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The Role of Money in Explaining Business Cycles for a Developing Economy:The Case of Pakistan [1]

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Abstract

This paper theoretically evaluates the role of money and monetary policy in propagating business cycle fluctuations of Pakistan's economy. We introduce the role of money via money in utility (MIU) and cash in advance constraint (CIA) in simple closed economy DSGE models and analyze monetary policy through a money growth rule as well as Taylor type interest rate rule. We establish the theoretical and empirical linkages between nominal and real variables of Pakistan's economy for post financial liberalization era. We find that the cash base economy models under money growth rule matches the data relatively better compared to cashless economy with Taylor rule.

JEL Classification: D58, E27, E52.

Key words: General Equilibrium Models, Modeling and Simulations, Monetary Policy

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Non-Technical Summary

There are a number of structural features of Pakistan economy that provide ample justification for studying and modeling the role of money in isolation. First, Pakistan ranks as one of the lowest in various financial access indicators such as number of per capita deposit accounts, loan accounts per thousand adults and bank branches per million adults among peer countries. This lack of financial access naturally leads to a high level of currency holding; a fact captured by Pakistan having higher level of currency in circulation when compared with peer nations. Second, the existence of a large informal sector induces economic agents to conduct a large number of transactions through cash instead of formal financial channels; high levels of currency in circulation are directly related to the size of informal sector. Hence, there is a clear need to study the role of money for the aggregate economy using both empirical analysis and theoretical models.

In order to establish a clear link between nominal and real side of economy over the course of business cycles, we conduct a comprehensive macro data analysis. The relationship between various measures of nominal and real side of the economy is explored. We find that:

- For the quarterly frequency, level of monetary aggregates show more association with fluctuations in output (GDP and LSM) as compared to inflation.
- In annual data, fluctuations in monetary aggregates are more closely associated with inflation as compared to output (GDP and LSM).
- The relationship between fluctuations in monetary aggregates and economic activity and inflation have stregnthened 2000 onwards.
- Nominal interest rates show positive association with inflation at both annual and quarterly frequency;
- There is almost no association between different lags of interest rates and inflation as well as GDP.
- The link between fluctuations in LSM and interest rates show that manufacturing sector is relatively more responsive to interest rate as compared to other sectors of the economy.

In addition to empirical analysis, we also theoretically evaluate the role of money and monetary policy in propagating business cycle fluctuations of Pakistan economy. Different theoretical models using different ways of introducing the role of money via money in utility (MIU) and cash in advance constraint (CIA) as well as with different formulation of monetary policy either through a money growth rule or Taylor type interest rate rule were employed for theoretical evaluation.

The results from our theoretical models show that inclusion of money and the way it is incorporated in models makes significant difference in model performance. The models with explicit role for money (MIU & CIA) under money growth rule exhibits better data matching as compared to model without money closed by a Taylor type interest rate rule in case of Pakistan. Furthermore, the impulse response functions of various models show that under given modeling structure and parameterization, the impact of monetary policy shock on inflation, GDP and LSM is limited and short lived.

1. Introduction

The monetary policy decisions are considered important factors of business cycle fluctuations in economic literature. The objective of this paper is to explore, for a developing nation like Pakistan, the role quantity of money and its price play in explaining output fluctuations.

To do so, this paper investigates the role of monetary aggregates and interest rate in propagating short run fluctuations of Pakistan economy over the period 1991-2012. Furthermore, after exploring empirical relationships between nominal and real side of economy, we use micro-founded Dynamic Stochastic General Equilibrium (DSGE) modeling setup with different specifications of monetary policy to theoretically explain these relationships.

In order to establish a clear link between nominal and real side of economy over the course of business cycles, we conduct a comprehensive macro data analysis. The relationship between various measures of nominal and real side of the economy is explored by using scatter plots, contemporaneous correlations, dynamic correlations, Granger causality tests and estimated vector autoregressions.

In particular, we analyze how different monetary aggregates i.e. M0, M1 & M2 and, interest rates i.e. policy rate, money market rate and T-bill rate dynamically affect and get affected by GDP and Large Scale Manufacturing. This data analysis has been conducted utilizing both annual and quarterly data series[3].

We find that quantitative instruments of monetary policy such as various monetary aggregates have far greater impact on the real side of Pakistan economy relative to the price indicators of monetary policy represented by various nominal interest rates.

The reason to study `money' is the way Pakistan economy is structured. First, Table C1 (see Appendix C) shows that among a set of peer developing countries, Pakistan ranks lowest in financial access as measured in terms of number of per capita deposit accounts indicator. Similarly the other two known indicators of financial access, loan accounts per thousand adults and bank branches per million adults, also reveal that financial access is relatively low in Pakistan. This lack of financial access to deposit accounts naturally leads to high level of currency holding; a fact captured by Pakistan having high levels currency in circulation when compared with peer nations (see Table C2 in the Appendix C).

Second, the existence of a large informal sector induces economic agents to conduct a large number of transactions through cash instead of formal financial channels; high levels of currency in circulation are directly related to the size of informal sector. The Table C3 shows that Pakistan has a relatively large and significant informal sector.

These structural features of Pakistan economy provide ample justification for studying and modeling in isolation the role of money in Pakistan economy.

[3] The business cycle component of different time series has been extracted by taking log and then applying the Hodrick Prescott filter for both annual and quarterly data.

In case, quantity of money matters more than its price for output have profound ramifications for New Keynesian DSGE models[4]. The DSGE models are widely being used in policy institutions in both developed and developing economies. However, the developed economies have taken the lead in estimating and using the DSGE models for policy-making.

In the recent past, there have been quite a few serious efforts to utilize these models in context of developing countries in general and Pakistan in particular. Garcıa-Cicco (2009), Florian and Montoro (2009), Peiris and Saxegaard (2007) and, Medina and Soto (2006 and 2007) are notable examples of the use of DSGE models for other developing countries. On the other hand, Choudhary and Pasha (2013), Haider et al. (2012), Choudhri and Malik (2012), Ahmad et al. (2012) and, Haider and Khan (2008) have used DSGE models to analyze economic issues in Pakistan.

However, all DSGE models based studies for Pakistan have certain limitations. For instance, Choudhary and Pasha (2013) and Ahmad et al. (2012) use real business cycle framework that abstracts from money and inflation dynamics. Moreover, models in these studies cannot be used for analysis of short run fluctuations as their parameters are based on calibrations from annual data. Choudhri and Malik (2012) lack appropriately estimated formulation of monetary policy and evaluation of simulated models. Haider et al. (2012) and Haider and Khan (2008) have not established any empirical linkages between nominal and real side of economy. This study seeks to fill this gap by first presenting stylized facts pertaining to implications of monetary policy actions for real side of economy and then presenting appropriately calibrated models.

In our models, we incorporate two alternate formulations of money holding: money in utility function and cash in advance constraint as well as two different ways to conduct monetary policy: monetary targeting and interest rate targeting in the basic New Keynesian DSGE framework.

The monetary targeting central bank conducts monetary policy through controlling supply of nominal money stock. Under this monetary policy approach, we stimulate money demand through money-in-utility-function (MIU) motives of holding money (Cooley and Hansen (1989, 1997 and 1998) and Svensson (1985)) and cash-in-advance constraint (CIA) (Clower (1967) and, Lucas and Stokey (1987)). The central bank is assumed to follow an autoregressive money supply growth rule in both the CIA and MIU approaches.

The interest rate targeting central bank uses Taylor rule-type interest rate reaction function in order to respond to fluctuations in inflation and output from their steady state values in a cashless economic environment. This approach implicitly assumes that central bank adjusts money supply to attain target level of interest rate. As a result, money becomes a redundant variable; generally not even explicitly included in the model. Some prominent examples of this approach in the literature are Smets and Wouters (2003 and 2007), Woodford (2003) and Clarida et al. (1999).

The rest of this paper is organized as follows. The next section presents our empirical findings on the impact of nominal variables on real economic activity. Section 3 discusses different model structures.

[4] These models are well known for having micro foundations, dynamic framework, capability to deal with stochastic shocks under rational expectations and overcoming the Lucas critique to some extent.

Section 4 discusses the calibration of various parameters, while Section 5 discusses our main results from different models and the last section concludes.

2. Stylized Facts' from Post Financial Liberalization Era

The empirical linkages between various macroeconomic variables discussed in this section are for the period 1991-2012. The choice of this particular time interval is based on the fact that starting from early 90s, Pakistani financial system underwent a set of structural reforms.

Before early 90s, the State Bank of Pakistan (SBP) used to conduct monetary policy through direct controls[5] e.g. variations in cash reserve ratio (CRR), statutory liquidity requirement (SLR) as bank rate was constant at 10% since 1977 to 1992[6]. The commercial banks were allocated credit ceilings under the credit plan by National Credit Consultative Council (NCCC).

The business cycle properties of any economy are generally captured by quarterly data. In Pakistan, national income accounts (NIA) are maintained only at annual frequency. However, there have been at least two serious efforts to work out quarterly national income accounts by Arby (2008) and Hanif et al.(2013). In this paper, we use quarterly series of national income accounts from Hanif et al. (2013) that provides data from first quarter of 1973 to last quarter of 2012.

We use data of real GDP, gross fixed capital formation, private consumption, large scale manufacturing index, and CPI inflation to represent real side of the economy. The nominal side of the economy is represented by M0, M1, M2, policy rate, 6-month T-bill rate and call money rate.

In order to further check the validity of co-movement patterns between real and nominal indicators coming from this quarterly data, we also conduct all the empirical exercises with annual data. This exercise with annual data has two main advantages. First, keeping in view that quarterly GDP and other national income account series at quarterly frequency are approximated, we need to confirm our findings by matching with actual data which is available at annual frequency only. Second, annual data allows investigation of stylized facts on a relative longer time horizon and we can infer about medium run implications of monetary policy.

In order to extract cyclical component from raw data, we seasonally adjust (for quarterly data only), take logarithms and detrend data using Hodrick Prescott filter with usual parameterization. The data constructed in this way represents short run fluctuations of a variable from its long run trend. While discussing and interpreting the results, we should always keep in mind that these facts pertain only to short run fluctuations from long run trend. For details about data sources and treatments, see Appendix 1.

This paper mainly focuses on relationships among economic activity, monetary aggregates, interest rates and inflation. In particular, we are interested in knowing how various specifications of money, monetary policy and interest rates affect (if at all) business cycle fluctuations and get affected by economic activity and inflation.

[5] While analyzing the desirability of such financial management system is clearly beyond the scope of this study, we believe that monetary policy under such administered environment cannot be modelled by optimization based models we intend to use in this study.
[6] For a detailed description of structural reforms, please see "Pakistan: Financial Sector Assessment 1990-2000" (2000), by State Bank of Pakistan and "Restructuring of Financial Sector in Pakistan" (2003) by Muhammad Nadeem Hanif.

2.1 Monetary Aggregates and Economic Activity

The most straight forward observation regarding the relationship between monetary aggregates[7]and GDP is that both nominal and real monetary aggregates are strongly procyclical at levels. The first two rows of scatter plots in Figure 1 and Figure 2 depict a clear positive relationship between monetary aggregates and GDP. The growth rate of monetary aggregates do not reflect a significant co-movement with GDP (3rd rows of scatter plots in Figure 1 & 2).

Similarly the real and nominal monetary aggregates at levels depict positive co-movement with LSM (Figure 3 & 4, row 1 & 2) whereas there is no clear link between monetary aggregate growth rates and LSM. The only exception is the annual M2 growth rate that shows slight positive co-movement with LSM (row 3 in Figure 3). These observations give confidence in *quarterly* GDP to use as a proxy of economic activity.

The contemporaneous correlations presented in Table A1 confirm these findings by showing that correlations of monetary aggregates with GDP and LSM are positive and statistically significant.

In order to better understand this strong pro-cyclical behavior of monetary aggregates, we try to investigate the direction of causation. The dynamic correlations between GDP and different lags of monetary aggregates (left panel of Figure 7A) are positive. This means that current GDP is positively associated with lagged monetary aggregates; indicating a leading indicator role being played by money. On the other hand, different leads of monetary aggregates also show positive correlations with GDP pointing out that higher income also causes higher money demand.

The dynamic correlations between monetary aggregates and LSM also show similar phenomenon as shown by the left panel of Figure 7B. The positive correlations at both leads and lags indicates two-way causality between money and economic activity.

The Granger causality test results presented in Table A2 and A3 seem to further endorse this two-way causality proposition. The nominal and real M1 and M2 Granger cause GDP and are Granger caused by GDP in the quarterly data (see Table A3). In annual data, real M1 and M2 show two-way Granger causality with GDP. The nominal monetary aggregates show a mixed pattern of causality in annual data; M1 causes GDP and M0 is caused by GDP. However, the growth of monetary aggregates seems to settle the issue of direction of causality. In quarterly data, growth of M1 and M2 Granger causes GDP whereas the converse is true in the annual data. This observation signals that in short run, monetary aggregates fluctuations cause fluctuations in GDP whereas in medium run (annual data) they are caused by GDP fluctuations.

In order to establish robustness of above mentioned relationships over time, we compare dynamic correlations calculated using full sample period (1990Q1-2012Q4) with the ones calculated using sample period 2000Q1-2012Q4. For the two sample periods, dynamic correlations of GDP and LSM fluctuations with leads and lags of monetary aggregates seem to preserve their overall shape (left panels in Figures 7A and 7B) and reflect a slight increase in magnitude in the recent time. This indicates that sensitivity of

[7] In our empirical analysis, averaging method was used for adjustment of outliers in the series of **M0** in 2001q2 and 2009q2, **M1** in 1998q1, q3 & q4, 2004q1 & q3 and **M2** in 2004q1 & q3. The series of M1 was adjusted for change of definition of demand deposits using splicing method from 2007 onwards.

economic activity to fluctuations in monetary aggregates has increased over time.

