we used data on reserve money (M0) rather than broad money (M2). In the multi-sector block, the values for shares of food and oil in CPI inflation (γ_f and γ_o) are taken from Ahmad and Pasha (2015). In order to calibrate the share of food production in domestic output (γ'_f), we used annual data for real GDP and its three major sectors: agriculture, industrial production and services. Each sector was divided into food and non-food components, using the weight for food from LSM index and a few assumptions. We calculated the export shares in production of food and core products (α'_f and α'_c) using data for the average shares of food and non-food exports in total exports, and assuming that export share in GDP equals the import share under the balanced trade assumption of the model.[9]

Nearly all of the behavioral parameters are estimated. Prior distributions for these parameters are shown in Table 3. Parameters restricted to be between 0 and 1 are assumed to have beta distribution while gamma (or inverse gamma) distribution is assumed for parameters constrained to be positive. Whenever possible, we use estimates or data for Pakistan to choose prior mean values of parameters. The standard deviation of each parameter is assumed to be between 25% and 30% of the mean value.

We first discuss the priors for the basic model. For the inverse elasticity of intertemporal substitution (σ), studies for Pakistan typically assume a value equal to one, but Ahmed et al. (2012) estimate the value to be 0.57. We choose a prior mean of 0.8, close to an average of these values. The prior mean for the inverse elasticity of labor supply (ν) is assumed to be 1.59 based on results in Ahmed et al. (2014). The value for the prior mean of the elasticity of substitution between domestic and foreign goods (η) is taken from Haider et al. (2013) and equals 1.12. We do not have much information on the habit formation coefficient (h) and inflation indexation parameter (κ), for developing economies. We simply assume that the prior means for both of these coefficients are 0.4. For Calvo price stickiness index (θ) we use a value of 0.25 as suggested by Choudhary et al. (2016). For the transaction cost coefficients for external debt and depreciation (ψ_1 and ψ_2), we let the prior means equal 0.2 and 0.7 based on evidence suggested by correlations between measures of risk premium (derived from UIP relation), exchange rate depreciation and foreign debt. For

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[9] These shares were calculated as $\alpha'_f = \frac{Food\ exports}{Total\ exports} \frac{\alpha}{\gamma'_f}$ and $\alpha'_c = \frac{Non-food\ exports}{Total\ exports} \frac{\alpha}{1-\gamma'_f}$.

the Taylor rule, we follow Ahmad and Pasha (2015) and Aleem and Lahiani (2011) and set the prior mean for interest rate smoothing coefficient (δ_R) equal to 0.60, and prior mean for interest rate response to inflation (δ_π) equal to 1.5.

In the government block, following Ahmad et al. (2016), prior mean for the money demand parameter (μ) is set equal to 0.06. For the fiscal revenue response to government debt (δ_{TR}), we assume the value of 0.15 which represents a lower side estimate from evidence on correlations between government revenue and debt using annual data. At the sector level, we assume that the Calvo price stickiness index for core goods (θ_c) is greater than the index for food products (θ_f), which in turn is greater than the oil products index (θ_o). We set the prior mean equal 0.6 for θ_c, 0.4 for θ_f and 0.2 for θ_o. We do not have strong beliefs about the DGP for different shocks and how it differs across shocks. We use the same priors for each shock. We assume beta distribution for the auto-regressive coefficients and let the prior mean and the standard deviation of each coefficient equal 0.5 and 0.1, respectively. The white-noise shock for each process is assumed to follow an inverse gamma distribution. We let both the prior mean and the standard deviation equal 0.01 for the standard error of each white-noise shock.

3.3. Results

Table 4 displays the posterior estimates for the basic model. The posterior mean value of the intertemporal elasticity ($1/\sigma$) is slightly lower and that for the elasticity of labor supply $(1/\nu)$ somewhat higher than the prior value. The posterior mean of the elasticity of substitution between home and foreign goods is smaller than our prior and is less than unity. The habit parameter determines the extent to which aggregate demand depends on the forward- and backward-looking components. The estimated value of this parameter is around 0.6 and suggests a significant role for both the forward- and backward-looking components. The indexation parameter determines the weights on the forward- and backward-looking components in the aggregate supply or Phillips curve relation for inflation. The estimated value for this parameter close to 0.3 implies that current inflation responds more to the expected value of future inflation than to past inflation. Estimates of transaction cost parameters are not too different than the priors and suggest that the transaction cost (or risk premium) increases in foreign debt and is negatively related to expected exchange rate change.

There is also considerable interest in identifying the Calvo parameter (θ). A lower value of this parameter indicates more flexible prices and a smaller impact of monetary policy on output. For developed economies, the typical estimated value of the Calvo parameter is around 0.75. Our estimate for this parameter is about 0.5, which is greater than our prior of 0.25, but still suggests greater flexibility of prices in Pakistan than developed economies. However, as discussed below, we find significant differences in the estimates of the Calvo parameter across sectors in the multi-sector model. Estimation of the monetary policy parameters indicates significant interest rate smoothing and a moderately strong interest rate reaction to inflation (the inflation coefficient is above 2).

Estimates of the parameters for domestic shocks reveal that the shock to preferences is more persistent and has higher variability than other domestic shocks. Foreign output and foreign interest rate shocks exhibit moderately high persistence. All foreign shocks, however, have significantly lower standard deviations than domestic shocks.

Posterior estimates for the general model are presented in Table 5 (the posterior and prior distributions for this model are compared in Figure 3). The table shows estimates of additional parameters specific to the general model as well as revised estimates of parameters common to both the basic and general models. An interesting finding is that price stickiness varies substantially across the sectors: the Calvo coefficient is 0.875 for core goods, 0.582 for food products and 0.164 for oil. There are also significant changes in the estimates of two parameters, the inverse elasticity of labor supply and the habit coefficient. Posterior means of both of these parameters in the general model are much higher than in the basic model. These results suggest that in the general model, the real wage does not change vary much and the backward-looking component has a strong influence on aggregate demand.[10] Estimates of the coefficients of the monetary policy rule in the general model indicate that interest rate smoothing and response to inflation is slightly stronger than in the basic model. The estimated value of the tax rule parameter is 0.12, which suggests a weak response of tax revenue to deviations of government debt from its target value.

4. Evaluation

To evaluate the performance of the new FPAS model developed in this project, we first briefly examine whether the impulse response functions (IRFs) generated by the basic or general versions of this model provide reasonable dynamic response of key macroeconomic variables to various shocks. We then explore the forecasting ability of the new FPAS model. Forecasting is not only an important evaluation tool, but would also represent a major application of the model for policy analysis. We compare the forecasting performance of the new FPAS model with that of the original FPAS model as well as of other forecasting models. We also examine how well the model's predictions match the actual data.

4.1. Impulse Response Functions

We consider the IRFs for both the basic and the general model. To facilitate comparison between the two models, we focus on shocks that are common to both models (IRFs for shocks specific to the general model are shown in Appendix III). For each of these shocks, Figure 4 shows the dynamic response of four major macroeconomic variables: rate of inflation, output, nominal interest rate and exchange rate depreciation. IRFs for both models show the expected pattern. For example, the monetary policy (interest rate) shock temporarily decreases the inflation rate and decreases output. The shock to domestic demand (consumer preferences) leads to an increase in both inflation and output in the short run. The impact of these shocks on inflation is less pronounced for the general model. In the general model, moreover, the dynamic effect of the shocks is generally more spread out. IRFs for foreign shocks also display the expected pattern of effects, and exhibit differences between the two models which are similar to IRFs for domestic shocks.

4.2. Forecast Comparison

The aim of the new FPAS model is to initially complement and ultimately replace the current FPAS model at the State Bank of Pakistan. The current model is actively used to provide policy analysis and macroeconomic forecasts of major macroeconomic variables. The output from this model is shared with the Monetary Policy Committee (MPC) of the State Bank of Pakistan to aid macroeconomic assesment and to provide input for monetary policy decisions. It is, therefore, important to examine if the new FPAS model

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[10] Note that our model, for simplicity, does not incorporate stickiness in the nominal wage rate, which (together with price stickiness) would also imply little variability in the real wage rate.

improves the forcast accuracy of the current FPAS model. As the current model is close to the basic version of the new FPAS model, we focus on this version for forecast comparisons.

Bayesian vector autoregressions (BVARs) are widely used at Central banks around the world for forecasting and policy inference. We thus explore how the new FPAS model performs in comparison with Bayesian VARs. A linearized DSGE model can be approximated by a VAR and implies cross-equation restriction on the VAR. Del Negro & Schorfheide (2004) propose a procedure which systematically relaxes these restrictions to construct a hybrid model (DSGE-VAR).We also undertake a forecast comparison bteween the new FPAS model and a comparable DSGE-VAR.

To make forecast comparisons, we use quarterly data for 7 variables for Pakistan (which were utilized to estimate the basic version). We estimate each model and construct in-sample rolling-window forecasts from 2009Q1 to 2015Q4. The rolling-window for model estimation and forecasting is set at 20 quarters. In each Iteration of the model, we derive 8-period ahead forecasts for the selected variables as well as root mean squared error (RMSE) of forecasts. Each model is therefore estimated 20 times for the 2009Q1-2013Q4 period.

4.2.1. Comparison with the Current FPAS Model

The current FPAS model is a reduced form DSGE model developed in-house at SBP, and is being used to forecast major macroeconomic variables and provide policy recommendations for consideration by the Monetary Policy Committee at the State Bank of Pakistan. The model is a New Keynesian DSGE model with real and nominal rigidities. The model consists of four blocks: (a) Aggregate demand block; (b) Aggregate supply block; (c) External sector block, and finally (d) policymaker's reaction function. To gain pragmatic usefulness for forecasting and monetary policy analysis, the current FPAS model eschews explicit modelling of micro foundations. It also relies on previous studies and judgement to calibrate model parameters. The key points of departure for the new FPAS model are that it embeds reasonable micro-foundations and estimates model parameters instead of calibrating them.

To evaluate the current FPAS model, Ahmed and Pasha (2015) compare the accuracy of inflation forecasts (in terms of root mean square error) with the best combination of econometric models suggested in Hanif and Malik (2015). They find that the inflation projections of the current FPAS model are superior to the combination of best alternatives, especially for normal or moderate inflation periods.

We also focus on inflation projections to compare the forecast performance of the current and new FPAS models. Table 6 and Figure 5 compare the root mean squareerror (RMSE) of the inflation forecasts of the current and new FPAS models for horizons of 1 through 8 quarters.The new model depicts significantly better forecast performance for all forecast horizons. The relative performance of the new model, moreover, improves as the length of the horizon increases.

4.2.2. Comparison with Bayesian VAR and DSGE-VAR

VARs are parameter rich models which provide good in sample fit, but lack stable inference and suffer from inaccurate out-of-sample forecasts. A potential solution to this problem is based on Bayesian econometric treatment for linear system of equations pionered by researchers at the University of Minnesota in the 1980s (Litterman, 1984; Doan, Litterman, & Sims, 1984). This solution combines the richly

parameterized unrestricted VAR model with researcher's specified parsimonious priors, and is helpful in controlling estimation uncertainty. Following this approach, we estimate a Bayesian VAR using typical Minnesota prior specification for the theoretical DSGE model. In this VAR, we use the same observable variables as the basic model (further explanation is provided in Appendix II).

We also estimate a DSGE-VAR. This model uses the new FPAS DSGE model to shape the prior odds and provide model identification consistent with the theoretical model (further details on the estimation procedure are given in Appendix II). The optimal weight on the DSGE model for the DSGE-VAR priors as well as the comparison of impulse responses of the DSGE-VAR and the DSGE constitute key dimensions for assessing the validity of economic restrictions implied by the structural model. DSGE-VARs are often used to test alternative model specifications and accertain robstness of DSGE model estimation (see Adjemian, Pariès, & Moyen, 2008). Various iterations of the DSGE-VAR are undertaken each with a different value of the DSGE-VAR prior, and model robustness is accertained on the basis of highest marginal density.[11] DSGE-VAR can also be used for forecasting and we explore how the forecast efficiency of this model compares with the new FPAS model.

Forecasts comparisons are conducted for three variables; (a) real GDP of Pakistan, (b) CPI inflation and (c) nominal interest rate. Figure 6 shows the RMSE for each model at different horizons, and Figure 7 displays the RMSE of the new FPAS model relative to that of the Bayesian VAR (three panel on the left) and DSGE-VAR (three pannels on the right) over the forecast horizon of 1 to 8 quarters. In Figure 7, dots below the horizontal line corresponding to value of 1.0, show superior forecasting performance for the new FPAS model at the indicated horizon. As illustrated, the new FPAS model clearly performs better than the Bayesian VAR for all forecast horizons. The forecast performance of the new FPAS is close to the DSGE-VAR model: at short horizons (1-2 quarters), it fares better in predicting output growth, but worse in forecasting the interest rate. Thus the DSGE-VAR model, which relaxes the cross-equation restriction on the VAR implied by the new FPAS model, does not contribute much to improving the overall forecasting ability of this model.

Comparison of in-sample forecasts of the new FPAS model and competing models based on a point estimate of forecast accuracy is a desirable first step, as the new FPAS model is to be used as a tool for policy assessment. One limitation of yardsticks of point accuracy of forecasts, such as RMSE, is that they do not account for sampling variability (Diebold, 2015).[12] We thus undertake an additional test of model prediction accuracy of the three competing models based on Diebold Mariano (1995). This test (DM test) is based on spectral analysis of forecast differential of two competing models and factors in the sampling uncertainty (see Appendix 2 for further discussion of the test). We conduct the DM test for 8 period ahead rolling-window forecasts of three variables; CPI inflation, nominal interest rate and total output.

The results of the DM test for the comparison of the forecast accuracy are displayed in Table 7 for the new FPAS model and Bayesian VAR model, and Table 8 for the new FPAS Model and DSGE-VAR model. For brevity, we do not present actual test statistics but simply indicate which model is preferred at each forecast

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[11]A high value for the DSGE VAR prior selected on the basis of highest marginal density criterion is desireable as it indicates that the DSGE model imposes useful theoritcal restrictions.

[12] Example of other measures of forecast accuracy include, for example, mean absolute forecast error (MAFE) and mean percentage forecast error (MPFE).

horizon.[13] DM test also indicates that the new FPAS model forecasts are better than Bayesian VAR forecasts for the three variables, CPI inflation, nominal interest rate and real GDP, at all forecast horizons. These results are very favorable to the New FPAS model as the strength of DSGE models is generally thought to lie in providing macroeconomic analysis of different policy scenarios rather than in forecasting macroeconomic variables. Test diagnostics also indicate that new FPAS model performs better in forecasting real GDP while DSGE VAR out-performs the new FPAS model in forecasts of both inflation and nominal interest rates at all forecast horizons.

4.2.3. Matching the Actual Data

Although relative RMSE providesa useful measure to gauge forecast performance within a set of macroeconomic models, it is also important to explore how well the forecasts matches actual data. We examine the match for both new FPAS and DSGE-VAR models. Scatter plots in Figures 8 through Figure 10 illustrate the comparison of forecasts from the two models with the realized value for the three variables, real GDP, inflation and the interest rate, over all forecast horizons. Scatter plot closer to the 45 degree line would represent a good match between the forecasts and the data. The divergence between the forecasts and the realized values for the three variables is generally not too large. As would be expected, the forecast performance tends to worsen as the the horizon increases. Also, the forecasts in some cases diverge more from the realized values at higher values (i.e., in th enorth-east part of the graph).

We also perform a standard test of forecast efficiency (Gürkaynak, R. and Wolfers, 2007). For this test, we estimate the following representation:

$$y_t = \alpha_i^h + \beta_i^h \hat{y}_{i,t}^h + \varepsilon_{i,t}^h \; , \tag{87}$$

where, y_t is the actual value in period t of a variable considered for forecasting, $\hat{y}_{i,t}^h$ is the forecast for model i at horizon h, and $\varepsilon_{i,t}^h$ is the error term. Ideally a good forecast implies that the difference between the predicted and realized values is minimal. The above regression equation tests whether the forecast values are close to the realized values. Good forecasting performance implies intuitively that the intercept (α_i^h) should equal zero, slope coefficients (β_i^h) should equal one, and there is a high R^2 statistic. If α_i^h approaches zero while β_i^h approaches one, then the above equation would imply the familiar forecast accuracy diagnostic based on RMSE.[14]

We estimate (87) using rolling-window forecasts for CPI Inflation, nominal interest rate and real GDP of Pakistan obtained from the New FPAS, Bayesian VAR and DSGE-VAR models. The results are shown in Tables 9-11. The intercept is close to zero in all cases, and for CPI inflation and real GDP forecasts, it is generally not significantly different from zero. The slope coefficient, however, differs from one and this difference is often quite large at longer horizons. Thus, as also suggested by Figures 8-10, forecasts deteriorate at higher values, and this deterioration is more pronounced when forecast horizons are long.

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[13] Readily available upon request.