2.2 Monetary Aggregates and Inflation

This sub-section discusses the relationship between monetary aggregates and inflation. Nominal M0 and M2 depict positive association in both quarterly and annual data (Figure 5 & 6, row 1). On the other hand, Nominal M1 does not show any significant association with inflation at neither annual nor quarterly frequency. It is interesting to note that for nominal M0 and M2 the correlation in annual data is roughly double the value of correlation observed in quarterly data. The contemporaneous correlations in Table A1 show that M0 and inflation have a significant positive correlation at both quarterly and annual frequency. However, M2 has a significant positive correlation with inflation only at annual frequency.

The dynamic correlations between inflation and different leads and lags of monetary aggregates in left panel of Figure 7C shows a positive correlation between inflation and lagged monetary aggregates. Comparing this dynamic correlation pattern with the one reflected by monetary aggregates and GDP (left panel, Figure 7A), we observe an important difference. The dynamic correlations between inflation and monetary aggregates are smaller in magnitude and less persistent as compared to the ones observed in the case of GDP and LSM.

This analysis based on contemporaneous unconditional correlations between monetary aggregates and, inflation and output at an aggregate level suggests a strong pass-through of money to output rather than prices. The behavior of three monetary aggregates is quite similar in lag periods.

Furthermore, comparison of dynamic correlations for two different sample periods (left panel, Figure 7C) reveals that sensitivity of inflation to fluctuations in monetary aggregates has increased considerably while there is no major change in the signs of correlations.

The Granger causality test results presented in Table A3 show that none of the nominal monetary aggregates Granger causes inflation. Instead, both level and growth rate of M0 are Granger caused by inflation[8]. This observation is consistent with our finding about M0 in dynamic correlations. Furthermore, the real monetary aggregates show two-way Granger causality with inflation. In annual data, nominal M1 at level and nominal M2 in growth rate Granger causes inflation.

Another important observation regarding the role of money in Pakistan is related to the concept of fiscal dominance. The presence of fiscal dominance, which primarily manifests itself through monetary aggregates, in Pakistan implies another strong link between monetary aggregates and aggregate performance of the economy. Choudhri and Malik (2012) find that in presence of fiscal dominance not only there is volatility in inflation; the response of inflation to various shocks also gets amplified.

2.3 Interest Rates and Economic Activity

In general, nominal interest rates show slightly positive but statistically insignificant correlation with GDP at both quarterly and annual frequencies (Figure 1 & 2, row 4, Table A1). Large Scale Manufacturing seems to be independent of the three indicators of nominal interest rates at quarterly

[8] The bivariate Granger causality results should be interpreted with the caveat that other missing variables might better explain the relationship between the two variables.

frequency (Figure 4, row 4). However, LSM shows considerable negative co-movement with nominal interest rate at annual frequency (Figure 3, row 4). But these correlations are statistically insignificant (Table A1). These observations lead us to conclude that nominal interest rate fluctuations have little contemporaneous impact on fluctuations in real economic activity.

However, this is not the case with real interest rates. The various indicators of real interest rates show negative co-movement with both GDP and LSM at both annual and quarterly frequencies. In addition, the contemporaneous correlations in Table A1 show that correlations between GDP and LSM and, the three indicators of real interest rates are negative.

An important point to note here is that the correlations between LSM and real interest rates are statistically significant and stronger than the correlations between GDP and real interest rates. This finding is consistent with the trend of higher than propotional share of the manufacturing sector in bank credit compared to agriculture and services sector in Pakistan. This finding points towards the fact that manufacturing sector is more responsive to interest rate based monetary policy as compared to the rest of the economy.

The dynamic correlation plots (middle panel of Figure 7A & Figure 7B) reflect very weak correlations between lags of nominal interest rates and GDP. However, strong positive correlations between nominal interest rates and GDP are clearly visible at lead periods. This hints at a phenomenon where short term nominal interest rate fluctuations are lagging instead of leading fluctuations in GDP. These dynamic correlations seem stable over the sample period as neither the signs nor the magnitudes show any considerable difference over the two sample periods defined in Figures 7A and 7B.

The Granger causality tests confirm this point of view. The Granger causality tests using quarterly data in Table A3 reports that none of the nominal interest rates Granger causes GDP yet all of them are Granger caused by GDP. However, annual data shows bidirectional causality between interest rates and GDP.

2.4 Interest Rates and Inflation

The different indicators of nominal interest rates show slightly positive contemporaneous co-movement with inflation at both annual and quarterly frequencies (Figure 5 & 6, row 4). Table A1 shows that these positive contemporaneous correlations are statistically significant for six month T-bill rate and money market rate.

Unlike the contemporaneous correlations, dynamic correlations show a negative association between lagged interest rate and inflation (Figure 7C, middle panel). However, these correlations are very weak; not less than -0.1 for all indicators of nominal interest rate. The lead periods of nominal interest rates and inflation depict positive correlation that is suggestive of validity of Fisher Effect[9] in Pakistan.

Comparison of these dynamic correlations over the two sample windows points out that these relationships are stable over time (middle panel, Figure 7C)

The real interest rates are uncorrelated with inflation at annual frequency and strongly negatively

[9] According to the Fisher Effect, there is a one-to-one correspondence between expected inflation and nominal interest rate i.e. $i = r + \pi^e$

correlated with inflation at quarterly frequency (Figure 5 & 6, row 5).

The Granger causality tests show that neither nominal nor real interest rates Granger causes inflation in both quarterly and annual data (Table A2 & A3). However, inflation does Granger causes nominal interest rates. This reinforces our prior observation regarding the validity of Fisher Effect.

2.5 Vector Autoregression Models

Vector autoregression (VAR) models have become one of the most important tools to assess the impact of monetary policy on various macroeconomic variables. In this section, we use VAR models to analyze the effects of money supply and interest rate shocks on output and inflation using quarterly data for the period 1990-2012.

We estimate VAR models using the following equation:

$$y_t = c + bt + \sum_{l=1}^{L} A_l y_{t-l} + \varepsilon_t \qquad (1)$$

where y_t is a vector of endogenous variables included in estimation with L lags. A_l is a matrix of parameters to be estimated, c and t represents constant and time trend as exogenous variables and b is a vector of coefficients associated with time trend. ε_t is a vector of error terms that are uncorrelated with their lagged values and other explanatory variables. In our case,

$$y_t = \begin{bmatrix} \ln GDP_t \\ ln\ Gvt\ Cons_t \\ \ln Pvt\ Cons_t \\ \ln \pi_t \\ \ln Pvt\ Inv_t \\ R_t \\ \ln \omega_t \end{bmatrix}$$

Where GDP_t $\mathrm{Gvt\ Cons}_t$, $\mathrm{Pvt\ Cons}_t$, π_t, $\mathrm{Pvt\ Inv}_t$, R_t and ω_t represent, seasonally adjusted, real gross domestic product, government consumption, private consumption, gross inflation[10], real private investment, gross nominal interest rate[11] and gross money growth rate[12,] at time t respectively. Whereas, ln represents the natural log.

We mostly follow the identification ordering used in Christiano et al. (2005). This identification assumes that investment, inflation, private and public consumption and GDP do not respond contemporaneously to monetary policy shock. In general, the stationarity of different time series is checked and ensured before using them in VAR models. However, we only take the natural log of different time series and do not perform any filtering or differencing before using the data in VAR models. We are aware that it is very likely that most of our time series are non-stationary.

However, there are number of studies that use VAR models on non-stationary data. Enders (2010) cites Sims (1980) and Sims, Stock and Watson (1990) where they oppose differencing time series for VAR even if they contain a unit root. The argument behind opposition of differencing is that the main objective of VAR is to investigate inter-linkages among various variables and not parameter estimations. The

[10] Gross inflation and money growth are defined as: $\pi_t = \frac{P_t}{P_{t-1}}$ and $\omega_t = \frac{M_t}{M_{t-1}}$.

[11] Six month T-bill rate

[12] M2

differencing might result in loss of important information content in data. DeCecio and Nelson (2007) and Christiano et al. (2005) have used non-stationary time series in their VAR models for comparison and estimation of DSGE models.

The impulse response functions from quarterly VAR model are presented in Figure 8[13][14].

In response to a nominal M2 growth rate shock[15], we see that output increases due to increase in money supply as reflected in the left panel of Figure 8. Similarly, inflation shows an initial dip in the impulse response function before increasing above its steady state value and eventually returning to its steady state path. The decline in interest rate causes an expansion in output. On the other hand, inflation shows a positive response to a negative interest rate shock but exhibits a dip after about five quarters.

We compute variance decompositions of GDP and inflation to assess relative importance of different variables for explaining variations in both of the variables. In Figure 13, we can see that variations in LGDP and INFQoQ are chiefly explained by their own fluctuations rather than any other variable. Money growth and interest rate play negligible role in explaining variations in GDP and inflation. Table A4 shows that, on average, TBR6 explains 5.95% and GM2 explains 2.10% of total variation in LGDP. Similarly, Table A5 shows that, on average, TBR6 explains 2.61% and GM2 explains 3.25% of total variation in quarter on quarter inflation.

2.6 Summary of 'Empirical Stylized Facts'

In order to focus on the main empirical facts of linkages between nominal and real side of Pakistan economy over the last two decades, let's recall the main findings discussed in this section:

2.6.1 Monetary Aggregates

- Nominal monetary aggregates are pro-cyclical and reflect two-way causality with real variables.
- In short run, monetary aggregates show more association with fluctuations in economic activity indicators.
- In medium run, fluctuations in monetary aggregates are more associated with inflation.
- Over the time, sensitivities of economic activity and inflation to fluctuations in monetary aggregates have increased.
- Role of montary aggregates in explaining total variations in output and inflation is limited.

[13] These responses are obtained for 20 quarters.

[14] VAR IRFs are not statistically significant from zero if zero line is contained between IRF standard error graphs (+/- 2SE dotted lines). The moment a standard error line intersects zero line, IRF becomes statistically insignificant. If we compare IRFs of output and inflation in Pakistan and USA (Christiano et al. (2005)), we might find a fair deal of resemblance in both economies' responses in terms of shapes of IRFs. However, significance criteria show that output IRF to monetary policy shocks is significant for more than 10 quarters in USA. The same is insignificant for the case of Pakistan.

[15] We also estimated VAR model with real money growth and found similar IRFs for interest rate and money growth shocks.

2.6.2 Interest Rates

- Nominal interest rate indicators show positive contemporaneous correlations with inflation; potentially reflecting tightening of interest rate based monetary policy in response to heating economic environment and vice versa.
- However, effectiveness of such policy appears to be quite limited as there is only negligible correlation between different lags of interest rate indicators and inflation as well as GDP.
- Fluctuations in LSM; however, show that manufacturing sector is relatively more responsive to interest rate based monetary policy as compared to the rest of the economy.
- Role of interest rates in explaining total variations in output and inflation is limited.

2.6.3 Overall

- According to our empirical analysis for period 1990-2012, we find that quantitative measures of money (M0, M2) appear to be more connected with the economic system as compared to price measures of money (six month T-bill rate,money market rate) in Pakistan.
- Furthermore, sensitivity of real economic variables towards monetary aggregates has increased over the last decade.
- Role of monetary aggregates and interest rates in explaining variations in output and inflation is limited.

3. Modeling Framework

For our basic modeling framework, we use a closed economy dynamic New Keynesian model (DNK) with households, intermediate good producing firms, final good producing firms and a central bank. This framework is fairly standard and closely resembles the models presented in Mc Candless (2008) and Walsh (2010). Each household derives utility from consuming final goods, leisure, and real money balances[16] and also invests in physical and financial assets. In addition, household also provides indivisible labour and rent out capital to intermediate good producing firms. The intermediate good producing firms produce differentiated goods which give them leverage to set prices in a monopolistically competitive environment. The final good producing firms package intermediate goods to produce the homogeneous final good and sell it to households in a perfectly competitive environment. Finally, the central bank controls money supply either through monetary aggregates or interest rate.

In order to remain consistent with our empirical findings, we will model monetary policy for both monetary targeting and interest rate rule based regimes. We found earlier that monetary aggregates have relatively strong linkages with fluctuations in economic activity and we will now be able to theoretically evaluate our empirical findings with different ways of incorporating money and monetary policy in a simple DSGE model framework.

[16] Only in case of MIU models

3.1 Firms

3.1.1 Final Good Producing Firms

The final good producing firms produce final good for consumption and investment by combining the differentiated goods produced by intermediate good producers according to the following Dixit-Stiglitz bundling technology:

$$y_t = \left[\int_0^1 (y_t^j)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} dj \right]^{\frac{\varepsilon_p}{\varepsilon_p - 1}} \qquad (2)$$

Here y_t^j, y_t and ε_p represent intermediate good produce by j^{th} intermediate firm, final output and constant elasticity of substitution between intermediate products, respectively. For given price and elasticity of substitution, the final good producers choose the quantity y_t^j of each intermediate good in such a way that maximizes their profit. The result of this profit maximization is the following demand function for the j^{th} intermediate good:

$$y_t^j = \left[\frac{P_t^j}{P_t} \right]^{-\varepsilon_p} y_t \qquad (3)$$

The equation (3) shows that the demand for intermediate good j is inversely related to its relative price and directly related to aggregate output. Aggregating across all intermediate goods and using equation (2), we get the aggregate price level

$$P_t = \left[\int_0^1 \left(P_t^j\right)^{1-\varepsilon_p} dj \right]^{\frac{1}{1-\varepsilon_p}} \qquad (4)$$

3.1.2 Intermediate Good Producing Firms

Intermediate goods producers demand capital and labour for given wages and rental rate of capital in competitive factors market. In addition, they set price of their differentiated product while exploiting some degree of monopoly and considering uncertainty regarding their ability to change prices in future. The Calvo (1983) model is used to capture the intermediate good producing firms' behavior under this uncertainty.

Demand for Labour and Capital

The intermediate good producing firms are assumed to follow a Cobb-Douglas production function with constant returns to scale (CRS)

$$y_t^j = \exp(a_t)\left(k_t^j\right)^{\theta}\left(h_t^j\right)^{1-\theta} \qquad (5)$$

where $a_t = \rho_a a_{t-1} + \varepsilon_t^a$ is a stochastic technology shock that affects all intermediate firms in the same way. ρ_a is the persistence parameter and $\varepsilon_t^a \sim N(0, \sigma^a)$ is an i.i.d. random shock to total factor productivity (TFP). The parameter θ is the share of capital in production. Also, k_t^j and h_t^j are physical capital and labour utilized by firm j respectively.

The intermediate good producers minimize total cost, $TC_t = W_t^j h_t + R_t^j k_t$ subject to available production function embodied in equation (5). Here W_t and R_t^k are nominal wage rate and nominal rental return rate on capital, respectively. The cost minimization implies following optimal capital to labour ratio:

$$\frac{k_t^j}{h_t^j} = \frac{\theta}{1-\theta}\frac{w_t}{r_t^k} \tag{6}$$

Here w_t and r_t^k are real wage rate and real rental return rate on capital, respectively. Since capital to labour ratio is same across all intermediate firms, we can rewrite the above equation as

$$\frac{k_t}{h_t} = \frac{\theta}{1-\theta}\frac{w_t}{r_t^k} \tag{7}$$

Using equation (7) and performing some simple algebraic manipulations gives equilibrium real marginal cost as a function of technology level and factor prices.

$$mc_t = \frac{1}{\exp(a_t)}(\theta)^{-\theta}(1-\theta)^{-(1-\theta)}(w_t)^{1-\theta}\left(r_t^k\right)^{\theta} \tag{8}$$

Pricing by Intermediate Good Producing Firms

The Calvo (1983) pricing model assumes that a firm cannot change the price of her product until she gets a random "green signal" from nature[17]. If a firm gets this signal, she re-optimizes her price to P_t^*, otherwise, it is kept fixed at the previous price level, P_{t-1}. In each period, the probability of receiving the green signal is $1-\varepsilon_p$. This means that with probability, ε_p, price of an intermediate good producers will remain fixed at previous period price level, P_{t-1}.

In other words, ε_p can be interpreted as a price stickiness index; where $\varepsilon_p = 0$ means perfectly flexible and $\varepsilon_p = 1$ means fixed prices. However, $\varepsilon_p \in (0,1)$ reflects the more relevant case of sticky prices. Keeping in view this uncertainty regarding price change and their downward sloping demand curve described in equation (3), the j^{th} intermediate good producing firm maximizes the following profit function with respect to P_t^*.

$$\Omega = E_t \sum_{k=0}^{\infty} \left(\beta\varepsilon_p\right)^k \left[P_t^* y_{t+k}^j - P_{t+k} y_{t+k}^j mc_{t+k}\right] \tag{9}$$

The solution to this dynamic optimization problem is the following optimal price

$$P_t^* = \left[\frac{\varepsilon_p}{(\varepsilon_p - 1)} \frac{E_t \sum_{k=0}^{\infty} \left(\beta\varepsilon_p\right)^k P_{t+k} y_{t+k}^j mc_{t+k}}{E_t \sum_{k=0}^{\infty} \left(\beta\varepsilon_p\right)^k y_{t+k}^j}\right] \tag{10}$$

Now using equation (4) and calvo probability, the overall price level of economy can be expressed as:

[17] This is Calvo's original terminology; this random "green signal" is referring to the fact that each firm in this set up can only adjust their prices with probability $1-\varepsilon_p$ where $\varepsilon_p \in [0,1]$.

$$P_t = \left[\varepsilon_p P_t^{1-\varepsilon_p} + (1-\varepsilon_p) P_t^{*1-\varepsilon_p}\right]^{\frac{1}{1-\varepsilon_p}} \qquad (11)$$

The equations (5), (7), (8), (10), (11) of production function, capital to labour ratio, marginal cost, optimal price and general price level constitute the fixed frame of our different models.

3.2 Monetary Targeting Models

Monetary targeting models allow explicit incorporation of money in economic decision making by households and central bank. On the basis of our empirical findings related to important role of money in explaining business cycle fluctuations, there exists a strong justification for presence of money in the model.

In literature, money in utility function (MIU) and cash in advance constraint (CIA) are the two most popular approaches of creating positive money demand in general equilibrium models.

MIU model assumes that holding money yields direct utility to household, even if it has no intrinsic value. As discussed in the introduction and in Table C1, C2 & C3 (see Appendix C), structural features constitute a straightforward motivation for utilizing the MIU approach. Among these features, limited access to financial services and relatively high level of currency holding are the important ones.

Similarly, one may also establish the need for 'cash-in-advance,' which assumes that households must have stock of money available to conduct transactions related to purchase of consumption goods. The main reason for the relevance of CIA type model in Pakistan is the existence of a large informal sector which induces economic agents to conduct a large number of transactions through cash instead of cashless financial channels.

Therefore, these structural features of Pakistan economy together with the empirical money-output link provides the motivation for using Money in Utility and Cash in Advance constraint models to study the role of money and monetary policy on the real side of Pakistan economy.

3.2.1 Money in Utility Function (MIU) Model

Originally due to Sidrauski (1967), this approach assumes that presence of real money balances yield direct utility to households. The model economy is assumed to consist of a continuum of identical households of unit mass indexed by $i \in [0,1]$. Each household i maximizes her lifetime expected utility function given by

$$U^i = E_t \sum_{t=0}^{\infty} \beta^t \left[lnc_t^i + Aln(1-h_t^i) + Dln\left(\frac{M_t^i}{P_t}\right)\right] \qquad (12)$$

Here $\beta \in (0,1), c_t^i, A, D, \frac{M_t^i}{P_t}$ and h_t^i represent discount factor, real consumption, weight of leisure in the utility function, money preference, real money balances and fraction of total time spent at work respectively.

The labour markets of developing economies are generally characterized by employment contracts over longer period of time, we assume labour is indivisible following Hansen (1985)[18]

$$U^i = E_t \sum_{t=0}^{\infty} \beta^t \left[lnc_t^i + \bar{A} h_t^i + Dln\left(\frac{M_t^i}{P_t} \right) \right] \qquad (13)$$

Each period i^{th} household receives a transfer of money equal to $(\varpi_t - 1)\left(\frac{M_{t-1}}{P_t} \right)$ from the central bank, where ϖ_t is the gross growth rate of aggregate money supply and M_t is per capita nominal stock of money in period t.

It is important to distinguish between M_t^i and M_t. M_t^i represents household specific nominal money stock and it is a choice variable in household optimization problem. On the other hand, M_t represents per capita nominal money stock; control variable by central bank that cannot be affected by decisions of a single household. However, both of these variables would be equal in the symmetric steady state.

The central bank controls money supply by following a simple money growth rule

$$M_t = \varpi_t M_{t-1} \qquad (14)$$

where

$$ln\,\varpi_{t+1} = (1-\rho_\varpi) ln\,\varpi + \rho_\varpi ln\,\varpi_t + \varepsilon_{t+1}^{\varpi} \qquad (15)$$

is an autoregressive stochastic variable, $\rho_\varpi \in (0,1)$ is the persistence of money supply and $\varepsilon_t^{\varpi} \sim N(0, \sigma_\varpi)$ is the money supply shock. Under given conditions, the households' real stock of money evolves according to the following law of motion:

$$\frac{M_t^i}{P_t} = \frac{M_{t-1}^i}{P_t} + (\varpi_t - 1)\frac{M_{t-1}}{P_t} \qquad (16)$$

The equation (16) shows that i^{th} households' stock of real money balances in time period t is a sum of previous periods' stock of real money balances, $\frac{M_{t-1}^i}{P_t}$ and net transfer of money from the central bank in

[18] In this setting, it is assumed that each period every household has a random probability α_t^i of getting employment contract. Every household provides a fixed amount of labour h_0 after getting employment. Since probability of getting employment is α_t^i and fixed amount of labour to be supplied is h_0 therefore expected labour supply in a given period is $h_t^i = \alpha_t^i h_0$ or $\alpha_t^i = \frac{h_t^i}{h_0}$

To ensure the convexity of consumption set, this set up assumes perfect employment insurance scheme in which each household gets same compensation irrespective of her employment status (for detail, see Hansen (1985)). Expected value of one period utility from leisure is given as

$\alpha_t A\ln(1-h_0) + (1-\alpha_t) A\ln(1-0)$. Using the relationship $\alpha_t^i = \frac{h_t^i}{h_0}$ and the fact that $\ln 1 = 0$, life time utility function becomes

$U^i = E_t \Sigma_{t=0}^{\infty} \beta^t \left[\ln c_t^i + \frac{A\ln(1-h_0)}{h_0} h_t^i \right]$. Using the shorthand notation $\frac{A\ln(1-h_0)}{h_0} = \bar{A}$, we get $U^i = E_t \Sigma_{t=0}^{\infty} \beta^t \left[\ln c_t^i + \bar{A} h_t^i \right]$

Since $1-h_0 < 1$, therefore $\ln(1-h_0) < 0$ and $\bar{A}$ is bound to be a negative number showing that labour supply creates disutility.

current period, $\left(\varpi_t - 1\right)\dfrac{M_{t-1}}{P_t}$.

After some algebraic manipulation, we get

$$\frac{M_t^i}{P_t} = \frac{M_{t-1}^i}{P_t} + \left(\frac{\varpi_t - 1}{\varpi_t}\right)\frac{M_t}{P_t} \tag{17}$$

The budget constraint faced by each household is

$$c_t^i + inv_t^i + \frac{B_t^i}{R_t P_t} = k_t^i r_t^k + w_t h_t^i + \frac{B_{t-1}^i}{P_t} + \kappa_t \tag{18}$$

and the capital accumulation constraint,

$$k_{t+1}^i = inv_t^i + (1-\delta)k_t^i \tag{19}$$

The right hand side of equation (18) shows household's income, which consist of wage earning $w_t h_t^i$, rental income $k_t^i r_t^k$, total return from bond holdings $\dfrac{B_{t-1}^i}{P_t}$ and profit κ_t from owning the intermediate good producing firms. On the other hand, the left hand side of equation (18) shows households expenditures on consumption c_t^i, physical assets investment inv_t^i and financial assets investment/borrowing $\dfrac{B_t^i}{R_t P_t}$.

By combining budget constraint, capital accumulation constraint and equation of motion for real balances, we get the following new budget constraint for the household:

$$c_t^i + k_{t+1}^i + \frac{B_t^i}{R_t P_t} + \frac{M_t^i}{P_t} = k_t^i r_t^k + w_t h_t^i + (1-\delta)k_t^i + \frac{B_{t-1}^i}{P_t} + \frac{M_{t-1}^i}{P_t} + \left(\frac{\varpi_t - 1}{\varpi_t}\right)\frac{M_t}{P_t} + \kappa_t \tag{20}$$

The household maximizes the utility function in equation (13) subject to constraint in equation (20) with respect to c_t^i, h_t^i, M_t^i, B_t^i and k_{t+1}^i. After some simplification, we get the following first order conditions for the household:

$$\frac{1}{c_t^i} = -\frac{\overline{A}}{w_t} \tag{21}$$

$$\frac{1}{c_t^i} = \beta E_t\left[\frac{1}{c_{t+1}^i}(1 + r_{t+1}^k - \delta)\right] \tag{22}$$

$$\frac{1}{c_t^i} = \beta E_t\left[\frac{R_t}{\pi_{t+1} c_{t+1}^i}\right] \tag{23}$$

$$\frac{1}{c_t^i} = \beta E_t\left[\frac{1}{c_{t+1}^i \pi_{t+1}} + \frac{D}{M_t^i / P_t}\right] \tag{24}$$

where $\pi_t = \dfrac{P_t}{P_{t-1}}$ is gross inflation.

Equation (21) reflects the intratemporal equilibrium between consumption and leisure takes place when marginal utilities of consumption and leisure are equated. Equation (22) shows that intertemporal equilibrium takes place when marginal utility of consuming today is equated with discounted marginal utility of consuming tomorrow (physical investment). Equation (23) describes the same relationship with reference to financial investment. Note that (22) and (23) could easily compared to yield

$$E_t\left[(1+r_{t+1}^k-\delta)\right]=E_t\,\frac{R_t}{\pi_{t+1}} \tag{25}$$

Here left side of the equation shows gross return from physical assets net of depreciation and on the right side we have gross return from financial assets net of inflation. In a frictionless economy, arbitrage activities equate the rates of return on physical and financial assets. The real and nominal interest rates are linked by the well known Fisher equation.

$$r_t = E_t\left[\frac{R_t}{\pi_{t+1}}\right] \tag{26}$$

Inflation and money growth rate are equal to each other in the long run steady state i.e. $\pi=\varpi$. Furthermore, the nominal money balances are normalized by division with price level. This normalization is necessary to be able to find steady state of real money balances. Using $M_t^i/P_t = m_t^i$ and $\pi_t = \frac{P_t}{P_{t-1}}$, equation (24) and equation (14) can be expressed as

$$\frac{1}{c_t^i} = \beta E_t\left[\frac{1}{c_{t+1}^i\pi_{t+1}}+\frac{D}{m_t^i}\right] \tag{27}$$

and

$$m_t = \frac{\varpi_t}{\pi_t}m_{t-1} \tag{28}$$

3.2.2 Cash in Advance Constraint Model

The idea of cash in advance constraint, introduced by Clower (1967), was initially used in general equilibrium models by Lucas and Stokey (1987) and Cooley and Hansen (1989). This approach assumes that each household must hold money to purchase consumption goods. The investment goods, however, are exempted from this restriction. So, consumption and investment goods can be classified as cash and credit goods respectively.