[14] In this case, $\varepsilon_{i,t}^h = y_t - \hat{y}_{i,t}^h$ and $\sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(\varepsilon_{i,t}^h\right)^2} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(y_t - \hat{y}_{i,t}^h\right)^2} = RMSE$

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Appendix I

Estimated Models

Basic Model

Equations (23)

Aggregate demand block (4 equations)

$$rw_t = \frac{\sigma}{1-h}c_t - \frac{\sigma h}{1-h}c_{t-1} + \nu n_t$$

$$y_t = x_{Y,t} + n_t$$

$$c_t = \frac{1}{1+h}E_t(c_{t+1}) + \frac{h}{1+h}c_{t-1} - \frac{1-h}{\sigma(1+h)}\left[r_t - E_t(\pi_{t+1}) + x_{H,t} - E_t(x_{H,t+1})\right]$$

$$y_t = \alpha\eta rp_{F,t} + (1-\alpha)c_t + \alpha y_t^* + \alpha\eta z_t$$

Aggregate supply block (6 equations)

$$\pi_{H,t} = \frac{\beta}{1+\beta\kappa}E_t(\pi_{H,t+1}) + \frac{\kappa}{1+\beta\kappa}\pi_{H,t-1} + \frac{(1-\theta)(1-\beta\theta)}{\theta(1+\beta\kappa)}rmc_{H,t}$$

$$rmc_{H,t} = rw_t + \alpha rp_{F,t} - x_{Y,t}$$

$$\pi_{F,t} = \frac{\beta}{1+\beta\kappa}E_t(\pi_{F,t+1}) + \frac{\kappa}{1+\beta\kappa}\pi_{F,t-1} + \frac{(1-\theta)(1-\beta\theta)}{\theta(1+\beta\kappa)}rmc_{F,t}$$

$$rmc_{F,t} = z_t - (1-\alpha)rp_{F,t}$$

$$rp_{F,t} = rp_{F,t-1} + \pi_{F,t} - \pi_{H,t}$$

$$\pi_t = \alpha\pi_{F,t} + (1-\alpha)\pi_{H,t}$$

IRR, foreign sector block (6 equations)

$$r_t = \delta_r r_{t-1} + (1-\delta_r)\delta_\pi E_t(\pi_{t+1}) + x_{R,t}$$

$$\dot{B}_t^* = \dot{B}_{t-1}^* + hi_t - c_t$$

$$hi_t = -\alpha rp_{F,t} + y_t + \alpha((1-\alpha)rp_{F,t} - z_t)$$

$$r_t = r_t^* + E_t\Delta s_{t+1} + tc_t$$

$$tc_t = -\psi_1(b_t^* + z_t) - \psi_2(\Delta s_{t+1} + \Delta s_t) + x_{TC,t}$$

$$\Delta s_t = z_t - z_{t-1} + \pi_t - \pi_t^*$$

Shocks (7 equations)

$$x_{H,t} = \rho_H x_{H,t-1} + e_{H,t}$$

$$x_{Y,t} = \rho_Y x_{Y,t-1} + e_{Y,t}$$

$$x_{R,t} = \rho_R x_{R,t-1} + e_{R,t}$$

$$x_{TC,t} = \rho_{TC} x_{TC,t-1} + e_{TC,t}$$

$$y_t^* = \rho_{Y*} y_{t-1}^* + e_{Y*,t}$$

$$\pi_t^* = \rho_{\pi*} \pi_{t-1}^* + e_{\pi*,t}$$

$$r_t^* = \rho_{R*} r_{t-1}^* + e_{R*,t}$$

Endogenous variables (23)

$rw_t, c_t, n_t, y_t, x_{Y,t}, r_t, \pi_t, x_{H,t}, z_t, \pi_{H,t}, rmc_{H,t}, \pi_{F,t}, rmc_{F,t}, rp_{F,t} (= p_{F,t} - p_{H,t}), x_{R,t},$

$\Delta s_t (= s_t - s_{t-1}), b_t^*, hi_t, tc_t, x_{TC,t}, y_t^*, \pi_t^*, r_t^*$

Exogenous variables

$e_{Y*,t}, e_{\pi*,t}, e_{R*,t}, e_{H,t}, e_{Y,t}, e_{R,t}, e_{TC,t}$

Parameters

$$\alpha, \sigma, \nu, \beta, \theta, \delta_\pi, \delta_R, h, \kappa, \psi_1, \psi_2, \rho_{XH}, \rho_{XY}, \rho_{XR}, \rho_{TC}, \rho_R, \rho_{Y*}, \rho_{\pi*}, \rho_{R*}$$

General Model with Multiple Sectors and Government

Equations (45)

Aggregate demand block (9 equations)

$$rw_t = \frac{\sigma}{1-h} c_t - \frac{\sigma h}{1-h} c_{t-1} + \nu n_t$$

$$n_t = y_t - \gamma_f' x_{Yf,t} - (1 - \gamma_f') x_{Yc,t}$$

$$c_t = \frac{1}{1+h} E_t(c_{t+1}) + \frac{h}{1+h} c_{t-1} - \frac{1-h}{\sigma(1+h)} \left[r_t - E_t(\pi_{t+1}) + x_{H,t} - E_t(x_{H,t+1}) \right]$$

$$y_t = \eta[(1 - \gamma_f')\alpha_c rp_{Fc,t} + \gamma_f' \alpha_f rp_{Ff,t} + (\gamma_f - \gamma_f')pr_{fc,t} + \gamma_o pr_{oc,t}]$$
$$+ [(1-\alpha_c')(1-\gamma_f') + (1-\alpha_f')\gamma_f']c_t + [\alpha_f'\gamma_f' + \alpha_c'(1-\gamma_f')](y_t^* + \eta z_t)$$

$$b_t = \frac{1}{\beta}(b_{t-1} + r_{t-1}) + \varphi_{GB} g_t - \varphi_{TRB} tr_t - \varphi_{RM} mg_t$$

$$rm_t = \frac{\sigma}{\mu} c_t - \frac{\beta}{\mu} r_t + \frac{1}{\mu} x_{M,t}$$

$$g_t = y_t + x_{G,t}$$

$$tr_t = y_t + \delta_{TR} b_{t-1} + x_{TR,t}$$

$$mg_t = rm_t - (rm_{t-1} + \pi_t)/\bar{\Pi}$$

Aggregate supply block (17 equations)

$$\pi_{Hc,t} = \frac{\beta}{1+\beta\kappa_c} E_t(\pi_{Hc,t+1}) + \frac{\kappa_c}{1+\beta\kappa_c} \pi_{Hc,t-1} + \frac{(1-\theta_c)(1-\beta\theta_c)}{\theta_c(1+\beta\kappa_c)} rmc_{Hc,t}$$

$$rmc_{Hc,t} = rw_t + \alpha_c rp_{Fc,t} + \gamma_f pr_{fc,t} + \gamma_o pr_{oc,t} - x_{Yc,t}$$

$$\pi_{Fc,t} = \frac{\beta}{1+\beta\kappa_c} E_t(\pi_{Fc,t+1}) + \frac{\kappa_c}{1+\beta\kappa_c} \pi_{Fc,t-1} + \frac{(1-\theta_c)(1-\beta\theta_c)}{\theta_c(1+\beta\kappa_c)} rmc_{Fc,t}$$

$$rmc_{Fc,t} = z_t + pr_c^* - (1-\alpha_c)rp_{Fc,t} + \gamma_f pr_{fc,t} + \gamma_o pr_{oc,t}$$

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$$rp_{Fc,t} = rp_{Fc,t-1} + \pi_{Fc,t} - \pi_{Hc,t}$$

$$\pi_{c,t} = \alpha_c \pi_{Fc,t} + (1-\alpha_c)\pi_{Hc,t}$$

$$\pi_{Hf,t} = \frac{\beta}{1+\beta\kappa_f} E_t(\pi_{Hf,t+1}) + \frac{\kappa_f}{1+\beta\kappa_f}\pi_{Hf,t-1} + \frac{(1-\theta_f)(1-\beta\theta_f)}{\theta_f(1+\beta\kappa_f)}rmc_{Hf,t}$$

$$rmc_{Hf,t} = rw_t + \alpha_f rp_{Ff,t} + (\gamma_f - 1)pr_{fc,t} + \gamma_o pr_{oc,t} - x_{Yf,t}$$

$$\pi_{Ff,t} = \frac{\beta}{1+\beta\kappa_f} E_t(\pi_{Ff,t+1}) + \frac{\kappa_f}{1+\beta\kappa_f}\pi_{Ff,t-1} + \frac{(1-\theta_f)(1-\beta\theta_f)}{\theta_f(1+\beta\kappa_f)}rmc_{Ff,t}$$

$$rmc_{Ff,t} = z_t + pr_{f,t}^* - (1-\alpha_f)rp_{Ff,t} + (\gamma_f - 1)pr_{fc,t} + \gamma_o pr_{oc,t}$$

$$rp_{Ff,t} = rp_{Ff,t-1} + \pi_{Ff,t} - \pi_{Hf,t}$$

$$\pi_{f,t} = \alpha_f \pi_{Ff,t} + (1-\alpha_f)\pi_{Hf,t}$$

$$\pi_{o,t} = \frac{\beta}{1+\beta\kappa_o} E_t(\pi_{o,t+1}) + \frac{\kappa_o}{1+\beta\kappa_o}\pi_{o,t-1} + \frac{(1-\theta_o)(1-\beta\theta_o)}{\theta_o(1+\beta\kappa_o)}rmc_{o,t}$$

$$rmc_{o,t} = z_t + pr_o^* + (\gamma_o - 1)pr_{oc,t} + \gamma_f pr_{fc,t} - x_{o,t}$$

$$pr_{o,t} = pr_{o,t-1} + \pi_{o,t} - \pi_{c,t}$$

$$pr_{f,t} = pr_{f,t-1} + \pi_{f,t} - \pi_{c,t}$$

$$\pi_t = (1-\gamma_f - \gamma_o)\pi_{c,t} + \gamma_f \pi_{f,t} + \gamma_o \pi_{o,t} + x_{\pi,t}$$

IRR, foreign sector block (6 equations)

$$r_t = \delta_R r_{t-1} + (1-\delta_R)\delta_\pi E_t(\pi_{t+1}) + x_{R,t}$$

$$b_t^* = b_{t-1}^* + hi_t - c_t$$

$$hi_t = y_t + \alpha_c[(1-\alpha_c)((1-\gamma_f - \gamma_o)-(1-\gamma_f'))]rp_{Fc,t} + \alpha_f[(1-\alpha_f)\gamma_f - \gamma_f']rp_{Ff,t}$$
$$+[(1-\gamma_f)(\alpha_f\gamma_f + \gamma_f') - \gamma_f(\alpha_c(1-\gamma_f - \gamma_o)+1-\gamma_f'+\gamma_o)]pr_{fc,t}$$
$$+\gamma_o[(1-\gamma_o)-(1-\gamma_f')-\gamma_f'-\alpha_c(1-\gamma_f - \gamma_o)-\alpha_f\gamma_f]pr_{oc,t}$$
$$-\alpha_c(1-\gamma_f - \gamma_o)pr_{c,t}^* - \alpha_f\gamma_f pr_{f,t}^* - \gamma_o pr_{o,t}^* - [\alpha_c(1-\gamma_f - \gamma_o)-\alpha_f\gamma_f - \gamma_o]z_t.$$

$$tc_t = -\psi_1(b_t^* + z_t) - \psi_2(\Delta s_{t+1} + \Delta s_t) + x_{TC,t}$$

$$r_t = r_t^* + E_t\Delta s_{t+1} + tc_t$$

$$\Delta s_t = z_t - z_{t-1} + \pi_t - \pi_t^*$$

Shocks (13 equations)

$$x_{H,t} = \rho_H x_{H,t-1} + e_{H,t}$$

$$x_{G,t} = \rho_G x_{G,t-1} + e_{G,t}$$

$$x_{TR,t} = \rho_{TR} x_{TR,t-1} + e_{TR,t}$$

$$x_{M,t} = \rho_M x_{M,t-1} + e_{M,t}$$

$$x_{Yc,t} = \rho_{Yc}x_{Yc,t-1} + e_{Yc,t}$$

$$x_{Yf,t} = \rho_{Yf}x_{Yf,t-1} + e_{Yf,t}$$

$$x_{o,t} = \rho_o x_{o,t-1} + e_{o,t}$$

$$x_{R,t} = \rho_R x_{R,t-1} + e_{R,t}$$

$$x_{\pi,t} = \rho_\pi x_{\pi,t-1} + e_{\pi,t}$$

$$x_{TC,t} = \rho_{TC}x_{TC,t-1} + e_{TC,t}$$

$$y_t^* = \rho_{Y*}y_{t-1}^* + e_{Y*,t}$$

$$\pi_t^* = \rho_{\pi*}\pi_{t-1}^* + e_{\pi*,t}$$

$$r_t^* = \rho_{R*}r_{t-1}^* + e_{R*,t}$$

Endogenous variables (45)

$rw_t, c_t, n_t, y_t, g_t, tr_t, b_t, rm_t, mg_t, x_{Yc,t}, x_{Yf,t}, r_t, \pi_t, x_{H,t}, x_{G,t}, x_{TR,t}, x_{M,t}, z_t,$

$\pi_{Hc,t}, rmc_{Hc,t}, \pi_{Fc,t}, rmc_{Fc,t}, rp_{Fc,t}(= p_{Fc,t} - p_{Hc,t}), \pi_c, pr_{fc,t}(= p_{f,t} - p_{c,t}),$

$pr_{oc,t}(= p_{o,t} - p_{c,t}), \pi_{Hf,t}, rmc_{Hf,t}, \pi_{Ff,t}, rmc_{Ff,t}, rp_{Ff,t}(= p_{Fc,t} - p_{Hf,t}),$

$\pi_f, \pi_{o,t}, rmc_{o,t}, x_{o,t}, x_{R,t}, \Delta s_t, b_t^*, hi_t, tc_t, x_{TC,t}, x_{\pi,t}, y_t^*, \pi_t^*, r_t^*$

Exogenous variables

$pr_{c,t}^*(= p_{c,t}^* - p_t^*), pr_{f,t}^*(= p_{f,t}^* - p_t^*), pr_{o,t}^*(= p_{o,t}^* - p_t^*), e_{H,t}, e_{Yc,t}, e_{Yf,t}, e_{o,t},$

$e_{G,t}, e_{TR,t}, e_{M,t}, e_{\pi,t}, e_{R,t}, e_{TC,t}, e_{Y*,t}, e_{\pi*,t}, e_{R*,t}$

Parameters

$\alpha_c, \alpha_f, \alpha_c', \alpha_f', \gamma_c, \gamma_f, \gamma_f', \sigma, v, \beta, \theta_c, \theta_f, \theta_o, \delta_\pi, \delta_R, \varphi_{GY}, \varphi_{GB}, \varphi_{TRB}, \varphi_{RMB}, \mu,$

$\delta_{TR}, h, \kappa(= \kappa_c = \kappa_f = \kappa_o), \psi_1, \psi_2, \rho_H, \rho_{Yc}, \rho_{Yf}, \rho_o, \rho_\pi, \rho_R, \rho_{TC}, \rho_M, \rho_G, \rho_{TR},$

$\rho_{Y*}, \rho_{\pi*}, \rho_{R*}$

Appendix II

Alternative Models for Forecasting and Test of Forecast Accuracy

Bayesian VAR (Minnesota Priors)

The theoretical specification of the Bayesian VAR (Minnesota priors) is elaborated below.
Consider the VAR(p) model;

$$y_t = \sum_{k=1}^{p} y_{t-k} A_k + u_t$$

where y_t is a vector of endogenous variables, A_k is the matrix that contains the coefficients, and $u_t \sim N(0, \Sigma_u)$.

The model specified in matrix form is $Y = X\Phi + U$.

The specification of Bayesian VAR estimation of the DSGE model is conducted in three stages. The first component of the prior is, by default, Jeffreys' improper prior:

$$p_1(\Phi, \Sigma) \propto |\Sigma|^{-(ny+1)/_2}$$

The second component of the prior is constructed from the likelihood of the T^* dummy observations (X^*, Y^*).

$$p_2(\Phi, \Sigma) \propto |\Sigma|^{-T^*/2} e^{-\left\{\frac{1}{2}Tr\left(\Sigma^{-1}(Y^*-X^*\Phi)'(Y^*-X^*\Phi)\right)\right\}}$$

Minnesota Prior specification for Estimation

Prior Hyperparameter	Value
τ	3
d	0.5
ϖ	1
λ	5
μ	2

The dummy observations are consturcted in line with Minnesota prior specification, i.e.;
- τ: The overall tightness of the priors,
- d: The decay factor for scaling down the coefficients of lagged values
- ϖ: The tightness for the prior on Σ
- Additional tuning parameters λ and μ.

The third component of the prior is constructed from the likelihood of T^- observations (X^-, Y^-) i.e. the training sample.
extracted from the beginning of the sample:

$$p_2(\Phi, \Sigma) \propto |\Sigma|^{-T^-/2} e^{-\left\{\frac{1}{2}Tr\left(\Sigma^{-1}(Y^--X^-\Phi)'(Y^--X^-\Phi)\right)\right\}}$$

The prior is therefore specified as;

$$p(\Phi, \Sigma) = p_2(\Phi, \Sigma)p_2(\Phi, \Sigma)p_2(\Phi, \Sigma)$$

$$p(\Phi, \Sigma) \propto |\Sigma|^{-(df^p+ny+1+k)/2} e^{\left\{-\frac{1}{2}Tr\left(\Sigma^{-1}(Y^p-X^p\Phi)'(Y^p-X^p\Phi)\right)\right\}}$$

Using Bayes rule the posterior distribution is given by;

$$p(\Phi, \Sigma|Y^+, X^+) \propto |\Sigma|^{-(df^p+ny+1+k)/2} e^{\left\{-\frac{1}{2}Tr(\Sigma^{-1}S^p)\right\}} \times |\Sigma|^{-k/2} e^{\left\{-\frac{1}{2}Tr\left(\Sigma^{-1}(\Phi-\Phi^p)'X^{p'}X^p(\Phi-\Phi^p)\right)\right\}}$$

DSGE-VAR Model

This discussion draws from Del Negro & Schorfheide, (2004) and Adjemian, Pariès, & Moyen, (2008). Consider the p order VAR representation for the $1 \times m$ vector of observed variables y_t

$$y_t = \sum_{k=1}^{p} y_{t-k} A_k + u_t \tag{A2.1}$$

where $u_t \sim N(0, \Sigma_u)$. Let z_t be the $mp \times 1$ vector$[y'_{t-1}, y'_{t-2}, \dots, y'_T]'$ and define $A = [A'_1, A'_2, \dots, A'_p]'$, the VAR representation can therefore be expressed as in matrix form;

$$Y = ZA + U \tag{A2.2}$$

where $Y = (y'_1, y'_2, \dots, y'_T)'$, $Z = (z'_1, z'_2, \dots, z'_T)'$ and $U = (u'_1, u'_2, \dots, u'_T)'$.

Dummy observations prior for the VAR can be constructed using the VAR likelighood function for $T = (\lambda T)$ artificial data simulated using the DSGE (Y^*, Z^*), combined with the diffuse priors. This prior is then given by:

$$p_0(A, \Sigma | Y^*, Z^*) \propto |\Sigma|^{-\lambda T - m + 1}/_2 e^{-1/_2 tr[\Sigma^{-1}(Y^{*'}Y^* - A'\Sigma^{-1}Z^{*'}Y^* - Y^{*'}Z^*A + A'Z^{*'}Z^*A)]} \tag{A2.3}$$

Implying that Σ conforms to an Inverted Wishart distribution and A coditional on Σ is Gaussian Normal. Assume also that observables are covariance stationary, Del Negro and Schorfheide (2004) show that DSGE theoretical autocovariance matrices for the given $n \times 1$ vector of model parameters θ denoted by $\Gamma_{YY}(\theta)$, $\Gamma_{ZY}(\theta)$, $\Gamma_{ZZ}(\theta)$ instead of the artificial sample moments $Y^{*'}Y^*$, $Z^{*'}Y^*$, $Y^{*'}Z^*$, $Z^{*'}Z^*$. In addition, the p-th order VAR approximation of the DSGE model provides the first moment of the prior distributions through the population least-square regression.

$$A^*(\theta) = \Gamma_{YY}(\theta)^{-1} \Gamma_{ZY}(\theta) \tag{A2.4}$$
$$\Sigma^*(\theta) = \Gamma_{YY}(\theta) - \Gamma_{YZ}(\theta)\Gamma_{ZZ}(\theta)^{-1}\Gamma_{ZY}(\theta) \tag{A2.5}$$

Conditional on the deep perameters of the DSGE θ and λ, the priors for the VAR parameters are given by:

$$vec\boldsymbol{A}|\Sigma, \theta, \lambda \sim N(vecA^*(\theta), \Sigma \otimes [\lambda T \Gamma_{ZZ}(\theta)]') \tag{A2.6}$$
$$\Sigma|\theta, \lambda \sim W^{-1}(\lambda T \Sigma^*(\theta), \lambda T - mp - m) \tag{A2.7}$$

Where $\Gamma_{ZZ}(\theta)$ is assumed to be non-singular and $\lambda \geq {mp + m}/_T$ for the priors to be proper. The *a-priori* density of A is defined by $n + 1$ parametrs θ and λ, which is likely to be less than the total number of VAR parameters. Finally we have to set the weight of the structural prior λ, which is independent from θ. Therefore the DSGE-VAR has the following structure;

$$p_0(A, \Sigma, \theta, \lambda) = p_0(A, \Sigma | \theta, \lambda) \times p_0(\theta) \times p_0(\lambda) \tag{A2.8}$$

where $p_0(A, \Sigma | \theta, \lambda)$ is defined above in A2.4, A2.5 and A2.6

The posterior distribution therfore takes the following form;

$$vec\boldsymbol{A}|\Sigma, \theta, \lambda, Y_T \sim N(vec\tilde{A}(\theta, \lambda), \Sigma \otimes V(\theta, \lambda)') \tag{A2.9}$$
$$\Sigma|\theta, \lambda, Y_T \sim W^{-1}\{(\lambda + 1)T\tilde{\Sigma}(\theta, \lambda), (\lambda + 1)T - mp - m\} \tag{A2.10}$$

where,

$$vec\tilde{A}(\theta, \lambda) = V(\theta, \lambda)^{-1}(\lambda T \Gamma_{ZY}(\theta) + Z'Y) \tag{A2.11}$$

$$\tilde{\Sigma}(\theta, \lambda) = \frac{1}{(\lambda + 1)T}[\lambda T \Gamma_{YY}(\theta) + Y'Y - (\lambda T \Gamma_{YZ}(\theta) + Y'Z)V(\theta, \lambda)^{-1}(\lambda T \Gamma_{ZY}(\theta) + Z'Y)] \tag{A2.12}$$

As the weight λ goes to infinity the projections of the DSGE VAR model project close on to the DSGE model. As the above expressions for θ and λ (i.e. the joint probability distribution) cannot be evaluated numerically we have to resort to MCMC methods for approximation of the posterior, we resort to Del Negro

and Schofheide (2004) for the specific MCMC algorithm, however we model λ as an estimated variable using the deep parameters θ.

Diebold Mariano Forecast Accuracy Test

The Diebold-Mariano (1995) (DM) forecast accuracy test is based on spectral analysis of forecast differential of two competing models. Let y_t denote the variable to be forecasted.[15] Also let $y^i_{t+h|t}$, denote the $h - step$ ahead forecasts of variable y for model i, $i = NewFPAS, DSGEVAR$ and $BayesianVAR$. Denote the forecast errors from the three models as

$$\varepsilon^{NewFPAS}_{t+h|t} = y_{t+h} - y^{NewFPAS}_{t+h|t}$$
$$\varepsilon^{DSGEVAR}_{t+h|t} = y_{t+h} - y^{DSGEVAR}_{t+h|t}$$
$$\varepsilon^{BVAR}_{t+h|t} = y_{t+h} - y^{BVAR}_{t+h|t}$$

The accuracy of each model forecast of variable y_t is captured by a squared error loss function $L\left(\varepsilon^i_{t+h|t}\right) = \left(\varepsilon^i_{t+h|t}\right)^2$. For simplicity, we discuss below the case of DM test comparing new FPAS model and DSGE VAR, and this discussion readily extends to a DM test for comparison of new FPAS and Bayesian VAR model.

The null hypothesis of the DM test is of equal forecast accuracy of two models i.e.

$H_0 : E\left[L\left(\varepsilon^{NewFPAS}_{t+h|t}\right)\right] - E\left[L\left(\varepsilon^{DSGEVAR}_{t+h|t}\right)\right] = E[d_t] = 0$

where d_t is the loss differential, the alternative hypothesis is;

$H_A : E\left[L\left(\varepsilon^{NewFPAS}_{t+h|t}\right)\right] - E\left[L\left(\varepsilon^{DSGEVAR}_{t+h|t}\right)\right] \neq 0$

DM statistic (S) is defined as;

$$S = \frac{\bar{d}}{\sqrt{LRV_{\bar{d}}}} \sim N(0,1)$$

where, the sample mean loss differential $\bar{d}$ is defined as:

$$\bar{d} = \frac{\sum_{t=t_0}^{T} d_t}{T_0}$$

And the long-run-variance $LRV_{\bar{d}} = \gamma_0 + 2\sum_{j=1}^{\infty} \gamma_j$, where $\gamma_j = cov(d_t, d_{t-j})$.

Diebold Mariano (1995) use spectral analysis by transforming the loss differential series d_t to its Fourier representation. The spectrum for d_t can be written as Fourier representation of the autocovariance function, γ_j; of interest is the fact that the autocovariance γ_j and spectral density are closely linked.[16] $LRV_{\bar{d}}$ is a consistent estimate of the asymptotic variance of the loss differential series (d_t).

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[15] As discussed in section 4 and 5, the forecast comparison of new FPAS model is conducted for three variables; CPI inflation, nominal interest rate and total output.

[16] Where the spectral density can be stated as; $f(\omega) = \sum_{h=-\infty}^{\infty} \gamma_j e^{-i2\pi\omega j}$

Appendix III

## IRFS for Shocks Specific to the General Model

Monetary Policy Shock
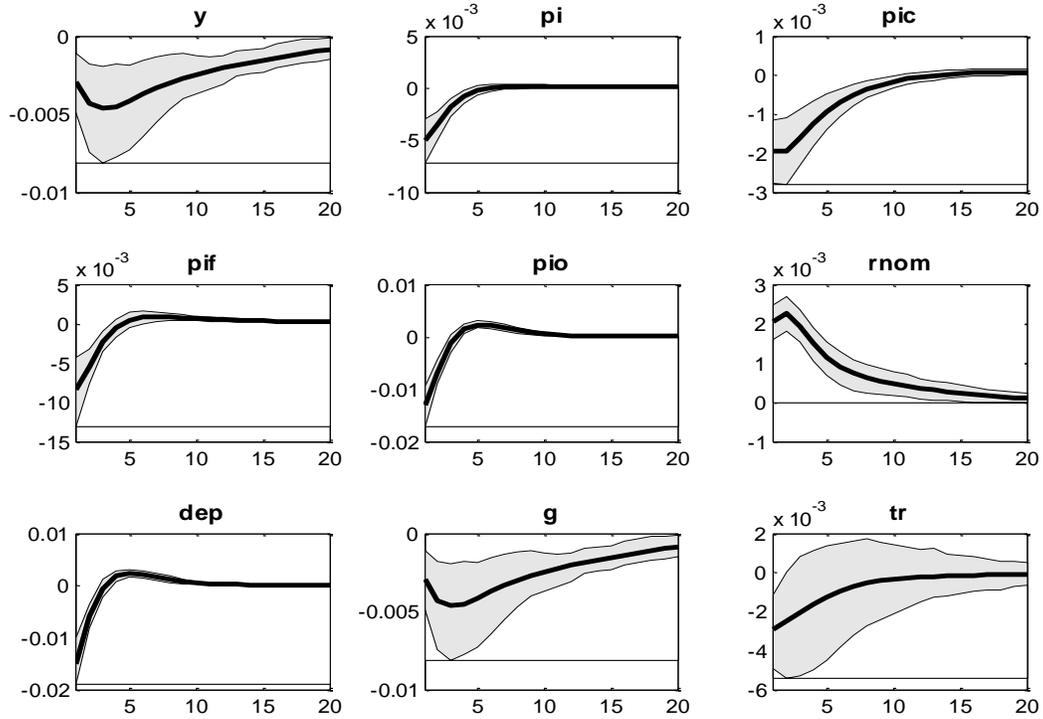

Consumer Preferences Shock
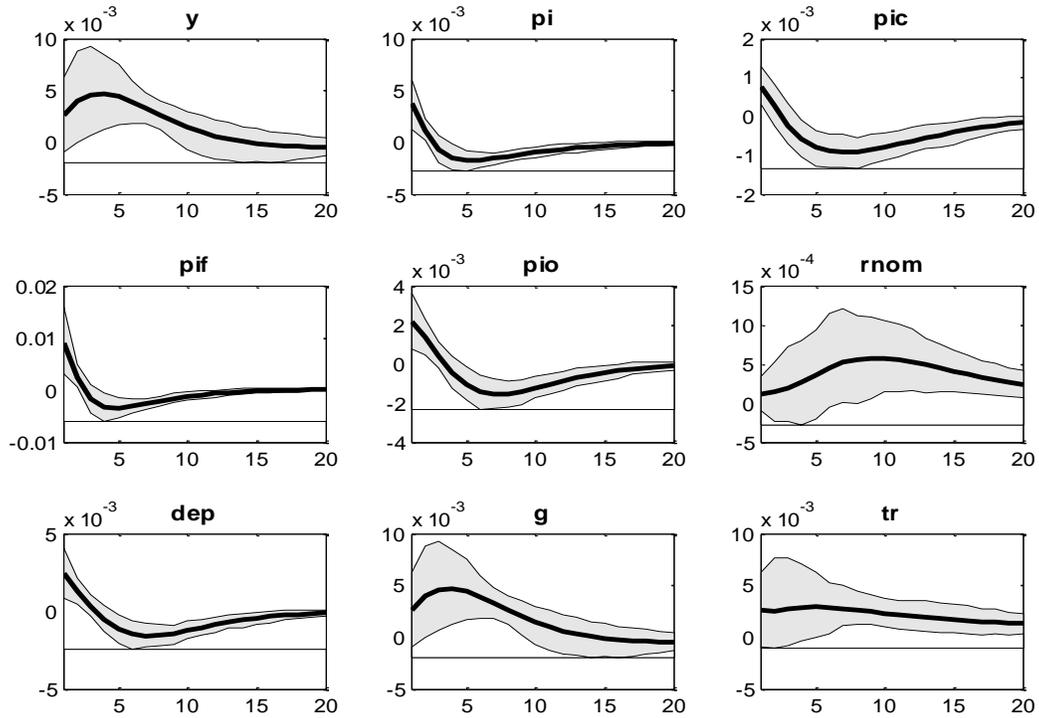
Food Productivity Shock


Core Productivity Shock



36
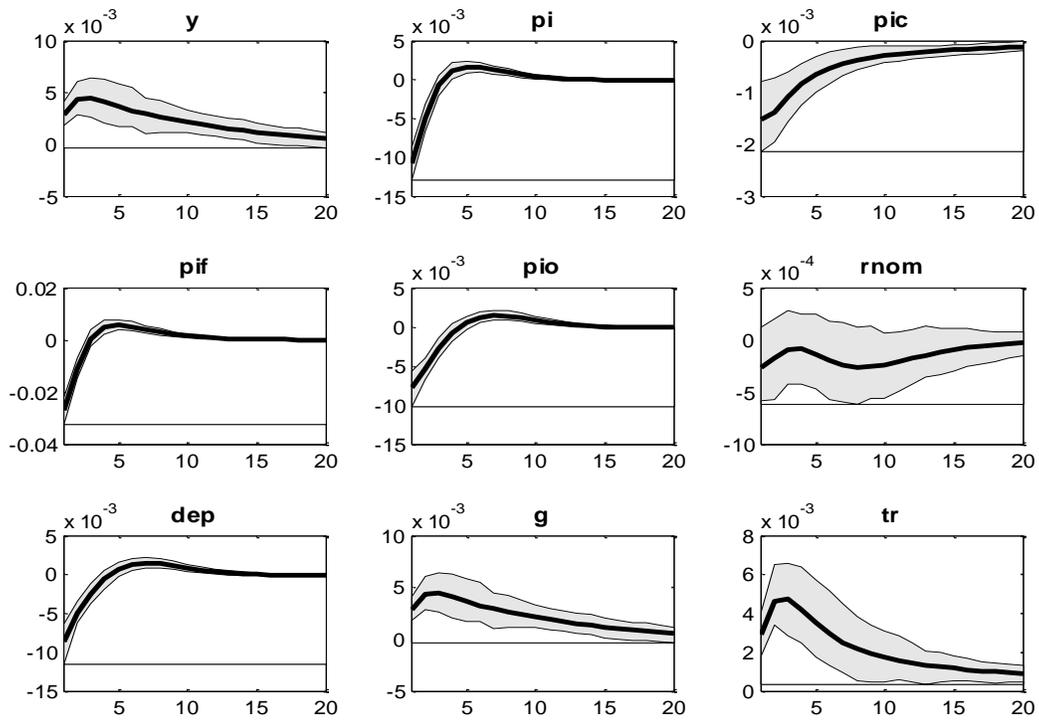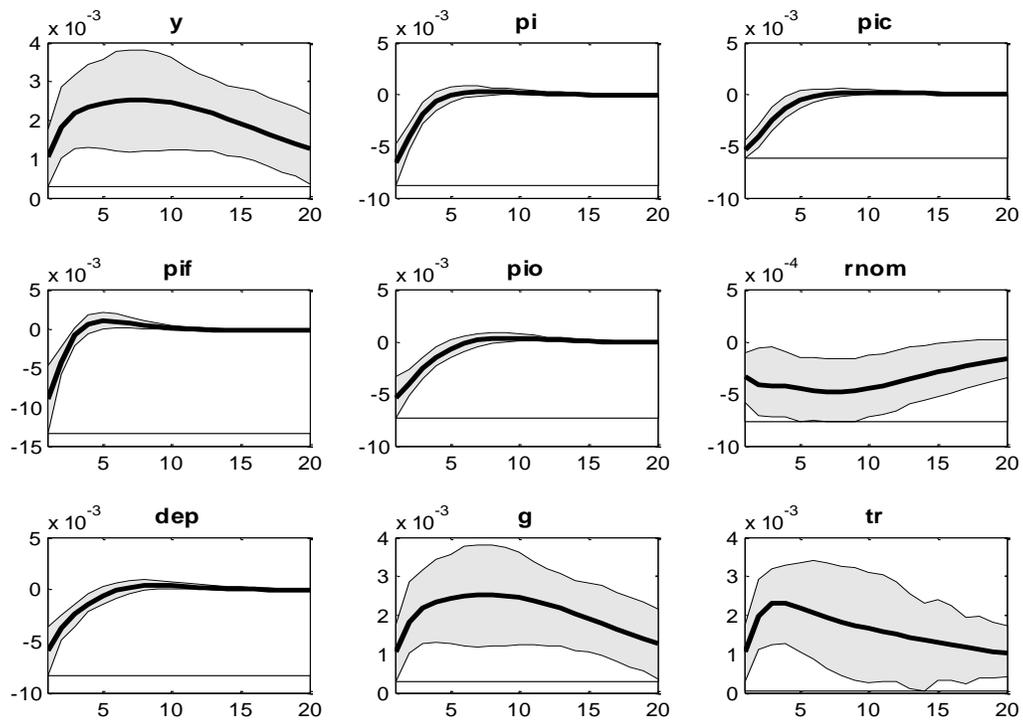
Foreign Interest Rate Shock