This restriction on consumption goods is termed as cash-in-advance constraint and symbolically, this can be expressed as

$$c_t^i = \frac{M_{t-1}^i}{P_t} \tag{29}$$

Normalizing by dividing both sides by P_{t-1},

$$\pi_t c_t = m_{t-1}^i \tag{30}$$

In this model, the central bank directly transfers money to households and real money balances evolve as in equation (17). The real money balances are no longer part of the utility function and household maximizes the following utility function

$$U^i = E_t \sum_{t=0}^{\infty} \beta^t \left[\ln c_t^i + \bar{A} h_t^i \right] \tag{31}$$

subject to cash-in-advance constraint in equation (29) and budget constraint in equation (20). The first order conditions for this model are:

$$E_t \frac{w_{t+1}}{w_t} = \beta \frac{R_t}{\pi_{t+1}} \tag{32}$$

$$\frac{\bar{A}}{w_t} = -\beta E_t \left[\frac{1}{\pi_{t+1} c_{t+1}^i} \right] \tag{33}$$

$$\frac{1}{w_t} = \beta E_t \left[\frac{1}{w_{t+1}} (1 + r_{t+1}^k - \delta) \right] \tag{34}$$

3.3 Interest Rate Targeting Model

In this model, the central bank operates by following a Taylor type interest rate rule by reacting to the fluctuations in output and inflation from their steady state values.

This way of modelling central banks' behaviour has become the workhorse of DSGE models for analyzing the role of monetary policy in both developed and developing economies. Even though, in our empirical section, we only found a weak link between interest rates and short run fluctuations in output, we still wanted to evaluate the role of short term interest rate as a tool of monetary policy in propagating business cycles in a developing economy. The best way to do this was to use the well established theoretical framework in literature of modelling monetary policy as a Taylor type interest rate rule in a simple New Keynesian DSGE model.

For this model, we assume that the economy is cashless and we briefly discuss households' behavior and monetary policy in this scenario.

3.3.1 Households

The households maximize utility function represented by equation (31) subject to combined budget constraint obtained by addition of budget constraint in equation (18) and capital accumulation constraint in equation (19)

The first order conditions of households are same as those found in equation (21), (22) and (23).

3.3.2 Monetary Policy

In this set up, the central bank conducts monetary policy through Taylor type interest rate rule by changing policy rate in response to fluctuations of output and inflation. The interest rate reaction function is given as:

$$R_t = (R_{t-1})^{\rho_R}\left[R\left(\frac{\pi_t}{\pi}\right)^{\psi_\pi}\left(\frac{y_t}{y}\right)^{\psi_y}\right]^{(1-\rho_R)}\exp(\varepsilon_t^R) \quad (35)$$

where $\rho_R \in (0,1)$, ψ_π and ψ_y represent degree of interest rate smoothing, response of monetary policy to inflation fluctuations and to output fluctuations, respectively. R, π, y and $\varepsilon_t^R \sim N(0,\sigma_R)$ are steady state values of nominal interest rate, inflation, output and the stochastic interest rate shock with mean 0 and standard deviation σ_R.

3.4 Aggregation and General Equilibrium

In all our models, we have assumed that all households are identical and belong to a continuum of unit mass. Therefore for all models, $\int_0^1 c_t^i di = c_t$, $\int_0^1 i_t^i di = i_t$, $\int_0^1 h_t^i di = h_t$, $\int_0^1 M_t^i di = M_t$ and $\int_0^1 k_t^i di = k_t$. The financial assets and liabilities cancel each other out at the aggregate level so that $\int_0^1 B_t^i di = 0$. The economy wide aggregate resource constraint takes the form

$$y_t = c_t + i_t \quad (36)$$

Hence, for all of our models the general equilibrium consists of allocation $\{y_t,\ c_t,\ M_t,\ h_t,\ i_t,\ k_t\}$ with sequence of prices $\{w_t, r_t^k, R_t, p_t\}$ that satisfy all first-order conditions of the household, the intermediate and final-goods-producing firms and the aggregate resource constraint for all realized and expected states of technological and monetary factors.

The derivation of steady state and log-linearization of the New Keynesian Phillips curve are presented in Appendix A and Appendix B respectively. Furthermore, all equations of different models are presented in Appendix E.

4 Calibration

The parameters of different models have been calibrated for quarterly frequency. In this paper, we have tried to use micro level evidence for calibration purposes wherever possible. However, in the absence of micro evidence we had to fall back on using macro data for calibration purposes. An important feature of our calibration exercise is that none of our parameters are fixed by matching dynamic properties of simulated models with data. Therefore, the dynamic properties of our simulated models are solely based upon calibration coming from data and not from data moments.

4.1 Households' Preferences

The discount rate, β is fixed at 0.97. This value shows that quarterly real rate of return in the economy is 3.1% and annual compounded return is 13%. The previous studies on Pakistan use a value close to unity for quarterly β that were calculated on the basis of average real interest rate (Ahmed et al. (2012)). The very high value of β indicates that on average, economic agents are extremely future-oriented or inclined towards investment in comparison to consumption. However, if we use such value, then the projected steady state investment to GDP ratio should be around 40% which is clearly in contrast with Pakistani data. The relationship between β and investment to output ratio is depicted in Figure A1.

The capital series is constructed by using total investment, therefore we fix value of β in a way that yields steady state investment to GDP ratio equal to 0.20[19] or 20 percent.

The preference for leisure $A = 1.27$ and indivisible labour coefficient $\overline{A} = -1.77$ are calibrated by matching model steady state hours with the empirical value found in data. The Labour Force Survey (LFS) data shows that on average, labour work for about 47.9 hours per week. This means that the average daily work hours are 6.8. This reflects that on average, 28% of total time is spent at work. Using this value in the steady state equation of h, we find that value of A is 1.27 and $\overline{A}$ is -1.77. It is important to note that these values of A and $\overline{A}$ are conditional not only upon h but also on calibration of other parameters e.g. $\beta,\ \theta$ and δ.

The money preference parameter D is estimated to be 0.0556 through GMM estimation[20] of the following Euler equation using annual data through method developed by Hansen and Singleton (1982)

$$\frac{1}{c_t} = \beta E_t \frac{1}{c_{t+1}\pi_{t+1}} + \frac{D}{m_t} \tag{37}$$

4.2 Production

The share of capital in production, θ, was calibrated using information from literature under insights from estimation of production function. First of all, we estimate constant returns to scale (CRS) Cobb-Douglas production function using quarterly data of real GDP, employed labour force and total capital to represent y_t, h_t and k_t respectively.

The employed labour force data was taken from various issues of Labour Force Survey whereas capital series was constructed using total gross fixed capital formation series under perpetual inventory method[21]. The gross fixed capital formation data was seasonally adjusted and assumptions of quarterly depreciation rate equal to 1.6% and average quarterly GDP growth equal to 1.3% were used in computation of initial value of accumulated capital series. The subsequent values of capital series were computed using the capital accumulation equation.

Table 1: Estimation of Production Function

$\log \frac{y_t}{h_t}$	c	$\theta \log \frac{k_t}{h_t}$
 SE t-stat	-2.17 (0.05) -42.94	0.66 (0.16) 4.13
$R^2 = 0.22$		

The above estimated equation yields $\theta = 0.66$ which is on the higher side compared to developed countries (for US, θ is usually taken as 0.36 or 0.33). We adjust this value slightly downwards by

[19] Even though, models presented in this paper assume that return on physical capital investment and financial capital investment are same. In our opinion, β should not be calculated only on the basis of interest rate in case of Pakistan. Interest rate may be a good proxy of overall return for developed economies where properly functioning capital markets make financial and real returns correlated and closer to each other. However, this is not the case in Pakistan where a big gap in the financial and real rate of return renders interest rate a poor proxy of overall return on investment in the economy.

[20] During estimation of the above equation, value of β was assumed close to 0.97.

[21] See Appendix 2 for details

choosing a value of 0.60 mainly for two reasons. First, estimation of production functions can be skewed due to measurement error as well as data quality. Second, the average of the same parameter over a range of developing countries estimated by Liu (2008) is closer to 0.50.

The persistence and standard deviation of the total factor productivity (TFP) shock, ρ_A, σ_A are estimated using the Solow residual series taken from production function estimated using quarterly data reported in Table 1. For persistence of technology shock ρ_A, we estimate the following equation:

Table 2: Estimated TFP Shock

$\log A_t$	c	$\rho_A \log A_{t-1}$
SE t-stat	0.00 0.00 1.76	0.99 0.03 28.85
$R^2 = 0.93$	$\sigma_A = 0.017$	

The standard deviation of technology shock, $\sigma_A = 0.0174$ has been computed by taking the standard deviation of the residuals of above equation.

The depreciation rate, δ, has been computed by using data from the Census of Manufacturing Industries (CMI (2005-06) that reveal annual depreciation rate of 6.5 percent. We compute the quarterly depreciation rate of 1.6% from the annual value. The calvo price stickiness index $\varepsilon_P = 0.25$, has been taken from Choudhary et al. (2011).

4.3 Monetary policy

4.3.1 Money Growth Rule

The steady state money growth and inflation are assumed to be equal in our models $(\pi = \varpi)$. The average annual inflation (YoY) for the period 1990-2012 is 9.2% (2.3% on quarterly basis) and the average growth rate of per capita M2 is 12.3% (2.9% on quarterly basis). We take a value that is close to mid-point of both these values by choosing 1.025 as the gross growth rate of money stock in each quarter. We use the quarter on quarter growth rate of M2 for estimation of persistence and standard deviation of money supply shock through the following equation:

Table 3: Estimated Money Growth Shock

$(GM2_t)$	c	$\rho_M (GM2_{t-1})$	$AR1$
SE t-stat	0.01 (0.00) 3.48	0.54 (0.12) 4.38	-0.32 (0.14) -2.27
$R^2 = 0.10$	$\sigma_M = 0.016$		

Similar to the technology shock, the standard deviation of money growth shock $\sigma_M = 0.016$ is computed by taking the standard deviation of residuals of the above equation.

4.3.2 Interest Rate Rule

In this section, we first briefly review the existing literature on the estimation of Taylor Rule for Pakistan by various authors. After the literature review we discuss our methodology and results of our Taylor Rule estimation for Pakistan economy

Ahmed and Malik (2011) have estimated Taylor rule with lagged interest rate, inflation, output gap and exchange rate over the period 1992Q2-2010Q4. For the closed economy setting of Taylor rule i.e. without exchange rate, they report parameters similar to our estimations.

Malik and Ahmed (2010) estimated Taylor rule for the sample period 1991-2006 using quarterly data without interest rate smoothing. They reported inflation coefficient of 0.51 but owing to omission of lagged interest rate, the model is mis-specified and exhibits acute autocorrelation problem (DW=0.89).

Aleem and Lahiani (2011) estimated forward-looking (inflation 4q and output gap 2q) Taylor rule using data of 6M T-bill rate, core inflation, index of manufacturing production, exchange rate and US interest rate for the period 1992Q1-2008Q1 using GMM. For closed economy versions of Taylor rule estimations (reflected by specifications 1, 2, 9, 12 and 13 presented in Table 1 & 2 of the paper), Taylor principle condition is fulfilled for all cases except for one. Finally preferred specification by authors also indicates compatibility with Taylor principle.

In this study, the Taylor rule has been estimated following Ireland (2000). We assume that central bank responds to fluctuations in output and inflation from their steady state values. The interest rate smoothing term is included to avoid large deviations in the interest rate. The deviation of inflation and per capita GDP from their steady states are computed by residuals of least square estimations. We regress the log of quarterly gross inflation on constant and take residuals of this regression μ_t^{π} as a proxy of fluctuations of inflation from steady state. Similarly, output gap, μ_t^{Y}, series is worked out by regressing log of per capita output on a constant and time trend and using the residuals of that equation.

The gross nominal 6-months T-bill rate has been used as a proxy of policy rate for this estimation. The series of interest rate shock is worked out by taking residuals of the above regression. In order to compute the persistence of interest rate shock, we regress the interest rate shock on constant and its lag. The standard deviation of interest rate shock is calculated by taking the standard deviation of residuals of the above estimated Taylor rule equation.

The Taylor rule parameters are calibrated through constrained estimation of the linearized version of equation (35). The results of the constrained estimation are summarized in Table 4[22].

Table 4: Estimation of Taylor Rule

$\log R_t$	$(1-\rho_R)\log R$	$\rho_R \log R_{t-1}$	$\psi_\pi \mu_t^\pi$	$\psi_y \mu_t^Y$
	0.01	0.93	3.84	1.03
S.E	(0.00)	0.03	2.01	0.67
t-stat	2.34	32.18	1.91	1.53
p-value	0.02	0.00	0.06	0.13
$R^2 = 0.93$ $\sigma_R = 0.01$				

[22] Estimation of Taylor rule has been discussed in detail in Appendix D.

4.4 All Calibrated Parameters

The calibrated values of all the structural parameters used in different models are reported in Table5. Similarly, all the exogenous shock related parameters are listed in Table 6.

Table 5: Structural Parameters

#	Parameter	Description	Value
1	β	Discount factor	0.97
2	h	Steady state work hours	0.28
3	A	Weight of leisure in utility	1.27
4	$\bar{A}$	Indivisible labour parameter	-1.77
5	D	Weight of real balances in utility	0.056
6	θ	Share of capital in production	0.60
7	δ	Depreciation rate	0.016
8	ε_P	Price stickiness index	0.25
9	π	Steady state inflation	1.025
10	ϖ	Steady state money growth	1.025
11	ρ_R	Interest rate smoothing coefficient	0.93
12	ψ_π	Response to inflation in Taylor rule	0.25
13	ψ_Y	Response to output gap in Taylor rule	1.05

Table 6: Shock Parameters

#	Parameter	Description	Value
1	ρ_A	Persistence of technology shock	0.99
2	ρ_M	Persistence of monetary shock	0.54
3	ρ_R	Persistence of interest rate shock	0.30
4	σ_A	Standard deviation of technology shock	0.017
5	σ_M	Standard deviation of monetary shock	0.016
6	σ_R	Standard deviation of interest rate shock	0.008

5 Model Evaluation

In economic literature, it is a standard practice to evaluate the performance of various DSGE models by their ability to match relevant second order moments from the data as well as the consistency of their impulse response functions with economic theory and any relevant empirical evidence. Therefore, in order to evaluate the performance of the three competing theoretical models, we compare second order moments obtained from different models with their empirical counterparts as well as the magnitude and amplification of different impulse response functions in response to different exogenous shocks.