Foreign Output Shock



37
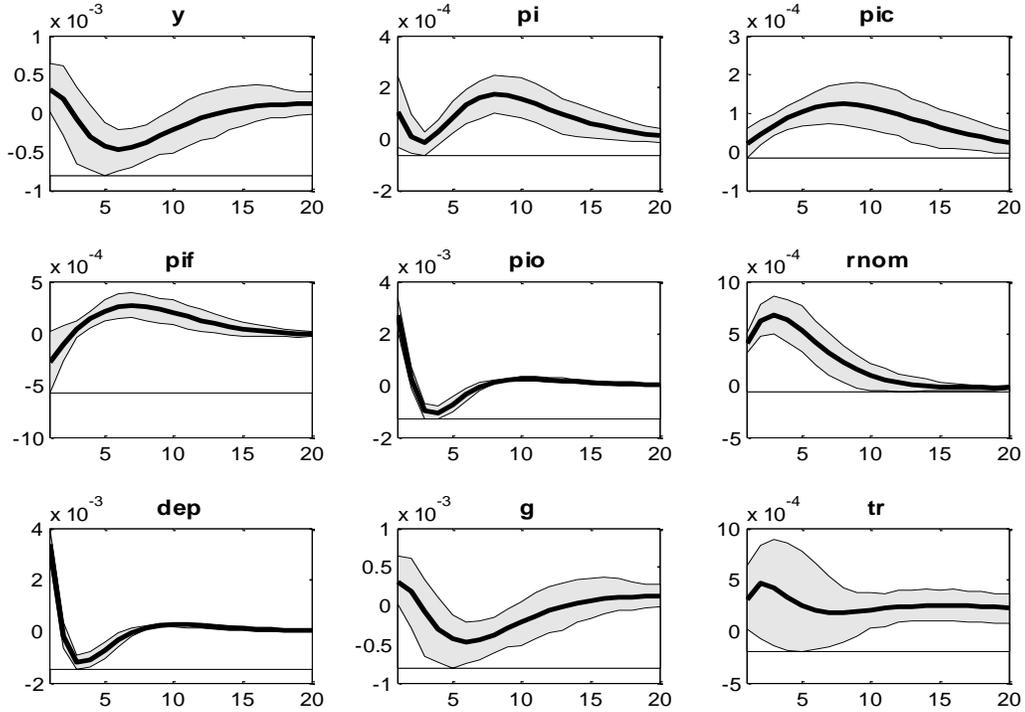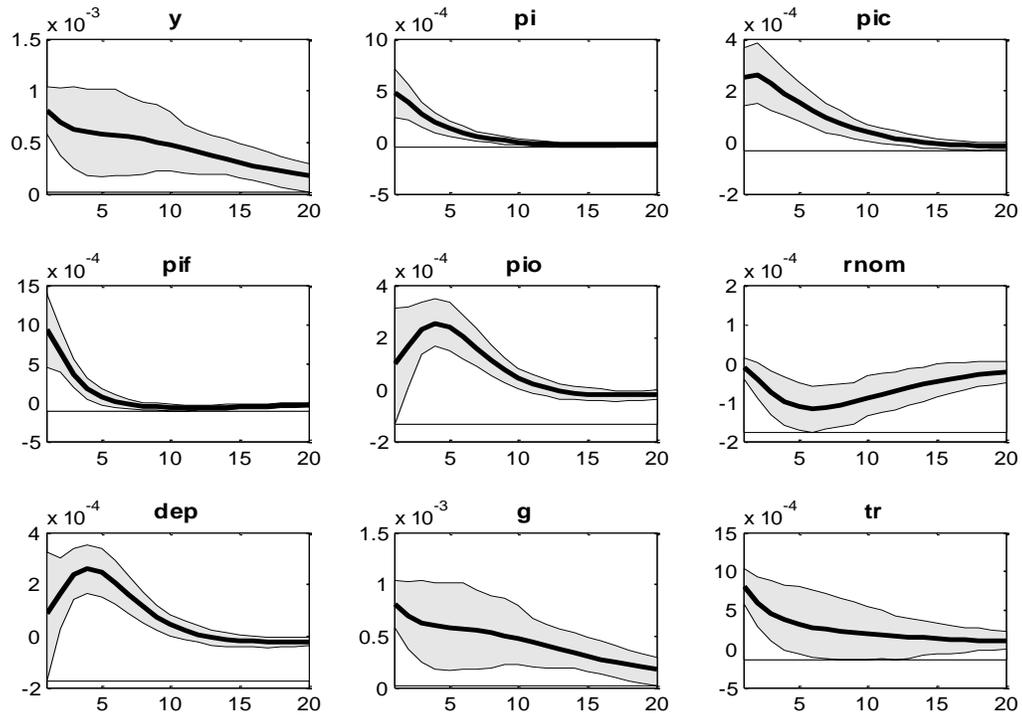
Oil Import Productivity Shock
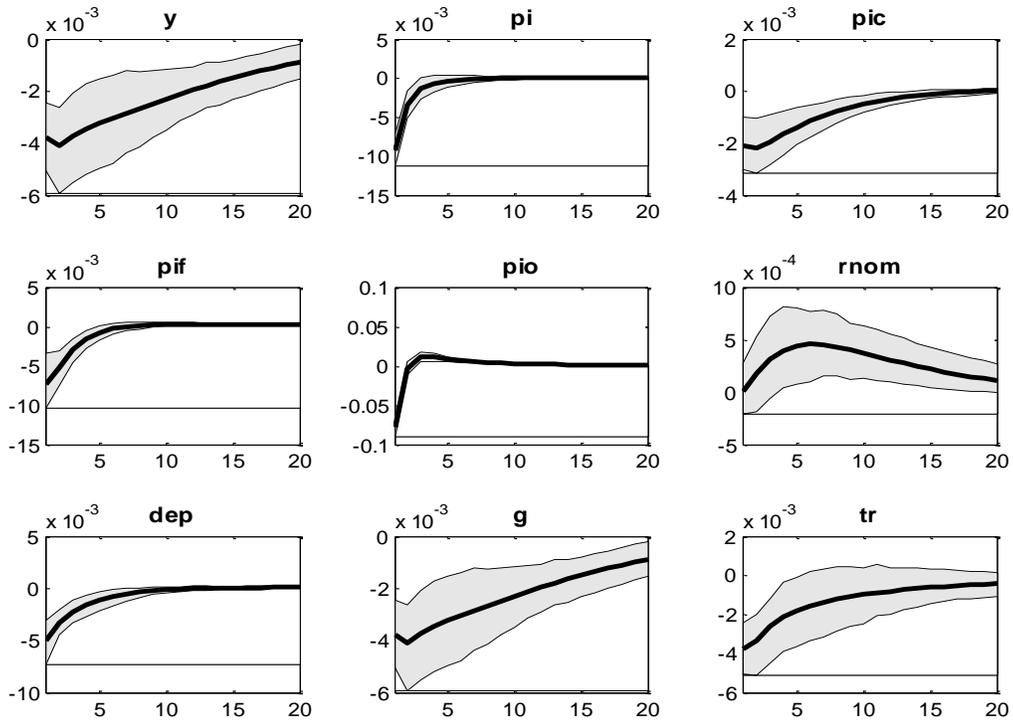
Fiscal Spending Shock


38
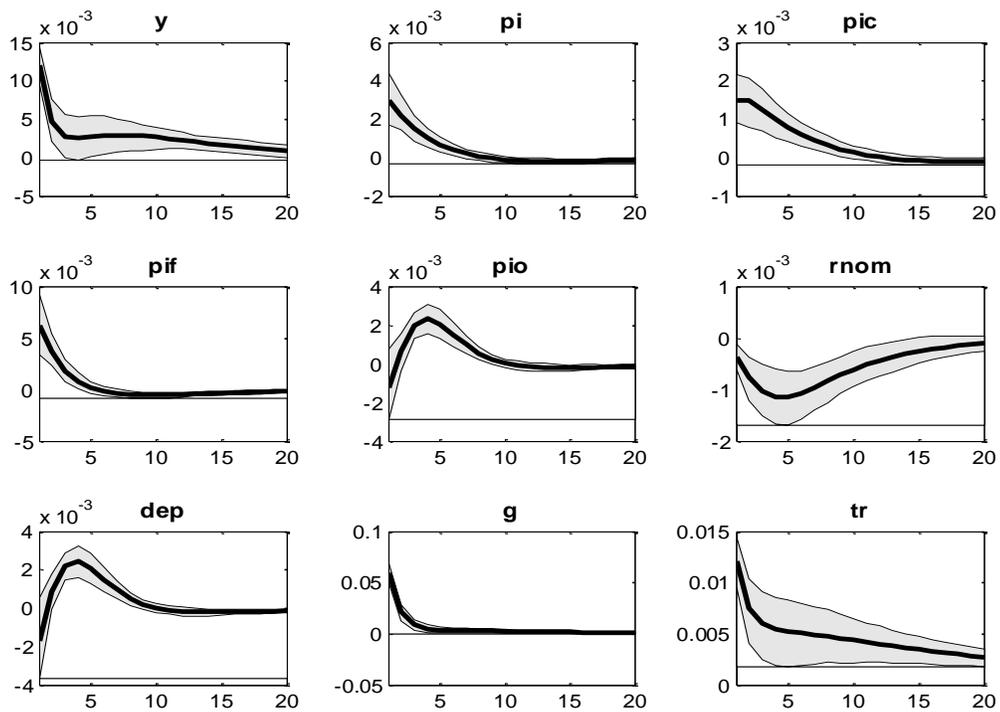
FX Risk Premium Shock
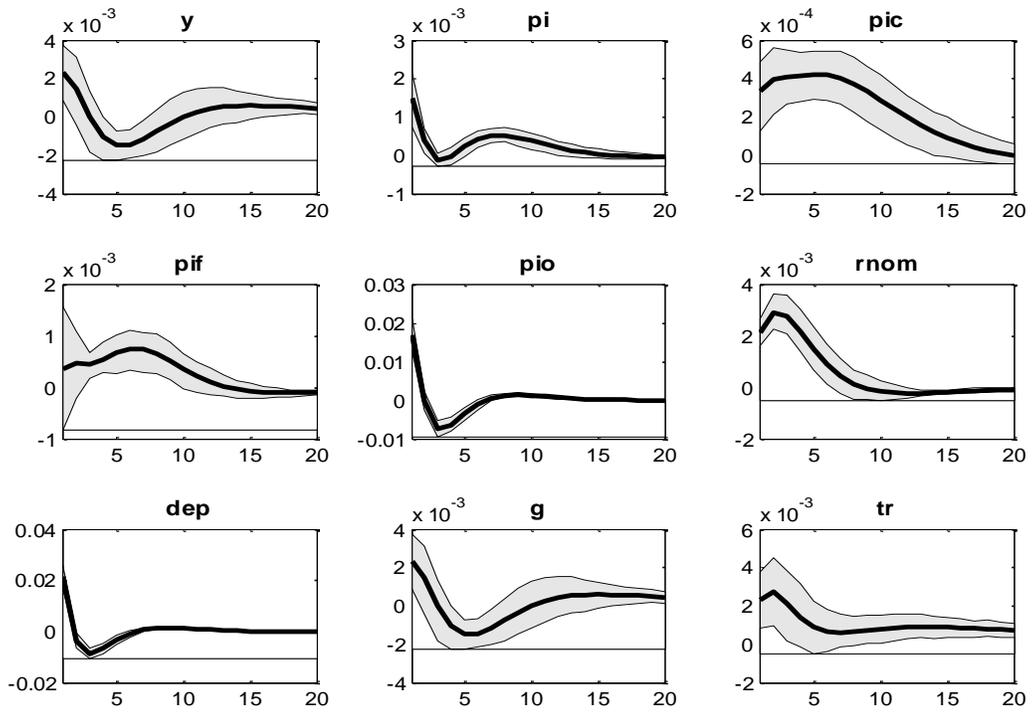
Overall Inflation Shock

39
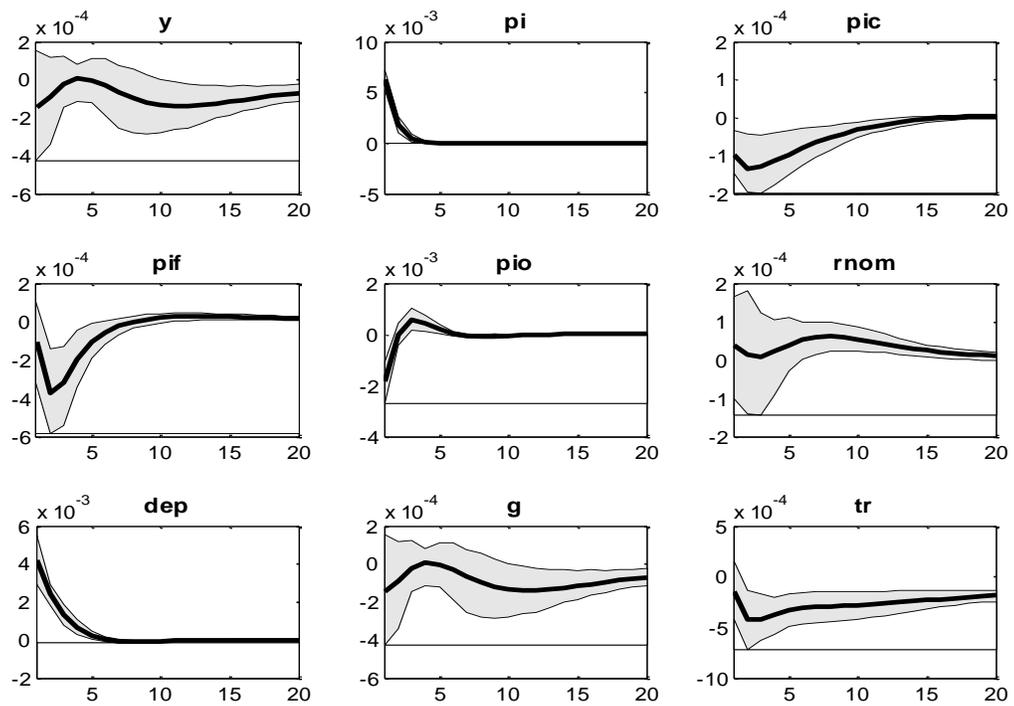
Overall Foreign Inflation Shock
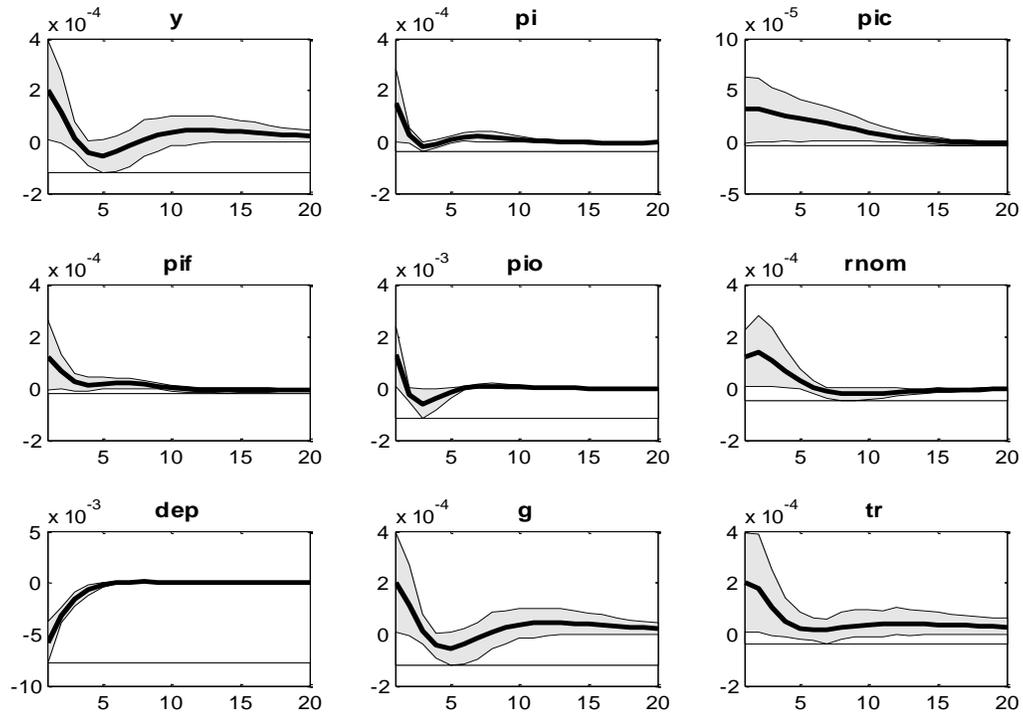
Relative Oil Price Shock