5.1 Simulated and Empirical Moments

The comparison of data and simulated moments is a standard practice in the literature. We obtain simulated moments of models by dynamically solving[23] all these models for quarterly calibrated parameters. On the other hand, empirical moments have been calculated using both quarterly and annual[24] data.

In order to compute quarterly moments from data, we use quarterly data on GDP, private investment, private consumption, gross inflation[25], gross nominal interest rate[26] and money[27]. The real per capita GDP, consumption and investment data were seasonally adjusted, logged and filtered using the Hodrick Prescott filter ($\lambda_q = 1600$) before computing empirical moments. The data on inflation and money growth rate was also seasonally adjusted, logged and HP filtered. The annual moments are calculated using annual data of same variables. The results of simulated and empirical moments are presented in Table 7.

The Money in Utility model with money growth shock underestimates the relative volatility of private consumption, nominal interest rate and money growth rate compared to the empirical counterpart using both annual and quarterly data. On the other hand the relative volatility of private investment and inflation reported by MIU model is quite close to the relative volatility reported in the data.

The Cash in Advance constraint model with money growth shock also underestimates the relative volatility of private consumption, nominal interest rate and money growth rate compared to the relative volatilities reported in the data. However, the CIA model comes quite close to matching the relative volatility of private investment and inflation as reported by quarterly data of Pakistan.

The model with monetary policy conducted through Taylor rule performs relatively worse in terms of matching the relative standard deviation of different variables with empirical counterparts reported in the second last column of Table 7. The only variable where the model's relative volatility comes close to matching empirical counterpart is private investment.

The second panel of Table 7 shows that Cash in Advance constraint model does a better job of matching the contemporaneous correlation of various macroeconomic variables with output compared to the other two model.

For example, the contemporaneous correlation between private consumption and GDP is found to be 0.54 for the CIA model , which is quite close to the empirical value of 0.51 from the quarterly data. On the other hand, for the MIU and Taylor rule model this correlation is found to be 0.92 and 0.88 respectively.

All three models, overestimate the contemporaneous correlation between GDP and private investment compared to the reported value of 0.30 and 0.59 from quarterly and annual data respectively. The value of contemporaneous correlation between private investment and GDP is found to be 0.99, 0.89 and 0.99 by MIU, CIA and Taylor rule model respectively.

Interestingly, all three models come relatively close in matching the correlation between inflation and

[23] using Dynare (Adjemian et al. (2011))
[24] The reason for inclusion of annual data moments is that national income account (NIA) data for Pakistan is not available on quarterly frequency. Although we use estimated quarterly data of NIA from Hanif et al. (2013), we use annual data moments as a check on moments calculated from estimated quarterly data.
[25] Quarter on quarter, calculated from **CPI**
[26] Money market rate
[27] per capita **M2**

output reported in the quarterly data. According to MIU model, the contemporaneous correlation between inflation and GDP is -0.10, while the CIA model gives a value of -0.16 for this correlation. The Taylor rule model on the other hand reports this correlation between inflation and output to be -0.07. All these values are generally close to the empirical counterpart of 0.04 from the quarterly data.

Table 7: Simulated and Empirical Moments

	Models			Empirical	
	MIU	**CIA**	**TR**	**Quarterly**	**Annual**
Relative Std Dev (GDP)					
GDP	1.00	1.00	1.00	1.00	1.00
Private investment	3.58	4.00	3.68	4.56	3.78
Private consumption	0.36	0.57	0.35	3.26	1.59
Inflation	0.54	0.50	1.67	0.65	1.16
Nominal interest rate	0.26	0.26	0.40	2.64	0.91
Money growth rate	0.49	0.48		3.70	0.74
Correlation with GDP					
GDP	1.00	1.00	1.00	1.00	1.00
Private investment	0.99	0.89	0.99	0.30	0.59
Private consumption	0.92	0.54	0.88	0.51	0.53
Inflation	-0.10	-0.16	-0.07	0.04	0.32
Nominal interest rate	0.11	0.06	-0.84	0.06	0.13
Money growth rate	0.11	0.06		0.12	-0.22
Autocorrelation					
GDP	0.75	0.75	0.77	0.65	0.66
Private investment	0.75	0.70	0.72	0.41	0.39
Private consumption	0.80	0.57	0.80	0.18	0.26
Inflation	0.33	0.36	0.00	0.67	0.30
Nominal interest rate	0.44	0.44	0.70	0.34	0.61
Money growth rate	0.44	0.44	0.00	0.39	0.96

The last panel of Table 7 reports the persistence of various macroeconomic variables from the three models as well as their empirical counterparts from both annual and quarterly data. In general, none of the models come very close to matching the persistence values of all macroeconomic variables from the data. However, Cash in Advance constraint model with money growth rate outperforms the MIU and Taylor rule model.

Overall, the MIU and CIA models do relatively better in terms of matching the volatilities of various macroeconomic variables with data counterparts. Furthermore, the CIA model with money growth rule comes close to matching contemporaneous correlation of some macroeconomic variables with GDP and the autocorrelation of different macroeconomic variables with empirical moments. The Money in Utility model and Taylor rule model do relatively worse in terms of matching the contemporaneous correlations of macroeconomic variables with GDP and autocorrelations of various macroeconomic variables with their empirical counterparts. All in all, based on moment matching exercise for relevant second order moments, Cash in Advance constraint model outperforms the other two models.

In addition to looking at relative volatility, contemporaneous correlation with output and autocorrelation of relevant macroeconomic variables, we also compare the simulated output and inflation with the actual deviation of these variables from trend in data,

Figure 10 shows the comparison of actual and simulated deviations of quarterly GDP and inflation from trend. The actual deviations from trend are computed using Hodrick-Prescott Filter whereas simulated deviations are computed using coefficients of policy functions coming from rational expectations solution of the three models and innovations in shock variables ε_a , ε_{ϖ} and ε_r .

These innovations were obtained from estimations described in Tables 2, 3 and 4. Considering the facts that the models are fairly simplified structures and we have taken only two shocks, we see that, to a large extent, models have been capable of capturing the direction of change in GDP and inflation.

However, all models over-predict the magnitude of change in GDP and inflation. This fact could be attributed to lack of various nominal and real frictions in our models.

5.2 Impulse Response Functions

After considering the second order moments for all three models and their empirical counterparts from both annual and quarterly data, we now turn our attention to the impulse response functions (IRFs) generated in response to various exogenous shocks for all three models

Figure 11 shows that a positive technology shock leads to a rise in output, investment, consumption, and real interest rate in all three models. One standard deviation shock in TFP causes almost 4% increase in output and almost 12 percent increase in investment relative to their steady states. On the other hand, inflation declines in response to a positive technology shock.

The impulse response function of various macroeconomic variables in response to a positive technology is almost identical for MIU, CIA and Taylor rule model. However, the response of nominal variables differ between the Taylor rule model and the other two models of money growth. The right panel in the second row of Figure 11 shows that both inflation and nominal interest rate declines quite significantly in response to a positive technology shock. The impulse response functions shown in Figure 11 are consistent with impulse response functions of these models in the literature.

The Figure 12 shows the impulse response functions in response to a monetary policy shock, which in case of MIU and CIA is a money growth shock and for the Taylor rule model it is an interest rate shock. Furthermore, the figure also shows that an expansionary monetary policy shock causes an increase in output, investment, consumption and inflation in the MIU and Taylor rule model. The magnitude and persistence of impulse responses vary for three models, as output increases by 1% in the MIU model and by 2% in the Taylor rule model. Also, the output in MIU model returns to its steady state value after around 5 quarters, while it returns to its steady state value after 3 quarters in the Taylor rule model.

The response of nominal variables in response to an expansionary monetary policy shock is similar for the two models with money growth rate (MIU & CIA). However, the model with interest rate rule shows different impulse responses of inflation from the other two models.

In addition to looking at impulse response functions of our three models, we also compare IRFs obtained from models with IRFs from Vector autoregressions as discussed before in section 2.5.

The left panel of Figure 9 shows that in response to 1% expansion in money growth, both CIA and MIU models produce similar response for inflation and fail to produce the initial decline in inflation following the money growth shock. The two models' response to inflation is very large and quicker when compared to benchmark VAR IRF for inflation.

In case of GDP, there is considerable difference in shape of IRFs from CIA and MIU models; CIA model IRF is closer to the VAR IRF in terms of shape and magnitude. On the other hand, MIU model overestimates the magnitude and speed of output response to money growth shock.

The comparison of Taylor rule model with VAR illustrates the impact of interest rate shock on output and inflation (Figure 9, right panel). For both output and inflation, Taylor rule model produces very large and quick initial response to expansionary 1% interest rate shock. However, very low level of persistence in these IRFs is reflected by steep decline in IRFs in the second period.

In general, we see that three models' IRFs capture the direction of change in line with empirical benchmark. However, the difference in magnitude and propagation in models' IRFs relative to VAR IRFs could be due to lack of real and nominal frictions in the DSGE models discussed in this paper.

5.3 Variance Decompositions

We compare variance decompositions of output and inflation from DSGE and VAR models to analyze relative importance of different shocks. Variance decompositions from three DSGE models and, VAR model are presented in Tables A4 and A5.

First we talk about variance decomposition of output presented in Table A4. In DSGE models, almost all of the variation in output is explained by TFP shock. For MIU and CIA models, monetary policy shock has negligible share; less than 1%. More than 99% of fluctuations in output are explained by TFP shock. In case of Taylor Rule based model, the share of monetary policy shock; although not negligible yet, is quite limited and not greater than 13%. VAR model also shows that monetary policy variables are less important for explanation of variation in output. Share of TBR6 is 5.95% and share of GM2 is 2.10%. DSGE and VAR models seem to agree on the point that nominal money growth and interest rate have less importance in explaining variations in output.

Variance decompostions for inflation are presented in Table A5. In DSGE models, monetary policy shock is mainly responsible for fluctuations in inflation. For MIU and CIA models, shares of monetary policy shock are 92.09% and 90.64%, respectively. TFP shock explains less than 10% of variations in infation in MIU and CIA models. In Taylor rule based model, monetary policy shock explains 72.53% and TFP shock explains 27.47% variations in inflation. These findings, however, are not in line with VAR model where share of TBR6 is 2.61% and share of GM2 is 3.25%. VAR results show that inflation itself (51.31%), government consumption (18.13%) and GDP (11.2%) mainly explain variations in inflation. There could be various reasons behind inability of our DSGE models to match VAR variance decompositions for inflation. First, we have modeled only one nominal friciton i.e. price rigidity. Second, calibrated value of Calvo price rigidity coefficient is quite low $(\varepsilon_P = 0.25)$. Third, number of shocks is limited to only two. Finally, we abstract real fricitons.

6. Conclusion

In this paper, we establish some empirical `facts' pertaining to inter-linkages between the nominal and real variables of Pakistan economy using a comprehensive set of empirical tools for both annual and quarterly data.

We find that all monetary aggregates are strongly pro-cyclical and some of them even act as a leading indicator of economic activity in Pakistan for the period 1990-2012. On the other hand, different nominal interest rates also co-moved positively with output and large scale manufacturing but real interest rates were countercyclical for the most part. Monetary aggregates and interest rates explain a quite limited portion of overall fluctuations in output and inflation.

In addition, we also (theoretically) evaluated the role of money and monetary policy in propagating business cycle fluctuations of Pakistan economy using different ways of introducing the role of money via money in utility (MIU) and cash in advance constraint (CIA) as well as with different formulation of monetary policy either through a money growth rule or Taylor type interest rate rule.

The results from our model simulations show that inclusion of money and the way it is incorporated in DSGE models makes significant difference in model performance. The cash economy models (MIU & CIA) under money growth rule exhibits better data matching potential as compared to cashless economy model closed by a Taylor type interest rate rule in case of Pakistan.

The impulse response functions of various DSGE models show that under given modeling structure and parameterization, the impact of monetary policy shock on Pakistan economy was limited and short lived during the period of the study.

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Appendix 1: Quarterly Data Sources

	Series Name	Symbol	Base	Unit	Source
1	Gross Domestic Product	GDP	Constant prices of 1999-00	Million Rs.	Hanif et al. (2013), Appendix D
2	Total Gross Fixed Capital Formation	ToTGFCF	Constant prices of 1999-00	Million Rs.	Hanif et al. (2013), Appendix E4
3	Private Gross Fixed Capital Formation	PvtGFCF	Constant prices of 1999-00	Million Rs.	Hanif et al. (2013), Appendix E5.
4	Government Gross Fixed Capital Formation	GvtGFCF	Constant prices of 1999-00	Million Rs.	Hanif et al. (2013), Appendix E6
5	Total Consumption	ToTCons	Constant prices of 1999-00	Million Rs.	Hanif et al. (2013), Appendix E1
6	Government Consumption	GvtCons	Constant prices of 1999-00	Million Rs.	Hanif et al. (2013), Appendix E3
7	Private Consumption	PvtCons	Constant prices of 1999-00	Million Rs.	Hanif et al. (2013), Appendix E2
8	CPI	P	Q1FY90=1	Index	Haver Analytics, N564PC@EMERGE
9	Call Money Rate (EoP)	MMR		%	N564RCE@EMERGE
10	6-Months T Bill Rate	TBR6		%	SBP
11	Policy Rate	POLR		%	SBP
12	Currency in Circulation	CiC		Million Rs.	SBP
13	Reserve Money	M0		Million Rs.	SBP
14	Narrow Money	M1		Million Rs.	SBP
15	Broad Money	M2		Million Rs.	SBP
16	Population	Pop		Million People	C564POP@IFS

*Annual data of series 1-7 (National Income Accounts at constant prices) were obtained from various issues of Economic Survey of Pakistan and converted to same base (1999-00) by using splicing method. Annual series of price level, inflation, interest rate and monetary aggregates were taken from same sources as described for their quarterly counterparts. Quarterly population was obtained from annual series using annual compounded growth rate method. Real interest rates were obtained by dividing gross nominal interest rates from gross YoY inflation. Real monetary aggregates were calculated by deflating from normalized CPI.