40
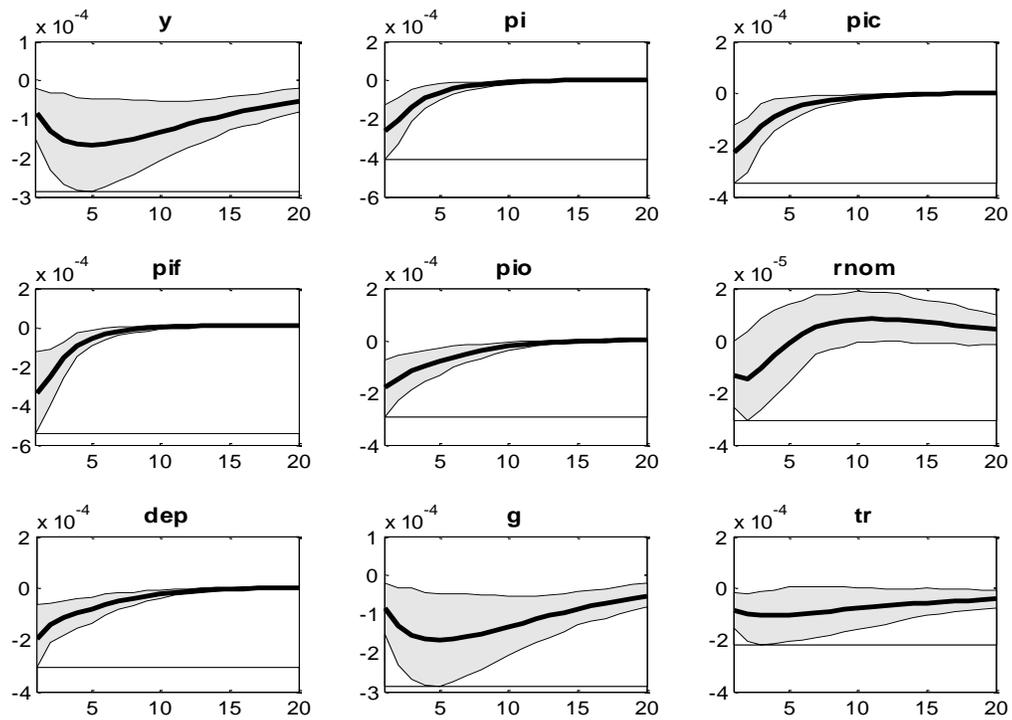
Relative Food Price Shock
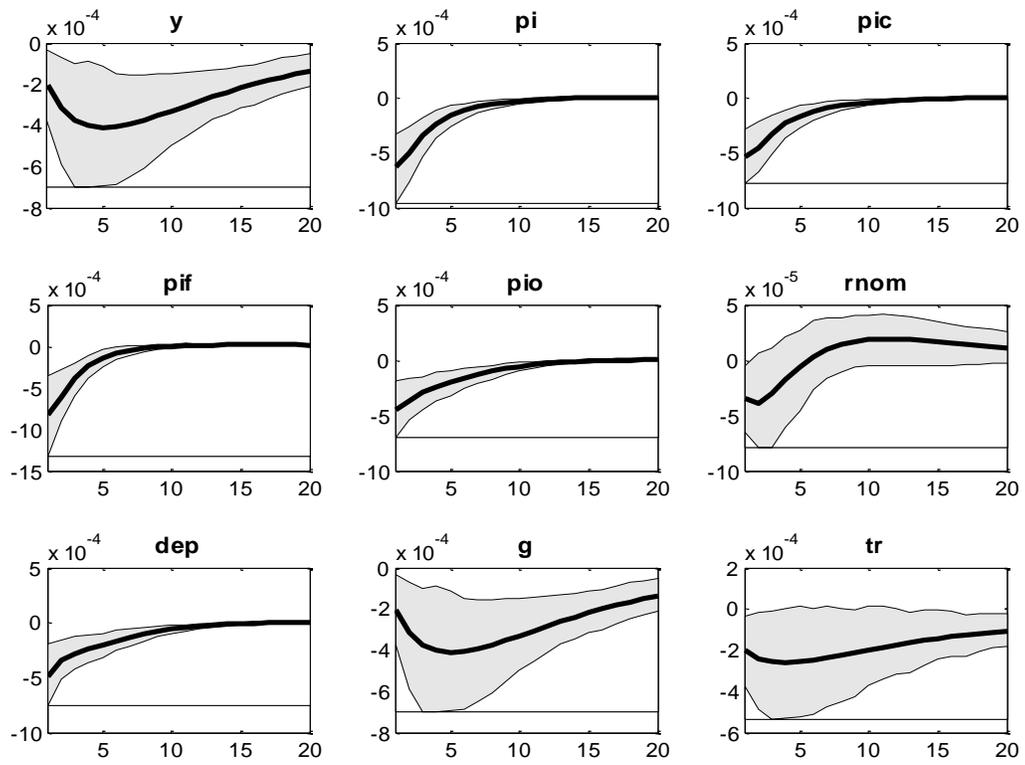
Table 1: Observed Variables: Definition and Relation to Model Variables

a) Variables for Basic Model

Observed Variables	Relation to Model Variables
$y_t^{obs} = \ln\left(GDP_t\right) - \ln\left(GDP_t^{HP-Trend}\right)$	$y_t^{obs} = y_t + e_{y,t}^{obs}$
$\pi_t^{obs} = \ln\left(\frac{CPI_t}{CPI_{t-1}}\right)$	$\pi_t^{obs} - Av(\pi_t^{obs}) = \pi_t$
$r_t^{obs} = \frac{(TBR_t - Av(TBR_t))}{400}$	$r_t^{obs} = r_t$
$\Delta s_t^{obs} = \ln\left(\frac{ER_t}{ER_{t-1}}\right)$ ER = PKR-USD exchange rate	$\Delta s_t^{obs} - Av(\Delta s_t^{obs}) = \Delta s_t$
$y_t^{*,obs} = \ln\left(GDP_t^{USA}\right) - \ln\left(GDP_t^{USA,HP-Trend}\right)$	$y_t^{*,obs} = y_t^*$
$\pi_t^{*,obs} = \ln\left(\frac{CPI_t^{USA}}{CPI_{t-1}^{USA}}\right)$	$\pi_t^{*,obs} - Av(\pi_t^{*,obs}) = \pi_t^*$
$r_t^{*,obs} = \frac{\left(TBR_t^{USA} - Av(TBR_t^{USA})\right)}{400}$	$r_t^{*,obs} = r_t^*$

b) Additional Variables for Extended Model

Observed Variables	Relation to Model Variables
$g_t^{obs} = \ln\left(RGE_t\right) - \ln\left(RGE_t^{HP-trend}\right)$ RGE = real government expenditures	$g_t^{obs} = g_t$
$tr_t^{obs} = \ln\left(RTR_t\right) - \ln\left(RTR_t^{HP-Trend}\right)$ RTR = real tax revenue	$tr_t^{obs} = tr_t$
$mg_t^{obs} = \ln\left(\frac{M_t}{M_{t-1}}\right)$ M = M0 money stock	$mg_t^{obs} - Av(mg_t^{obs}) = mg_t$
$\pi_{c,t}^{obs} = \ln\left(\frac{CPI_t^{core}}{CPI_{t-1}^{core}}\right)$	$\pi_{c,t}^{obs} - Av(\pi_{c,t}^{obs}) = \pi_{c,t}$
$\pi_{f,t}^{obs} = \ln\left(\frac{CPI_t^{food}}{CPI_{t-1}^{food}}\right)$	$\pi_{f,t}^{obs} - Av(\pi_{f,t}^{obs}) = \pi_{f,t}$
$\pi_{o,t}^{obs} = \ln\left(\frac{CPI_t^{oil}}{CPI_{t-1}^{oil}}\right)$	$\pi_{o,t}^{obs} - Av(\pi_{o,t}^{obs}) = \pi_{o,t}$
$pr_{f,t}^{*,obs} = \ln\left(PRF_t\right) - \ln\left(PRF_t^{HP-Trend}\right)$ $PRF = \frac{World\ Food\ Index}{CPI^{USA}}$	$pr_{f,t}^{*,obs} = pr_{f,t}^*$
$pr_{o,t}^* = \ln\left(PRO_t\right) - \ln\left(PRO_t^{HP-Trend}\right)$ $PRO = \frac{Avg.\ of\ Dubai,\ WTI\ and\ Brent}{CPI^{USA}}$	$pr_{o,t}^{*,obs} = pr_{o,t}^*$

Table 2: Calibrated Parameters

	Parameter	Value
Basic Model		
Discount factor	β	0.99
Share of imports in consumption	α	0.20
Steady state CPI gross inflation rate	$\bar{\Pi}$	1.02
Government Block		
Share of imports in government expenditures	α_G	0.20
Ratio of government revenues to debt	φ_{TRB}	0.40
Ratio of government expenditures to debt	φ_{GB}	0.55
Ratio of real money to debt	φ_{RMB}	0.35
Multi-Sector Block		
Share of food in CPI	γ_f	0.35
Share of oil in CPI	γ_o	0.07
Share of exports in production	α'	0.129
Share of exports in core production	α'_c	0.148
Share of exports in food production	α'_f	0.273
Share of food in production	γ'_f	0.41
Interest rate reaction to output fluctuations	δ_y	0.5
Interest rate reaction to real exchange rate fluctuations	δ_z	2.5

Table 3: Prior Distributions for Parameters

	Parameters	type	mean	stdev.
Basic model				
Inverse intertemporal elasticity	σ	Gamma	0.800	0.250
Inverse elasticity of labor supply	ν	Gamma	1.590	0.500
Domestic-foreign substitution elasticity	η	Gamma	1.120	0.350
Habit formation coefficient	h	Beta	0.400	0.100
Inflation indexation coefficient	κ	Beta	0.400	0.100
Calvo price stickiness parameter	θ	Beta	0.250	0.070
Transaction costs coef. for external debt	ψ_1	Gamma	0.200	0.065
Transaction costs coef. for depreciation	ψ_2	Gamma	0.700	0.200
Interest rate response to inflation	δ_π	Normal	1.500	0.500
Interest smoothing coefficient	δ_R	Beta	0.600	0.150
Government Block				
Money demand coefficient	μ	Gamma	0.060	0.020
Response of tax revenue to debt	δ_{TR}	Gamma	0.150	0.050
Multi-Sector Block				
Calvo parameter for core	θ_c	Beta	0.600	0.200
Calvo parameter for food	θ_f	Beta	0.400	0.120
Calvo parameter for oil	θ_o	Beta	0.200	0.065

Table 4: Posterior Estimates for the Basic Model

	Parameters	Prior Mean	Posterior Mean	Posterior Mode
Behavioral Parameters				
Inverse intertemporal elasticity	σ	0.800	0.848	0.774
Inverse elasticity of labor supply	ν	1.590	1.121	0.903
Domestic-foreign substitution elasticity	η	1.120	0.778	0.716
Habit formation coefficient	h	0.400	0.569	0.59
Inflation indexation coefficient	κ	0.400	0.282	0.257
Calvo price stickiness parameter	θ	0.250	0.494	0.471
Transaction costs coef. for external debt	ψ_1	0.200	0.22	0.214
Transaction costs coef. for depreciation	ψ_2	0.700	0.56	0.522
Interest rate response to inflation	δ_π	1.500	2.081	2.077
Interest smoothing coefficient	δ_R	0.600	0.770	0.772
Shocks				
Productivity shock, AR(1) coef.	ρ_Y	0.500	0.524	0.543
Preference shock, AR(1) coef.	ρ_H	0.500	0.753	0.772
Interest rate shock, AR(1) coef.	ρ_R	0.500	0.361	0.352
Transaction cost shock, AR(1) coef.	ρ_{TC}	0.500	0.456	0.478
Foreign output shock, AR(1) coef.	ρ_{Y*}	0.500	0.746	0.754
Foreign inflation shock, AR(1) coef.	$\rho_{\pi*}$	0.500	0.321	0.306
Foreign interest shock, AR(1) coef.	ρ_{R*}	0.500	0.796	0.805
Productivity shock, stdev.	σ_Y	0.010	0.012	0.01
Preference shock, stdev.	σ_H	0.010	0.025	0.023
Interest rate shock, stdev.	σ_R	0.010	0.004	0.004
Transaction cost shock, stdev.	σ_{TC}	0.010	0.013	0.012
Foreign output shock, stdev.	σ_{Y*}	0.010	0.006	0.005
Foreign inflation shock, stdev.	$\sigma_{\pi*}$	0.010	0.008	0.007
Foreign interest shock, stdev.	σ_{R*}	0.010	0.002	0.002

Table 5: Posterior Estimates for the Extended Model

	Parameters	Prior Mean	Posterior Mean	Posterior Mode
Behavioral Parameters				
Inverse intertemporal elasticity	σ	0.800	0.887	0.978
Inverse elasticity of labor supply	ν	1.590	0.451	0.403
Domestic-foreign substitution elasticity	η	1.120	0.712	0.693
Habit formation coefficient	h	0.400	0.917	0.934
Inflation indexation coefficient	κ	0.400	0.197	0.168
Transaction costs coef. for external debt	ψ_1	0.200	0.151	0.129
Transaction costs coef. for depreciation	ψ_2	0.700	0.37	0.369
Interest rate response to inflation	δ_π	1.500	2.408	2.426
Interest smoothing coefficient	δ_R	0.600	0.946	0.95
Money demand coefficient	μ	0.060	0.1	0.096
Response of tax revenue to debt	δ_{TR}	0.150	0.129	0.12
Calvo parameter for core	θ_c	0.600	0.875	0.876
Calvo parameter for food	θ_f	0.400	0.582	0.567
Calvo parameter for oil	θ_o	0.200	0.164	0.144
Shocks				
Preference shock, AR(1) coef.	ρ_H	0.500	0.728	0.762
Interest rate shock, AR(1) coef	ρ_R	0.500	0.502	0.495
Transaction cost shock, AR(1) coef.	ρ_{TC}	0.500	0.618	0.646
Inflation shock, AR(1) coef.	ρ_π	0.500	0.299	0.284
Foreign output shock, AR(1) coef.	ρ_{Y*}	0.500	0.758	0.765
Foreign inflation shock, AR(1) coef.	$\rho_{\pi*}$	0.500	0.331	0.318
Foreign interest shock, AR(1) coef.	ρ_{R*}	0.500	0.796	0.804
Govt. expenditure shock, AR(1) coef.	ρ_G	0.500	0.348	0.342
Govt. revenue shock, AR(1) coef.	ρ_{TR}	0.500	0.313	0.293
Core productivity shock, AR(1) coef.	ρ_{Yc}	0.500	0.635	0.683
Food productivity shock, AR(1) coef.	ρ_{Yf}	0.500	0.501	0.5
Oil shock, AR(1) coef.	ρ_o	0.500	0.796	0.826
Foreign food price shock, AR(1) coef.	ρ_{PRf*}	0.500	0.563	0.570
Foreign oil price shock, AR(1) coef.	ρ_{PRo*}	0.500	0.531	0.536

Table 5: Posterior Estimates for the Extended Model (cont.)