Real interest rates and real monetary aggregates are denoted by adding **'R'** before symbol of nominal variable e.g. real policy rate is denoted by RPOLR and real M2 is denoted by RM2.

Growth rates of monetary aggregates are denoted by adding **'G'** before symbol of variable e.g. growth of M2 is denoted by GM2.

Natural logarithm is denoted by adding **'L'** before variable name e.g. LGDP denotes natural log of GDP.

Cyclical component after removing HP filter trend is denoted by adding **'HP'** before variable name e.g. HPLGDP denotes HP filtered log of GDP.

Appendix 2: Data for Estimation of Production Function and TFP

Quarterly Data

In order to estimate production function, we use quarterly real GDP, million hours worked and capital series as proxies of output, labour and capital. We calculated the series of capital stock using perpetual inventory method using both total GFCF and private GFCF. In perpetual inventory method, initial capital is calculated as

$$K_0 = \frac{GFCF_0}{\hat{Y} + \delta}$$

where

$\hat{Y} =$ Average of QoQ Real GDP Growth Rate and,
$\delta =$ Quarterly Depreciation Rate.

After calculation of initial period capital, subsequent periods capital is calculated through law of motion for capital stock:

$$K_{t+1} = (1 - \delta) K_t + I_t$$

Total hours worked are calculated from Labor Force Survey of Pakistan over fiscal years 1991 to 2011 by the following formula:

$$\sum_{i=1}^{njk} 13 \left(wwh_{ijk} \times weight_{ijk} \right)$$

Here wwh_{ijk} represents weekly worked hours of i[th] individual in j[th] quarter of k[th] year. Weight links the sample to population. Weight gives the number of household an individual is representing in the population. Hours data for missing year is calculated by spline interpolation (piecewise polynomial interpolation).

Annual Data

Same procedure as in previous section has been adopted except for the proxy of labour. We use employed labor force data taken from various issues of Economic Survey of Pakistan.

Figure 1: Annual GDP and, Monetary Aggregates & Interest Rate

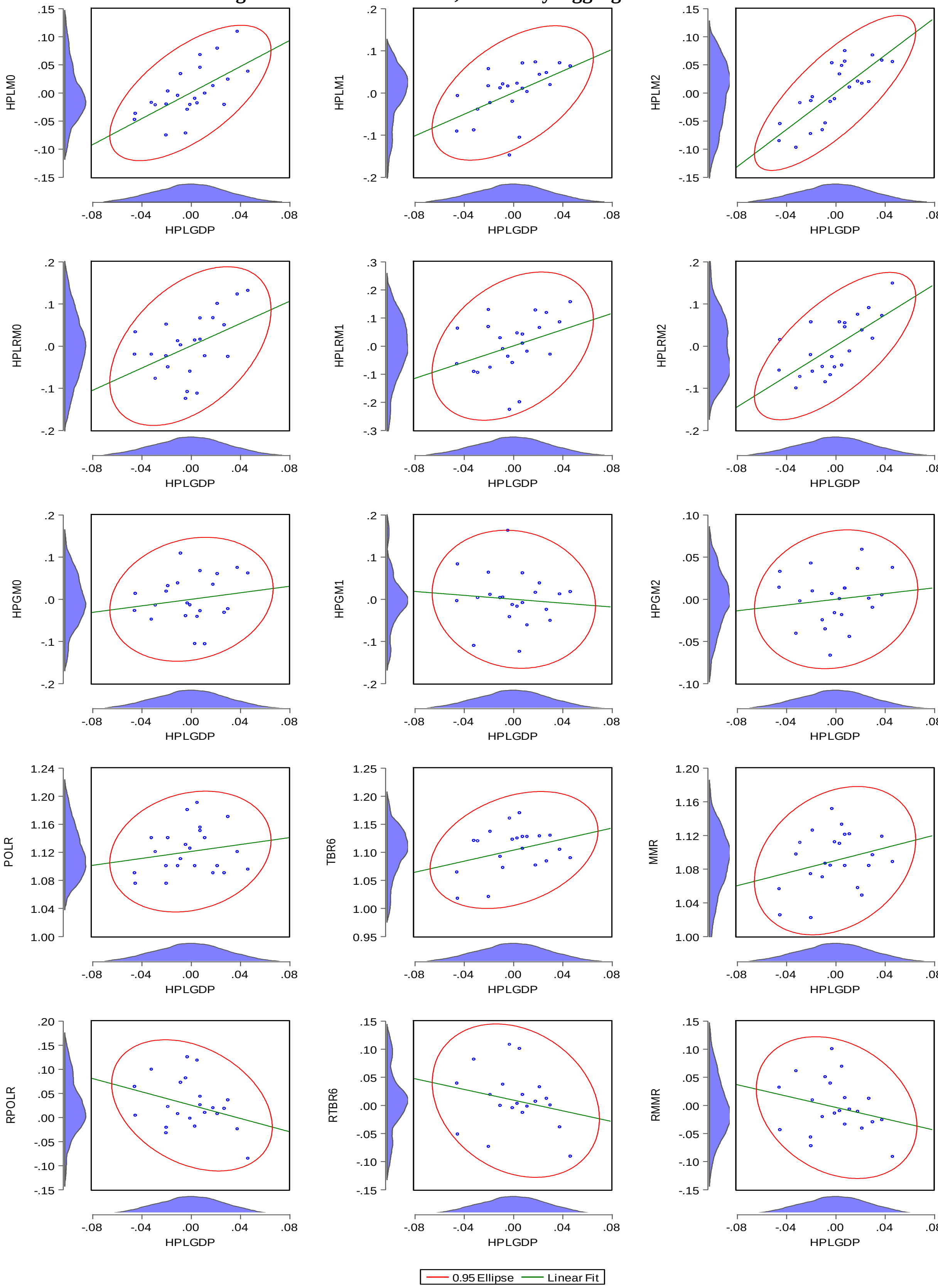

Figure 2: Quarterly GDP and, Monetary Aggregates & Interest Rate

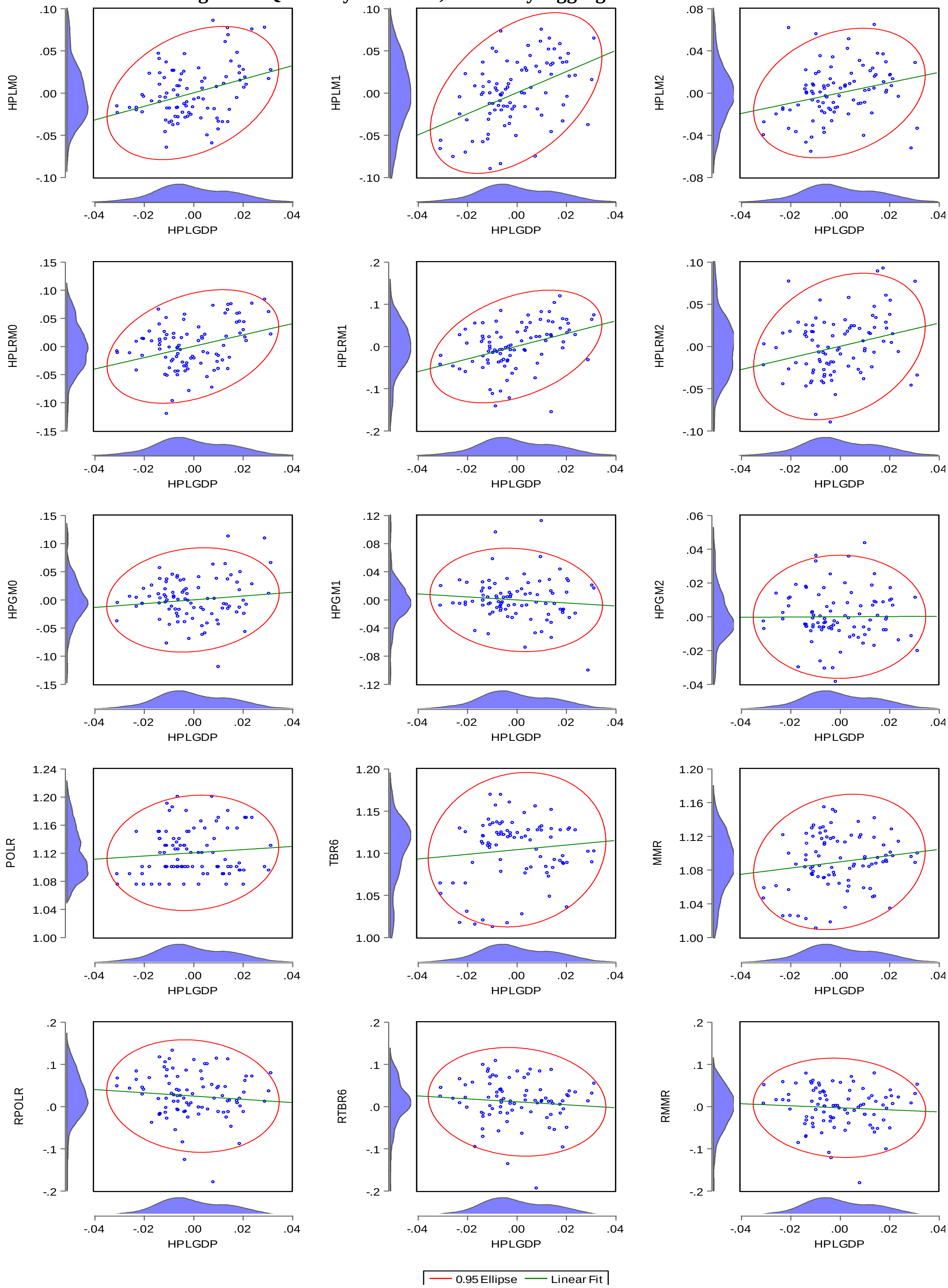

Figure 3: Annual LSM and Monetary Policy Indicators

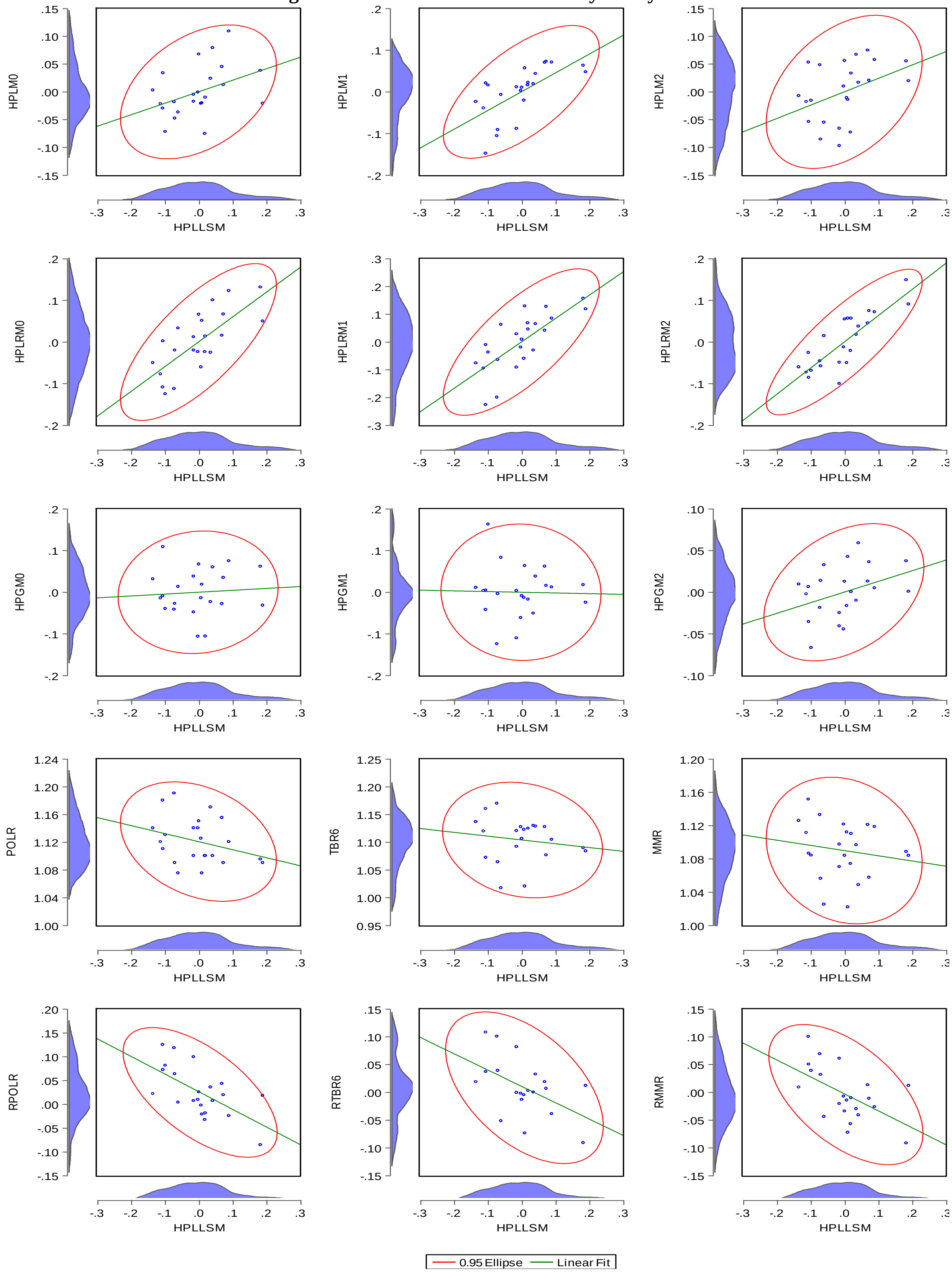

Figure 4: Quarterly LSM and Monetary Policy Indicators

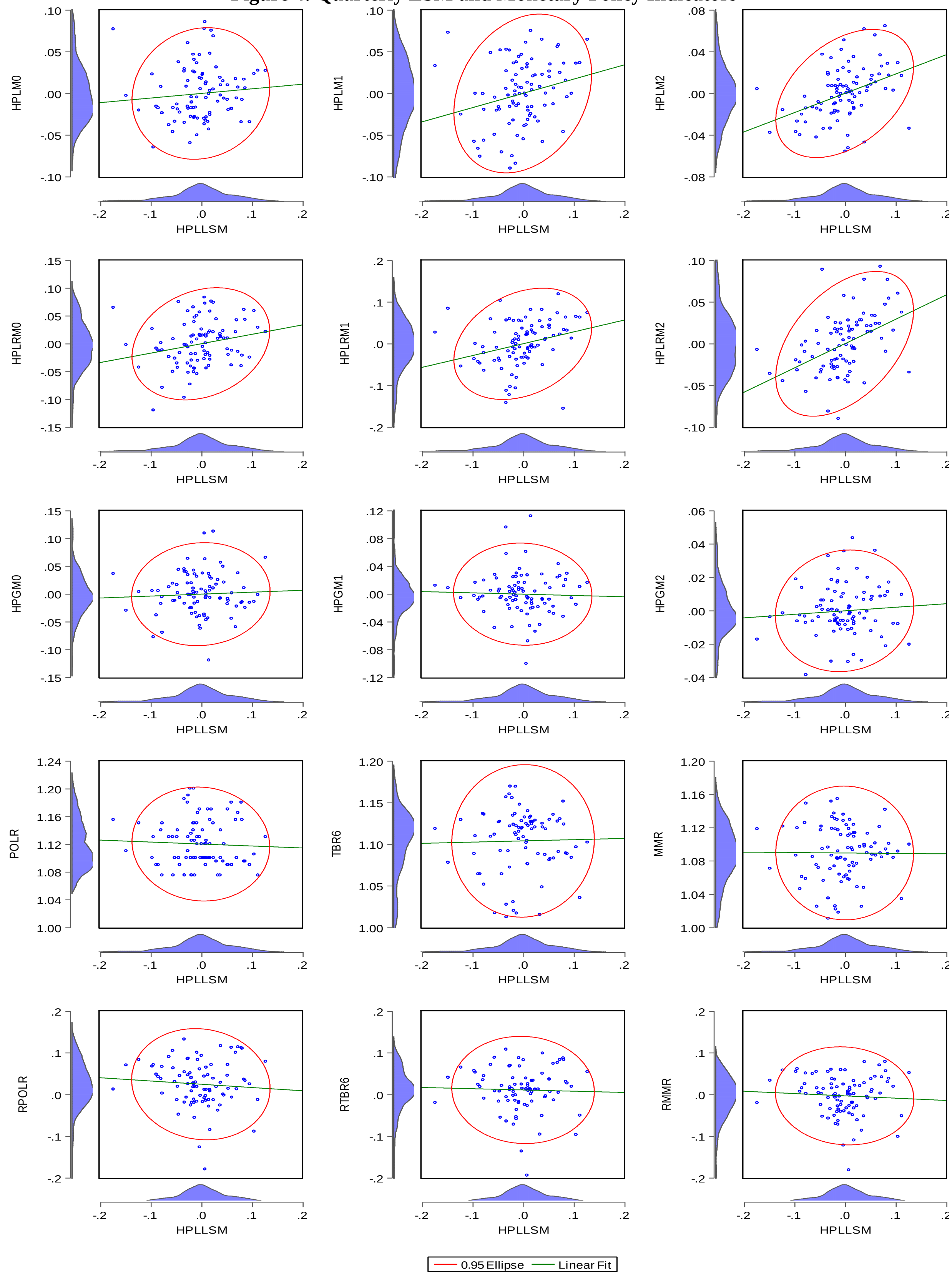

Figure 5: Annual Inflation and other Variables

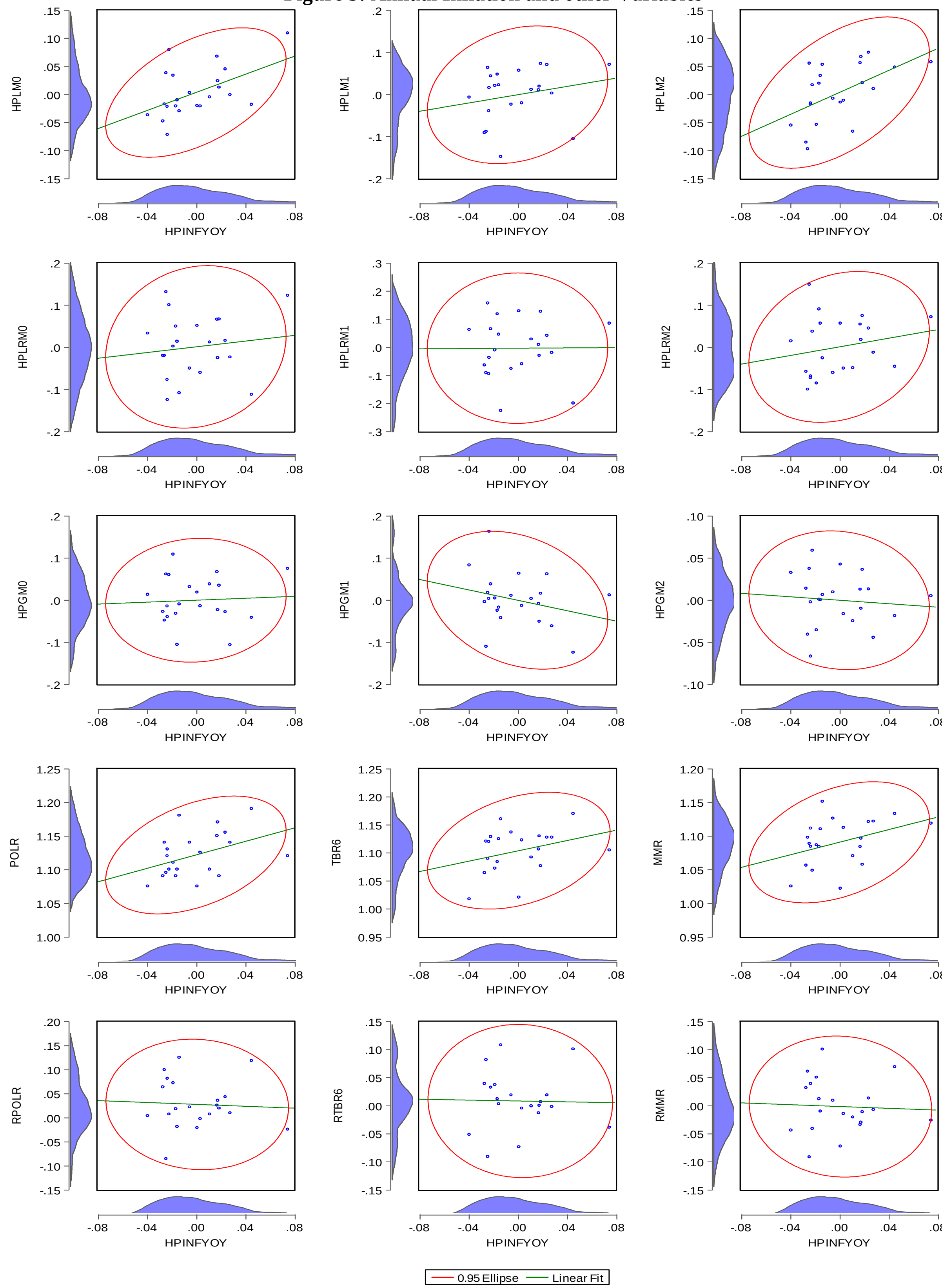

Figure 6: Quarterly (QoQ) Inflation and other Variables

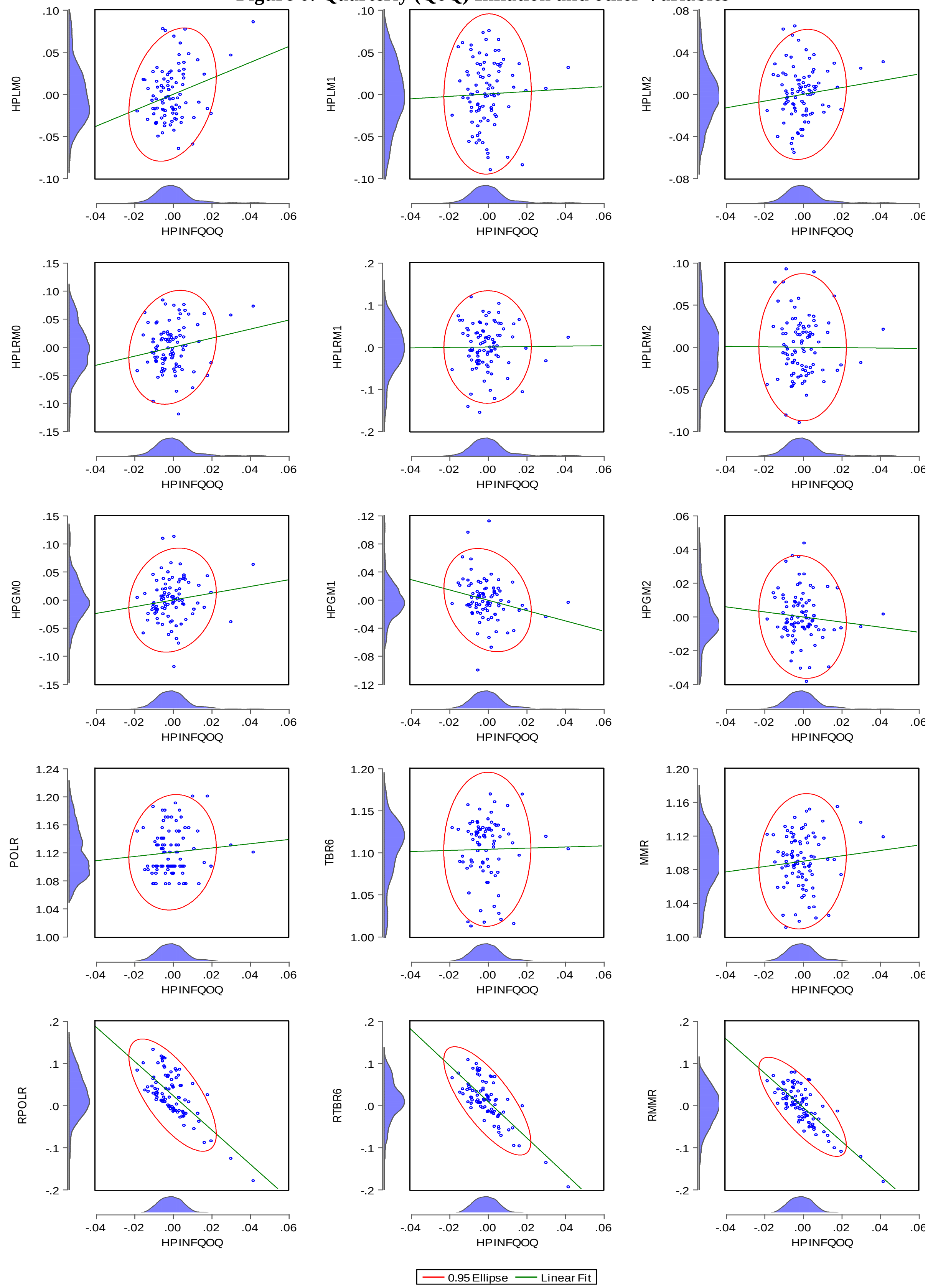

Figure 7A: Dynamic Correlations of GDP and Nominal Variables

Figure 7B: Dynamic Correlations of LSM and Nominal Variables

Figure 7C: Dynamic Correlations of Inflation and Nominal Variables

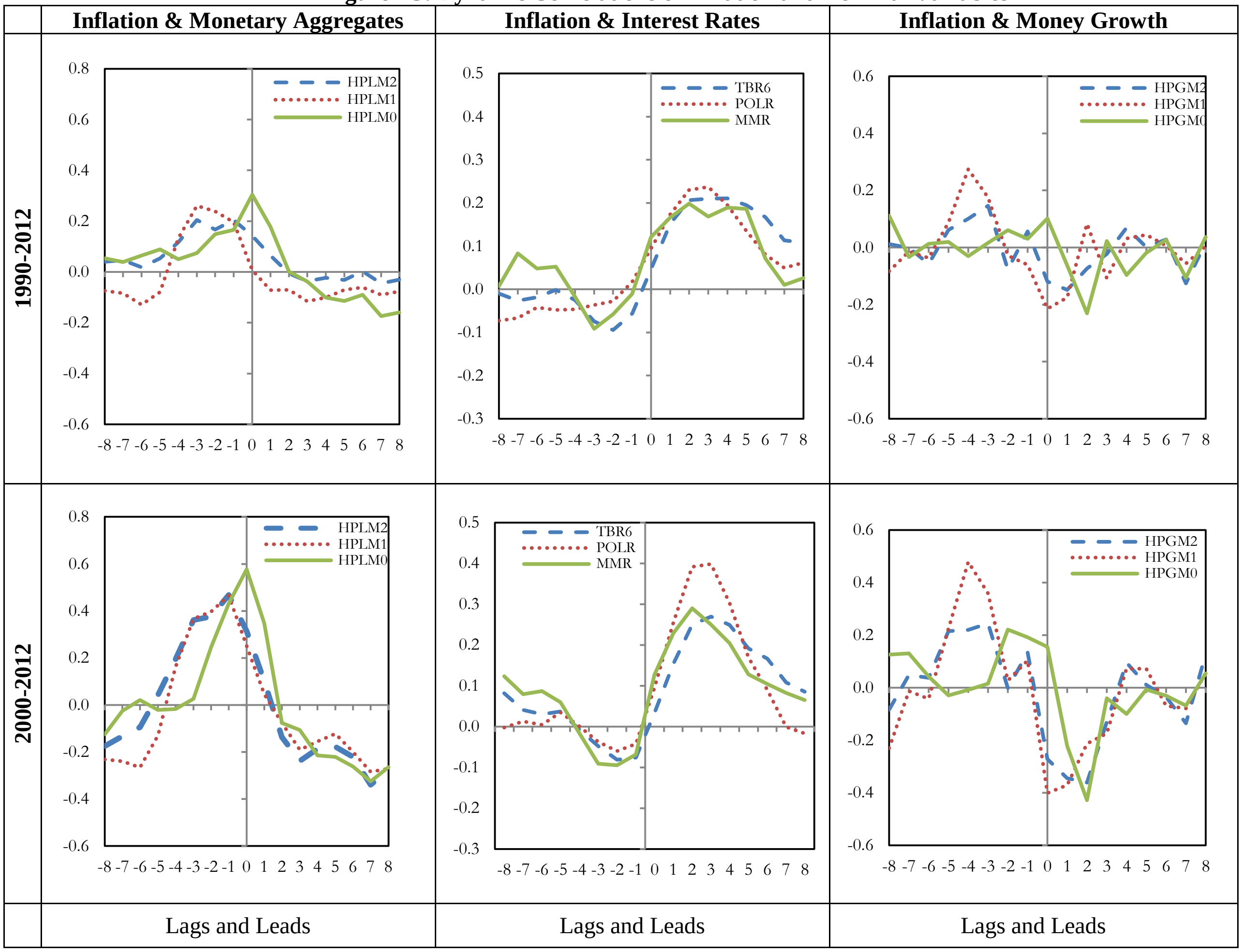

Figure 8: IRFs from VAR Models