	Parameters	Prior Mean	Posterior Mean	Posterior Mode
Preference shock, stdev.	σ_H	0.010	0.059	0.051
Interest rate shock, stdev.	σ_R	0.010	0.004	0.004
Transaction cost shock, stdev.	σ_{TC}	0.010	0.013	0.012
Inflation shock, stdev.	ρ_π	0.010	0.006	0.006
Foreign output shock, stdev.	σ_{Y*}	0.010	0.006	0.005
Foreign inflation shock, stdev.	$\sigma_{\pi*}$	0.010	0.008	0.007
Foreign interest shock, stdev.	σ_{R*}	0.010	0.002	0.002
Govt. expenditure shock, stdev.	σ_G	0.010	0.047	0.046
Govt. revenue shock, stdev.	σ_{TR}	0.010	0.063	0.061
Money growth shock, stdev.	σ_M	0.010	0.005	0.004
Core productivity shock, stdev.	σ_{Yc}	0.010	0.127	0.11
Food productivity shock, stdev.	σ_{Yf}	0.010	0.11	0.087
Oil shock, stdev.	σ_o	0.010	0.094	0.087
Foreign food price shock, stdev.	σ_{PRf*}	0.010	0.07	0.069
Foreign oil price shock, stdev.	σ_{PRo*}	0.010	0.161	0.157

Table 6: RMSE Comparison of the new and current FPAS model

Forecast Horizon	New FPAS Model	Current FPAS Model
h = 1	0.79	0.96
h = 2	1.10	1.54
h = 3	1.05	1.85
h = 4	1.03	2.25
h = 5	1.03	2.36
h = 6	0.98	2.43
h = 7	0.99	2.84
h = 8	0.78	2.59

Table 7: Forecast Accuracy Comparison of New FPAS model with BVAR

	CPI Inflation Forecast		N. Interest Rate Forecast		Real GDP Forecast	
	New FPAS	Bayesian VAR	New FPAS	Bayesian VAR	New FPAS	Bayesian VAR
h=1	●		●		●	
h=2	●		●		●	
h=3	●		●		●	
h=4	●		●		●	
h=5	●		●		●	
h=6	●		●		●	
h=7	●		●		●	
h=8	●		●		●	

● Indicates better forecast performance in comparison, based on Diebold Mariano (1995)

Table 8: Forecast Accuracy Comparison of New FPAS model with DSGE VAR

	CPI Inflation Forecast		N. Interest Rate Forecast		Real GDP Forecast	
	New FPAS	DSGE VAR	New FPAS	DSGE VAR	New FPAS	DSGE VAR
h=1		●		●	●	
h=2		●		●	●	
h=3		●		●	●	
h=4		●		●	●	
h=5		●		●	●	
h=6		●		●	●	
h=7		●		●	●	
h=8		●		●	●	

● Indicates better forecast performance in comparison, based on Diebold Mariano (1995)

Table 9: CPI Inflation Forecasts

   a) Bayesian VAR Forecast Accuracy

	Forecast Horizon					
	h=1	h=2	h=3	h=4	h=5	h=6
Slope	-0.0886	-0.121**	-0.159**	-0.295***	-0.163***	-0.107**
	(0.094)	(0.055)	(0.056)	(0.080)	(0.028)	(0.045)
Intercept	0.00394*	0.0027	0.00225	0.000423	-0.00184	-0.00219
R^2	0.028	0.145	0.26	0.374	0.356	0.171
Observations	21	21	21	21	21	21

   b) DSGE VAR Forecast Accuracy

	Forecast Horizon					
	h=1	h=2	h=3	h=4	h=5	h=6
Slope	0.250*	0.0323	0.131	0.731	-0.697	4.759
	(0.128)	(0.408)	(0.608)	(1.183)	(2.207)	(6.004)
Intercept	0.00267	0.00323*	0.00320*	0.00182	0.0006	0.0018
R^2	0.06	0	0.002	0.019	0.004	0.056
Observations	21	21	21	21	21	21

   c) New FPAS Model Forecast Accuracy

	Forecast Horizon					
	h=1	h=2	h=3	h=4	h=5	h=6
Slope	0.228*	-0.371	-0.711**	-1.980**	-4.070***	-0.666
	(0.125)	(0.381)	(0.293)	(0.770)	(1.105)	(0.733)
Intercept	0.00275	0.00406**	0.00393*	0.00317	0.00246	-0.000103
R^2	0.049	0.037	0.067	0.21	0.392	0.008
Observations	21	21	21	21	21	21

Note: Standard errors in parentheses; *** $p<0.01$, ** $p<0.05$, * $p<0.1$

Table 10: Real GDP Forecasts

   a) **Bayesian VAR Forecast Accuracy**

	Forecast Horizon					
	h=1	h=2	h=3	h=4	h=5	h=6
Slope	-0.025	-0.169***	-0.0572	0.0614	0.0957	0.0906
	(0.204)	(0.048)	(0.067)	(0.079)	(0.085)	(0.065)
Intercept	-0.006	-0.007	-0.007	-0.005	-0.005	-0.003
R²	0.001	0.093	0.018	0.038	0.116	0.096
Observations	21	21	21	21	21	21

   b) **DSGE VAR Forecast Accuracy**

	Forecast Horizon					
	h=1	h=2	h=3	h=4	h=5	h=6
Slope	0.304	0.108	0.00172	-0.338	-0.671	-1.662
	(0.246)	(0.250)	(0.327)	(0.492)	(0.840)	(1.566)
Intercept	-0.007	-0.007	-0.007	-0.006	-0.006	-0.006
R²	0.057	0.006	0	0.012	0.017	0.031
Observations	21	21	21	21	21	21

   c) **New FPAS Model Forecast Accuracy**

	Forecast Horizon					
	h=1	h=2	h=3	h=4	h=5	h=6
Slope	0.513*	0.216	0.14	0.743	1.422	2.366
	(0.249)	(0.242)	(0.309)	(0.686)	(0.928)	(2.863)
Intercept	-0.005	-0.006	-0.006	-0.005	-0.004	-0.002
R²	0.179	0.028	0.006	0.056	0.081	0.046
Observations	21	21	21	21	21	21

Note: Standard errors in parentheses; *** $p<0.01$, ** $p<0.05$, * $p<0.1$

Table 11: Nominal Interest Rate Forecasts

a) Bayesian VAR Forecast Accuracy

	Forecast Horizon					
	h=1	h=2	h=3	h=4	h=5	h=6
Slope	0.363***	0.192	0.08	-0.000213	-0.0473	-0.0765*
	(0.125)	(0.125)	(0.093)	(0.065)	(0.051)	(0.043)
Intercept	0.004***	0.005***	0.005***	0.006***	0.006***	0.005***
R^2	0.279	0.123	0.036	0	0.029	0.081
Observations	21	21	21	21	21	21

b) DSGE VAR Forecast Accuracy

	Forecast Horizon					
	h=1	h=2	h=3	h=4	h=5	h=6
Slope	0.860***	1.086***	0.819***	0.684	1.005	1.695*
	(0.112)	(0.208)	(0.273)	(0.401)	(0.611)	(0.913)
Intercept	0.0007	0.0002	0.0029	0.004**	0.004**	0.004***
R^2	0.792	0.487	0.161	0.08	0.094	0.128
Observations	21	21	21	21	21	21

c) New FPAS Model Forecast Accuracy

	Forecast Horizon					
	h=1	h=2	h=3	h=4	h=5	h=6
Slope	0.853***	0.528	0.275	-0.394	-1.117**	-1.576***
	(0.114)	(0.354)	(0.445)	(0.321)	(0.513)	(0.474)
Intercept	0.0004	0.003	0.005**	0.008***	0.009***	0.008***
R^2	0.725	0.148	0.018	0.041	0.167	0.267
Observations	21	21	21	21	21	21

Note: Standard errors in parentheses; *** p<0.01, ** p<0.05, * p<0.1

Figure 1: Observed Variables for Basic Model, 2001-2015
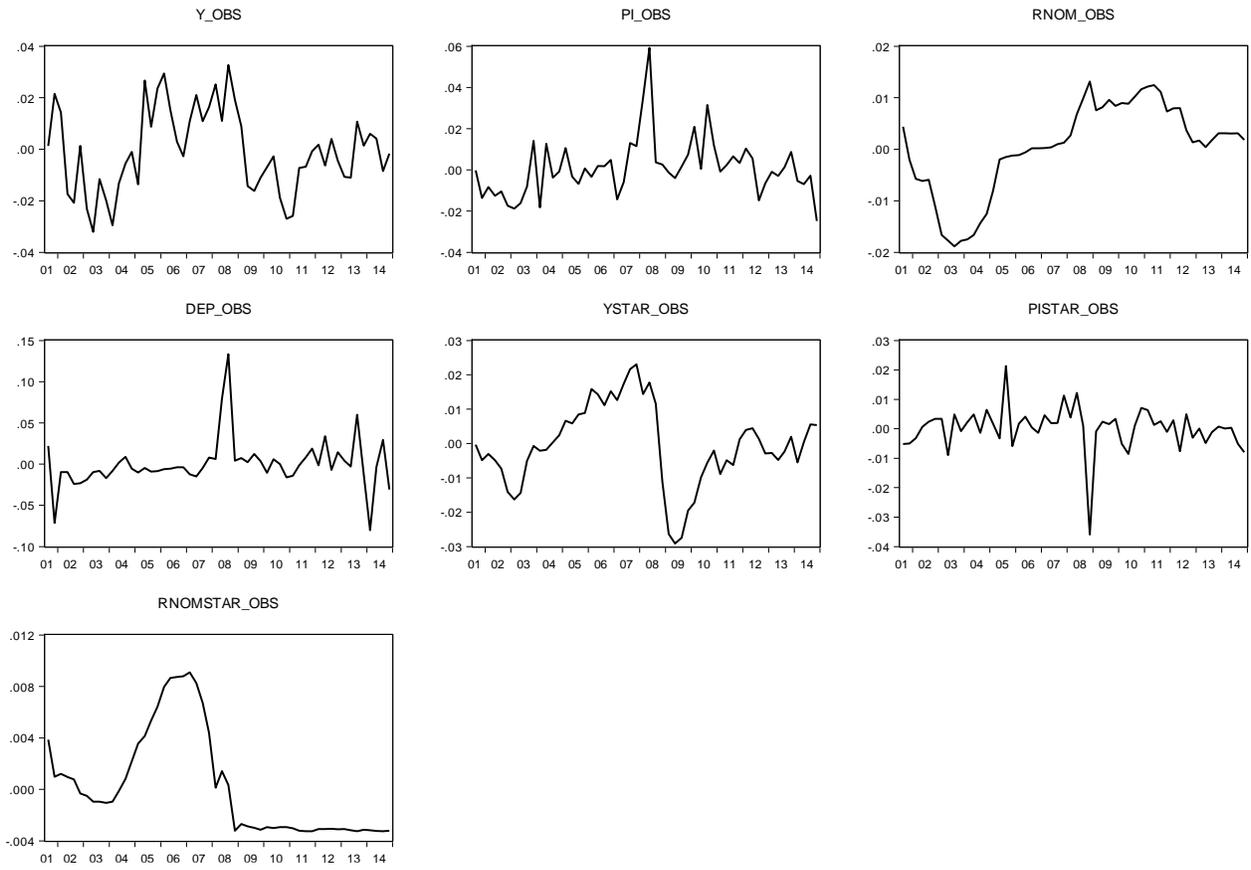
Figure 2: Additional Observed Variables, 2001-2015
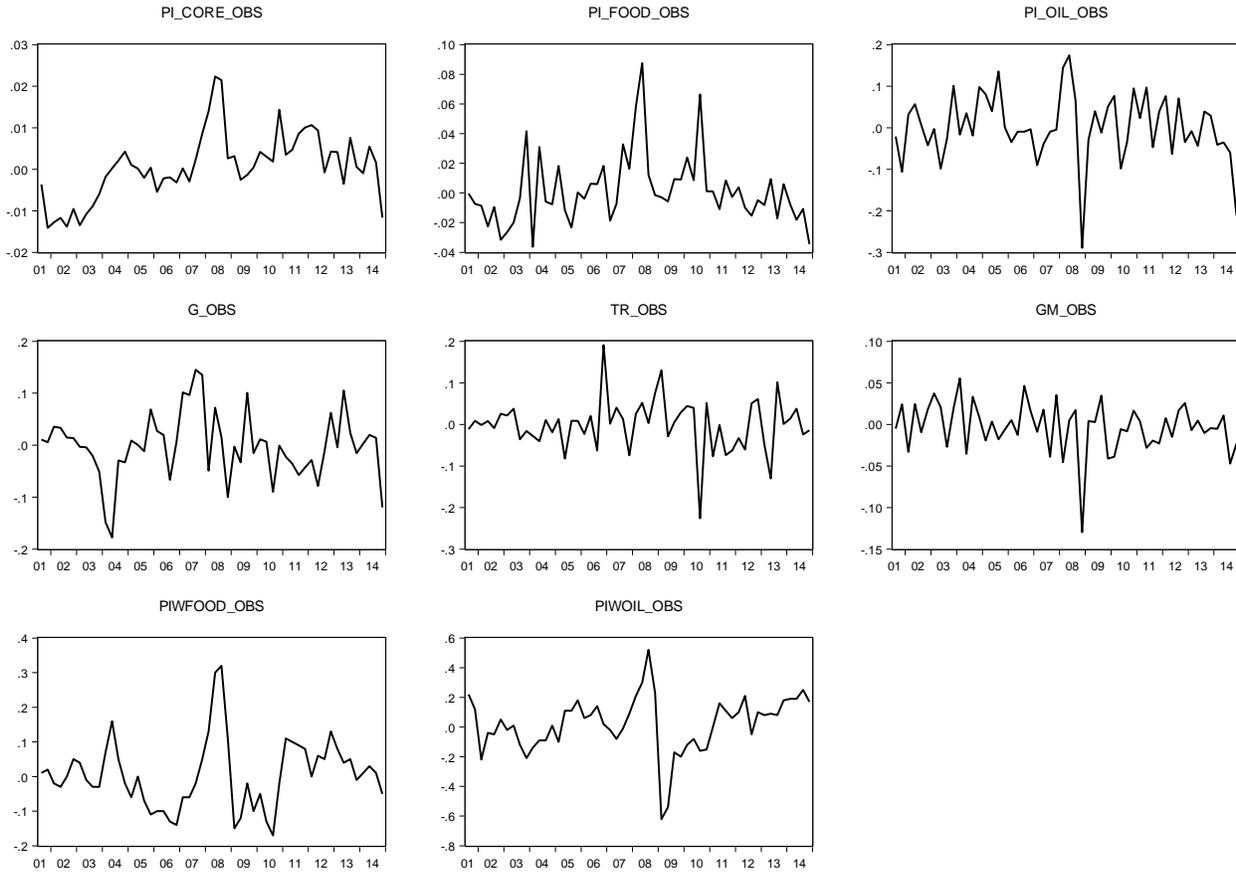
Basic Model Estimation Information

Number of MH draws	200,000
Discarded	20%
Number of chains	2
Acceptance ratio for 1st chain	31.57%
Acceptance ratio for 1st chain	31.50%
Log likelihood	1392.491

Extended Model Estimation Information

Number of MH draws	200,000
Discarded	20%
Number of chains	2
Acceptance ratio for 1st chain	29.81%
Acceptance ratio for 1st chain	29.87%
Log likelihood	2035.164

Figure 3: Posterior and Prior Distributions for the General Model
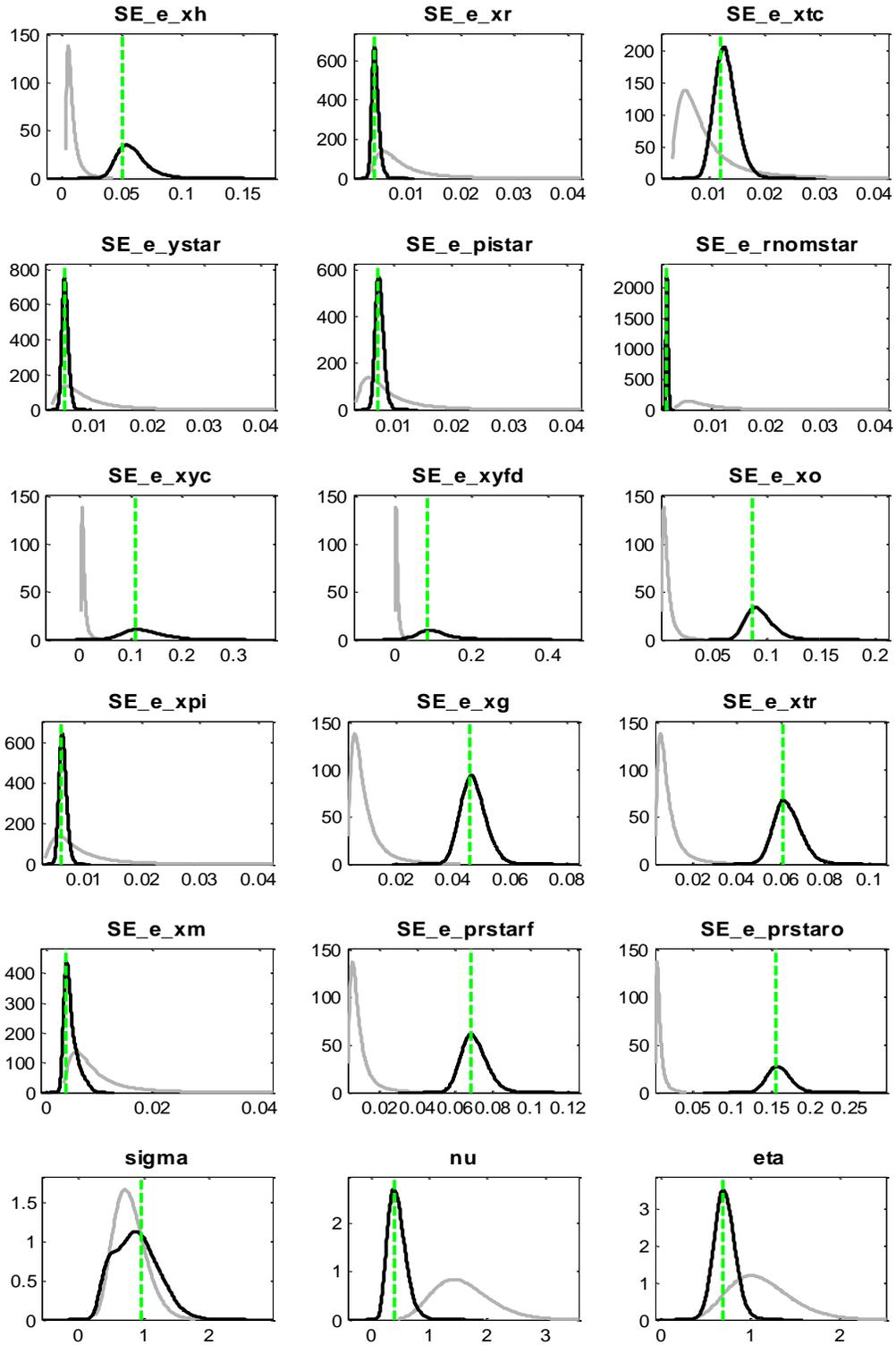
Figure 3: Posterior and Prior Distributions for the General Model (cont.)
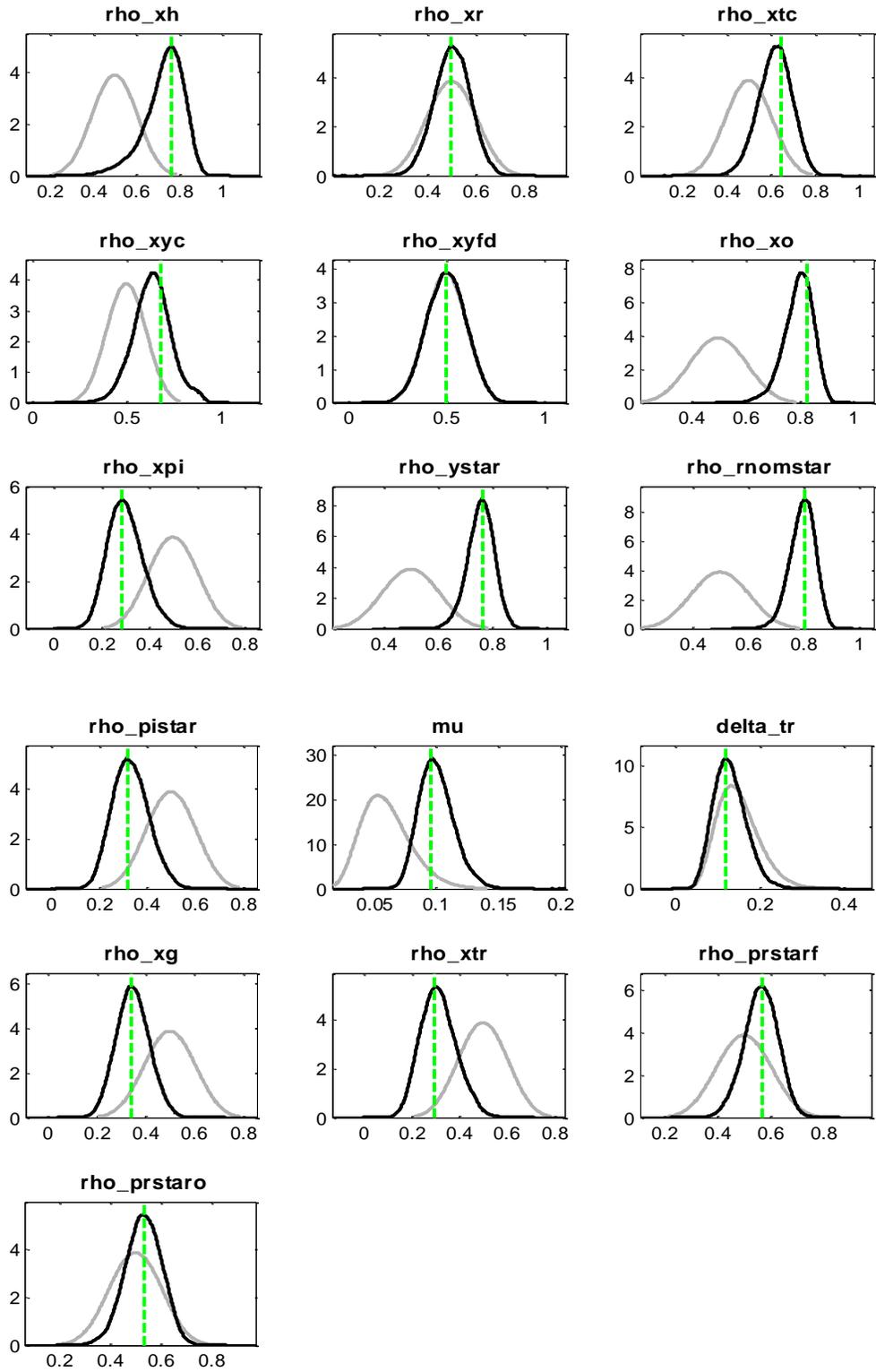
Figure 3: Posterior and Prior Distributions for the General Model (cont.)
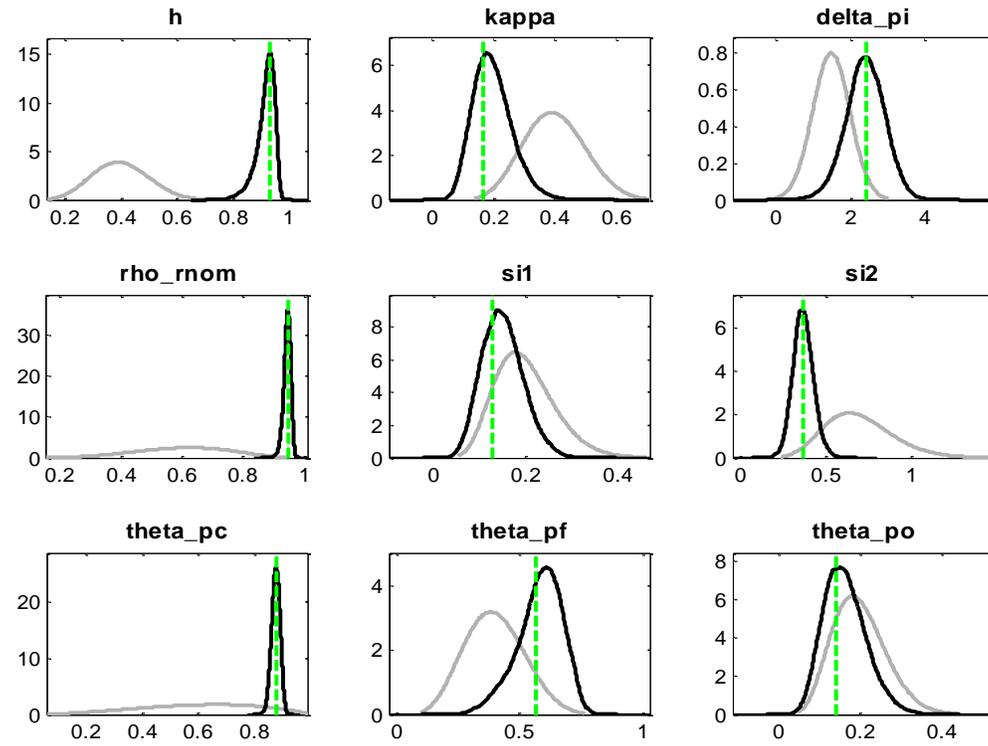
Figure 4: IRFs for Common Shocks to the Basic and General Model

Interest Rate Shock
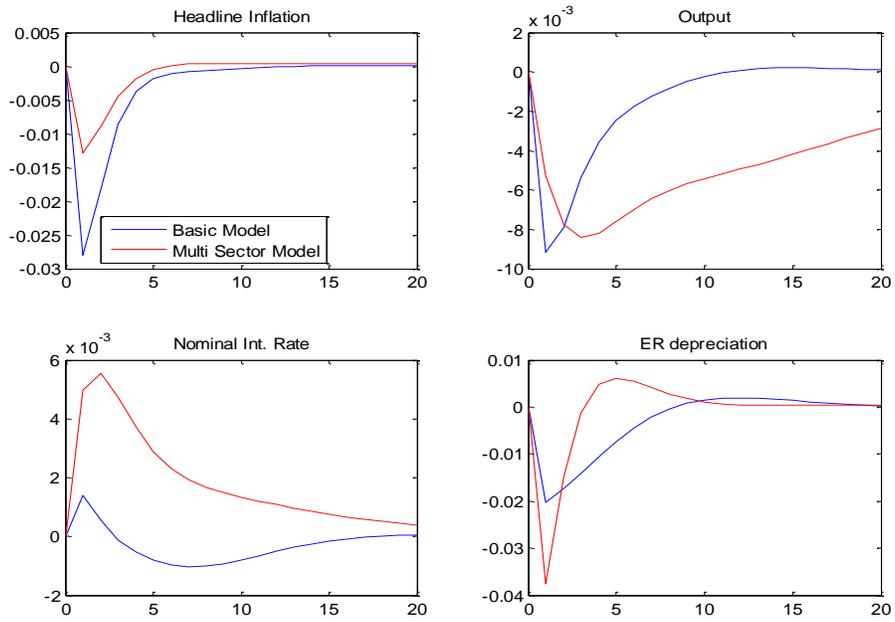
Consumer Preferences Shock
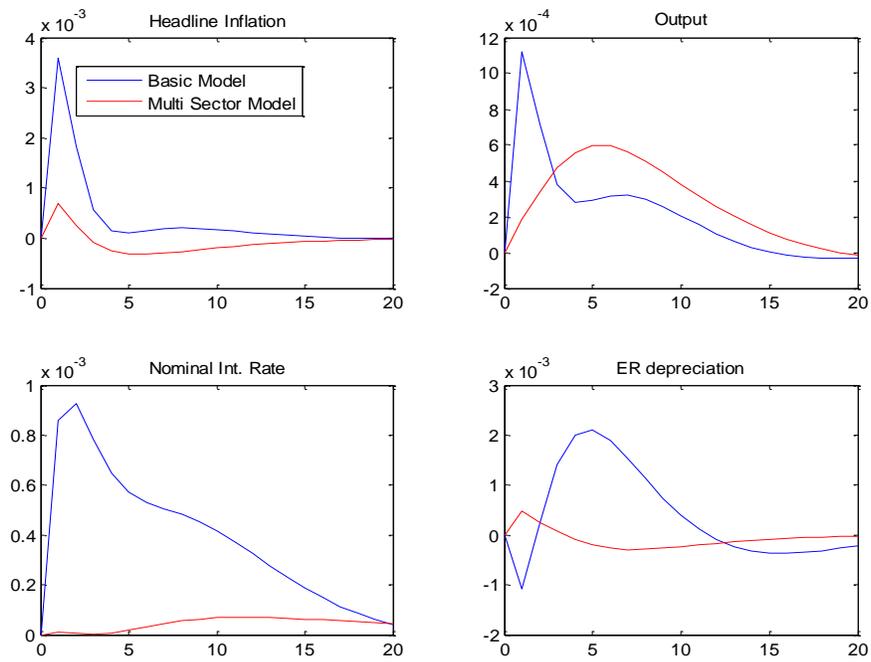
Figure 4: IRFs for Common Shocks to the Basic and General Model (cont.)

FX Risk Premium Shock
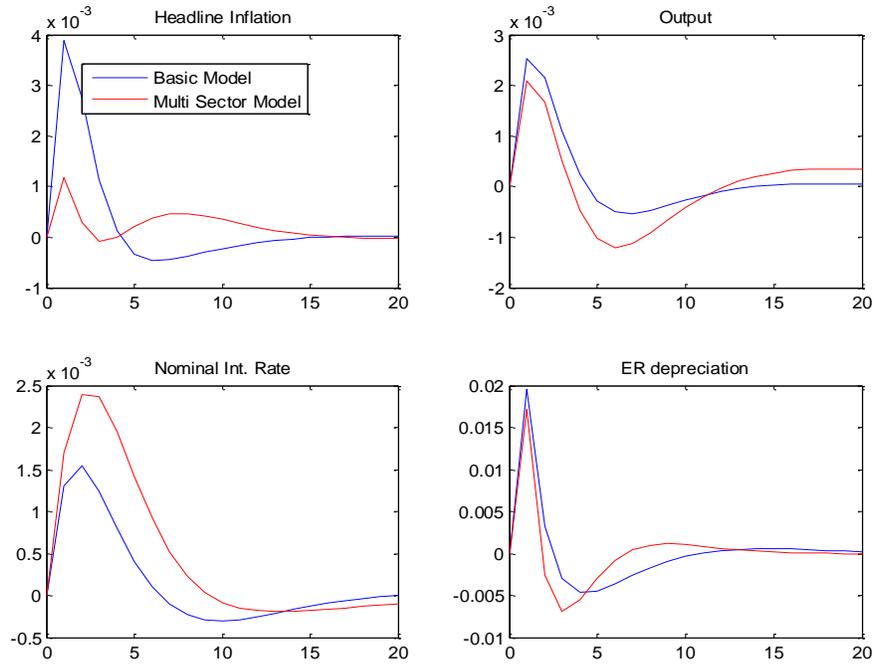
Foreign Output Shock
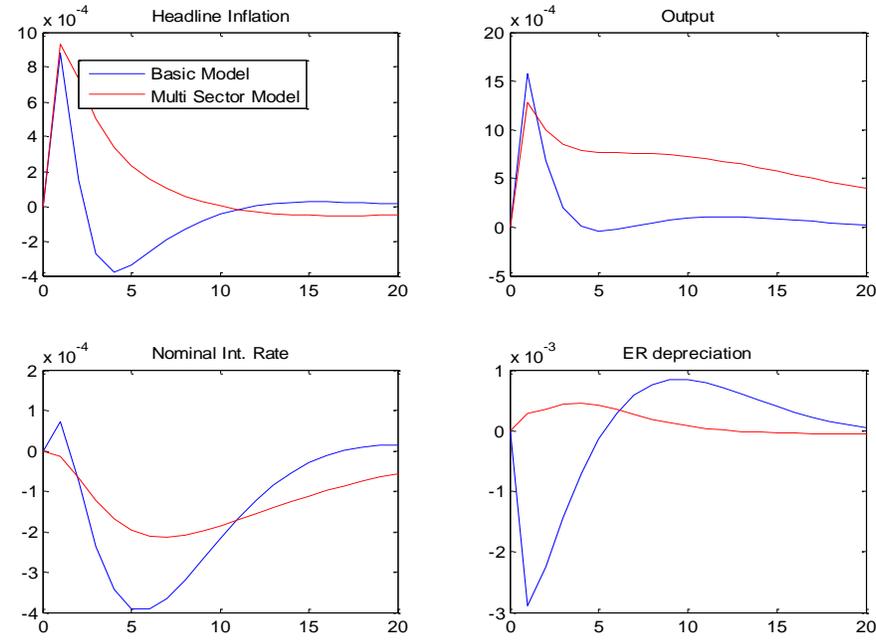
Figure 4: IRFs for Common Shocks to the Basic and General Model (cont.)

Foreign Interest Shock
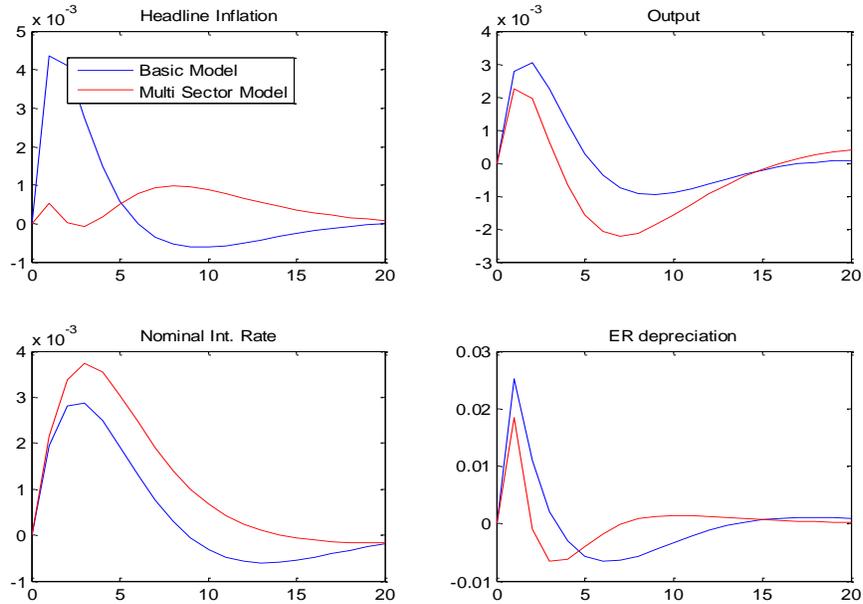
Foreign Inflation Shock
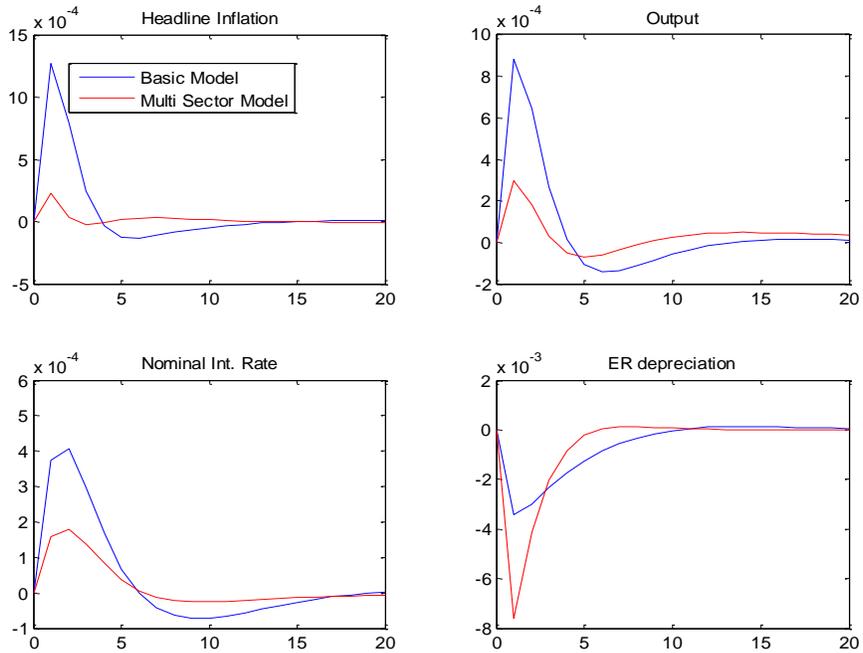
Figure 5: Comparison of Forecast performance of New IGC model and FPAS model
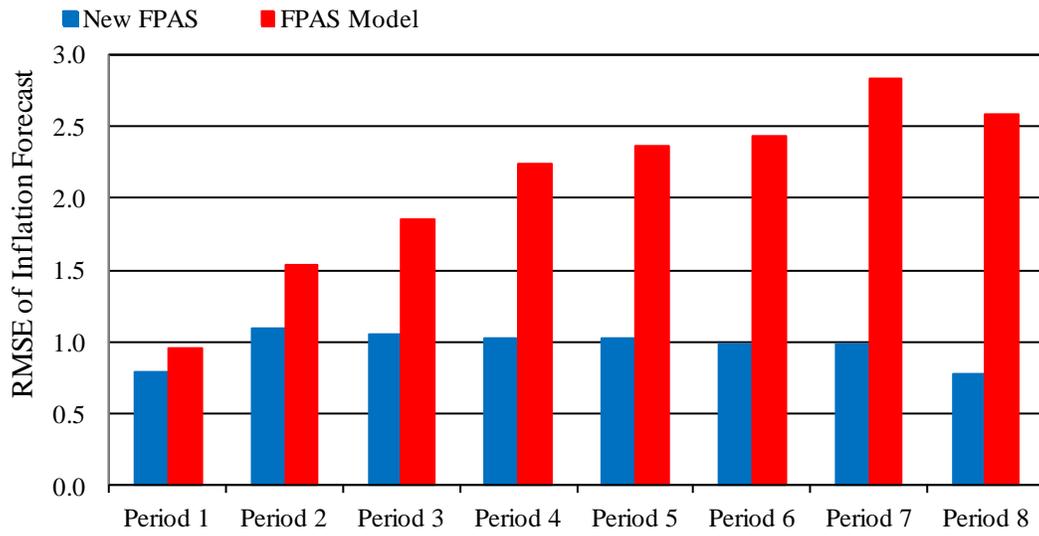
**Figure 6. RMSEs of Competing Forecasts**
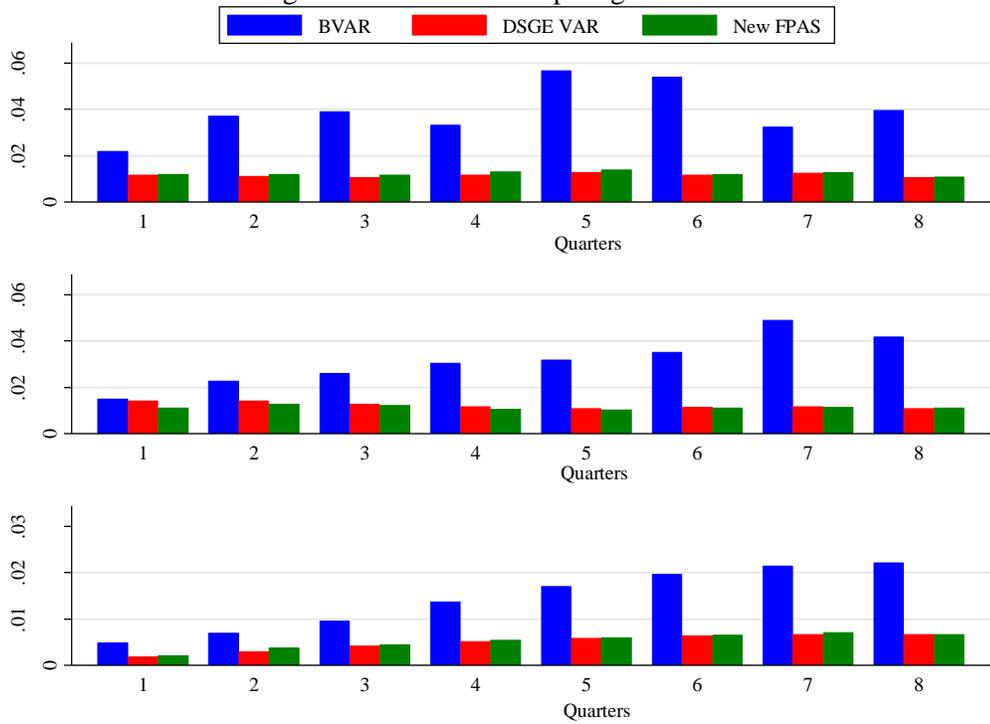
Figure 7. Relative Forecast RMSEs

New FPAS Relative to BVAR

New FPAS Relative to DSGE-VAR
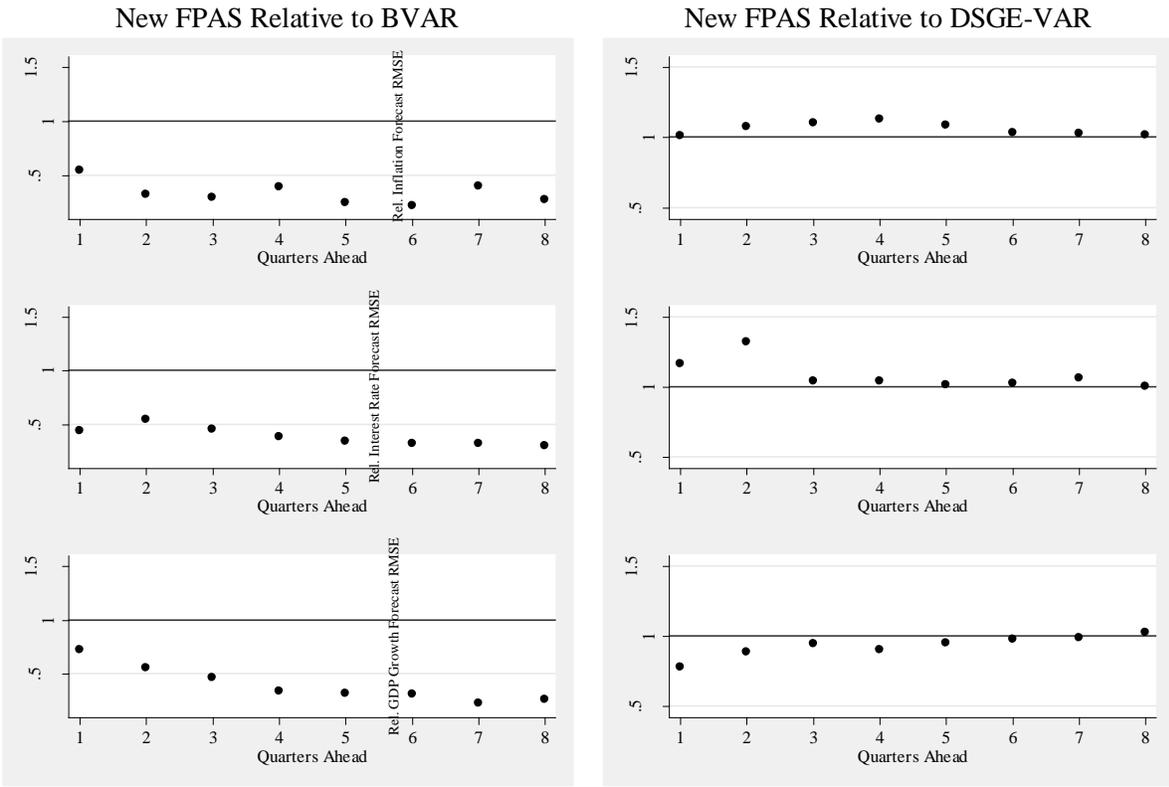
Figure 8. Models Forecast Comparison with Realized Values - Nominal Interest Rate
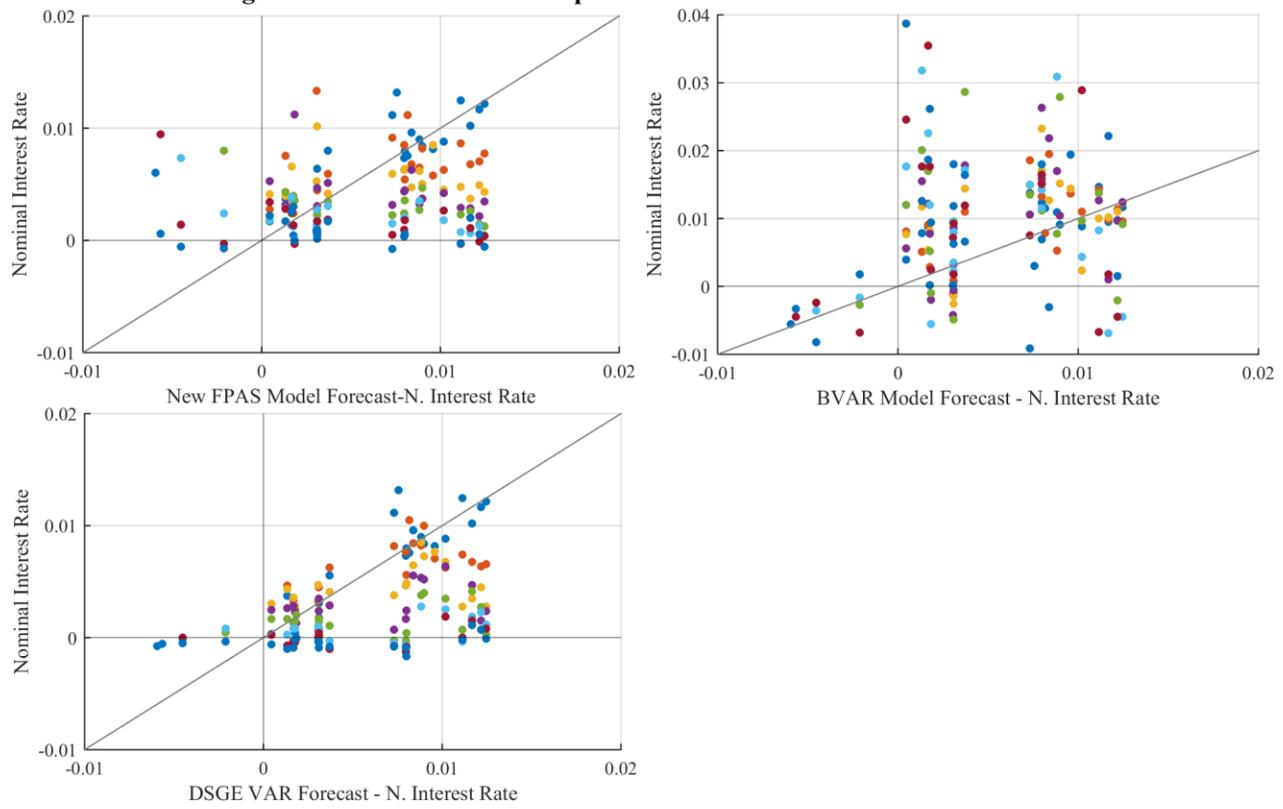
Figure 9. Models Forecast Comparison with Realized Values - CPI Inflation
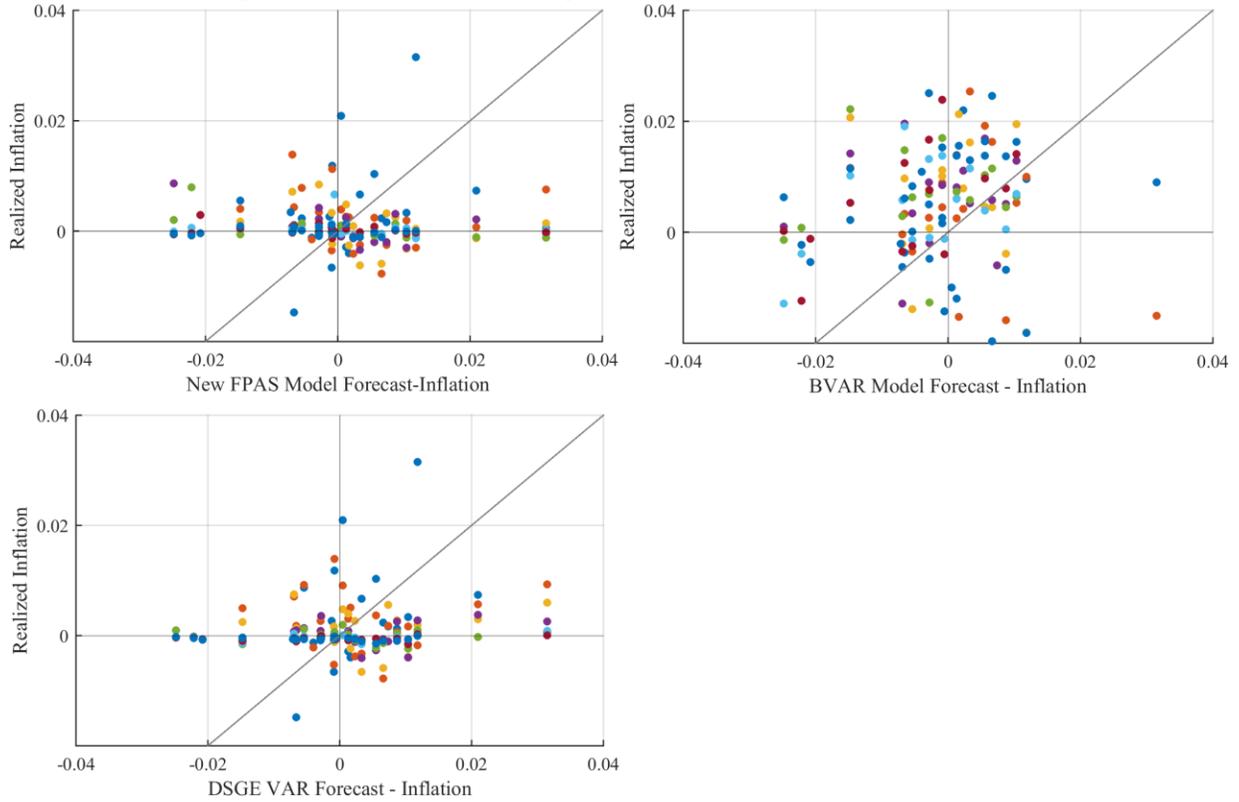

Figure 10. Models Forecast Comparison with Realized Values - Real GDP