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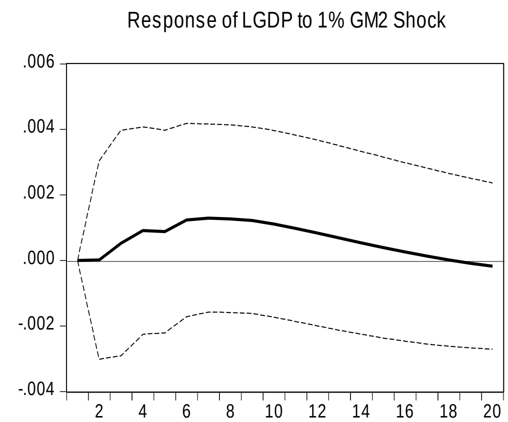
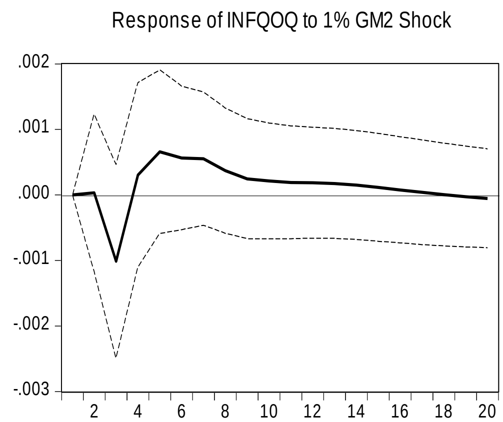
<tr><td>Nominal GM2</td><td>
</td><td>
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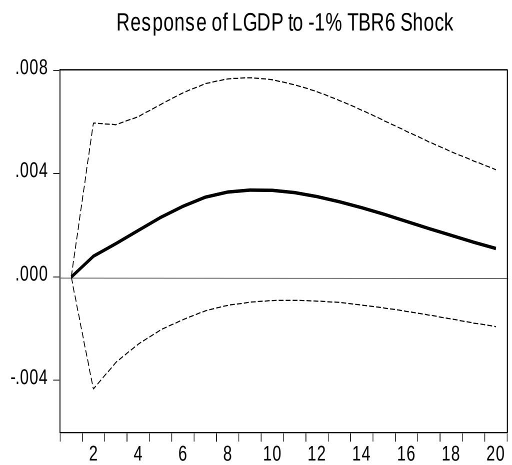
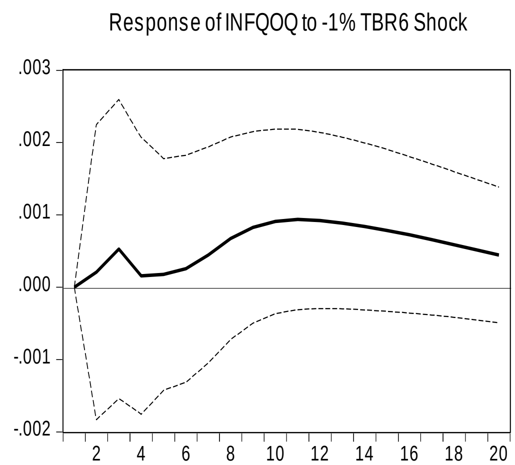
<tr><td>Interest Rate</td><td>
</td><td>
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<tr><td></td><td>Quarters</td><td>Quarters</td></tr>
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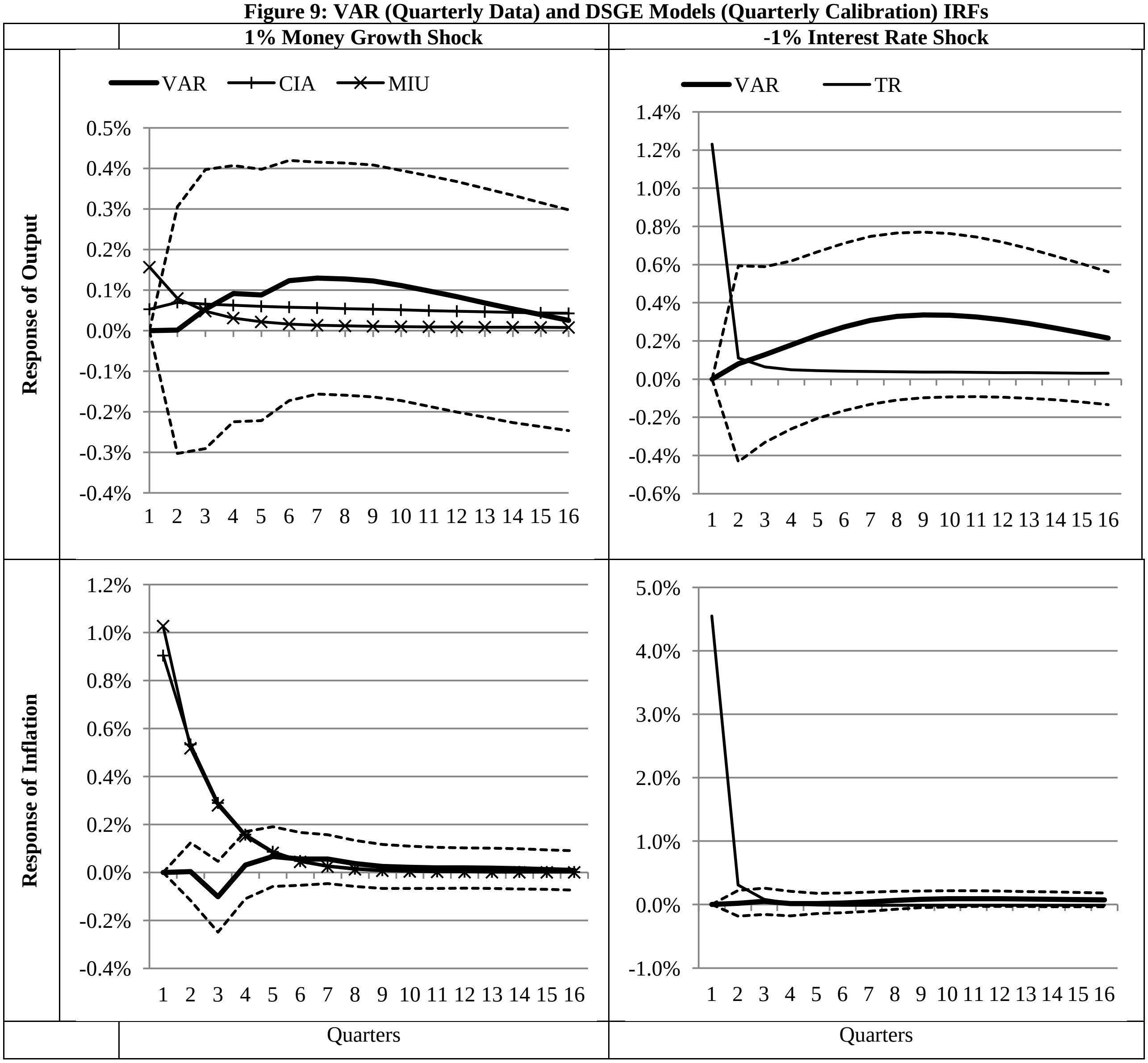
Figure 9: VAR (Quarterly Data) and DSGE Models (Quarterly Calibration) IRFs



*** Dotted lines show +/- 2 standard errors for VAR IRFs.**

Figure 10: Simulated Output and Inflation

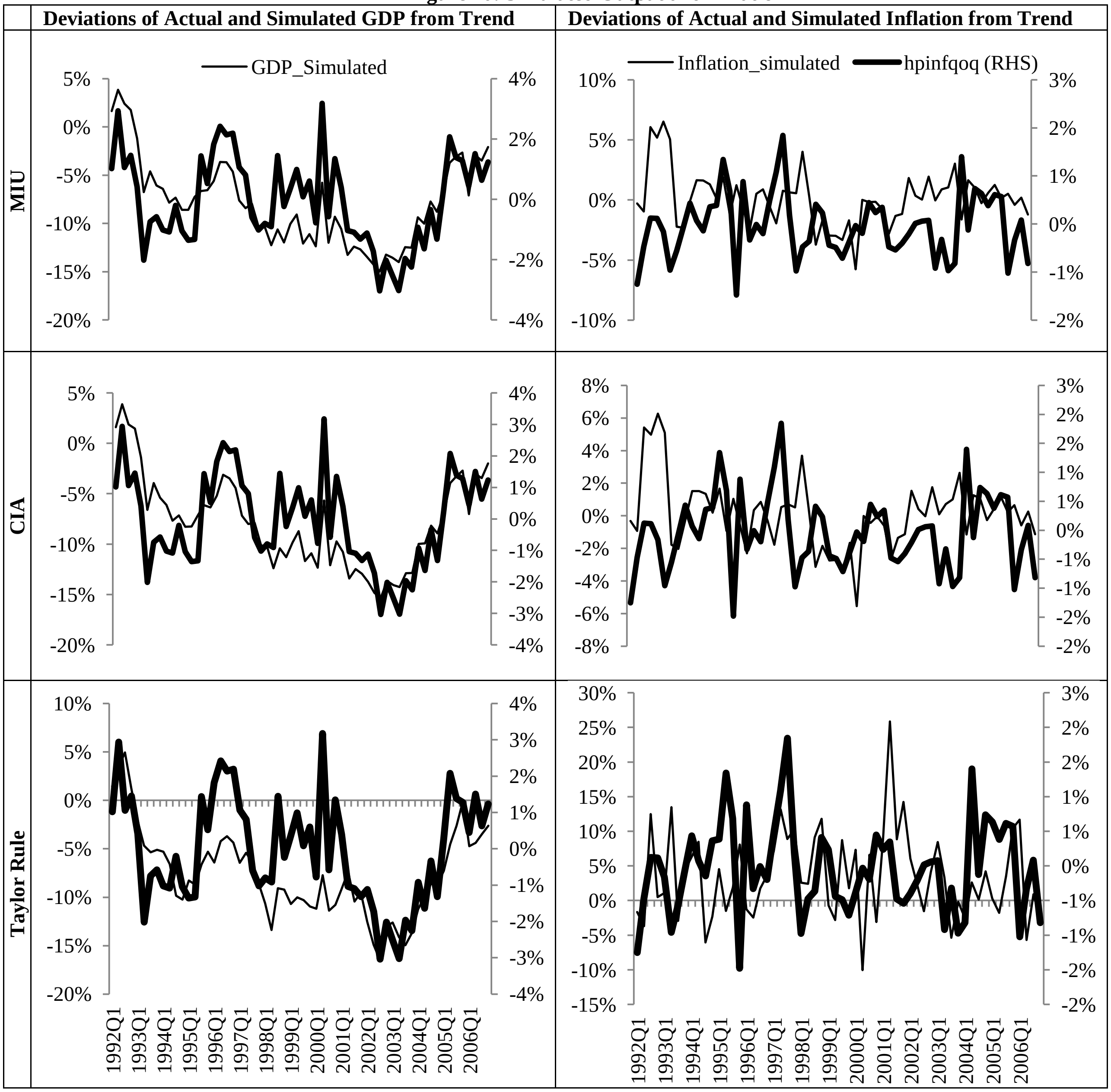


Figure 11: Impulse Response Functions to a Technology Shock

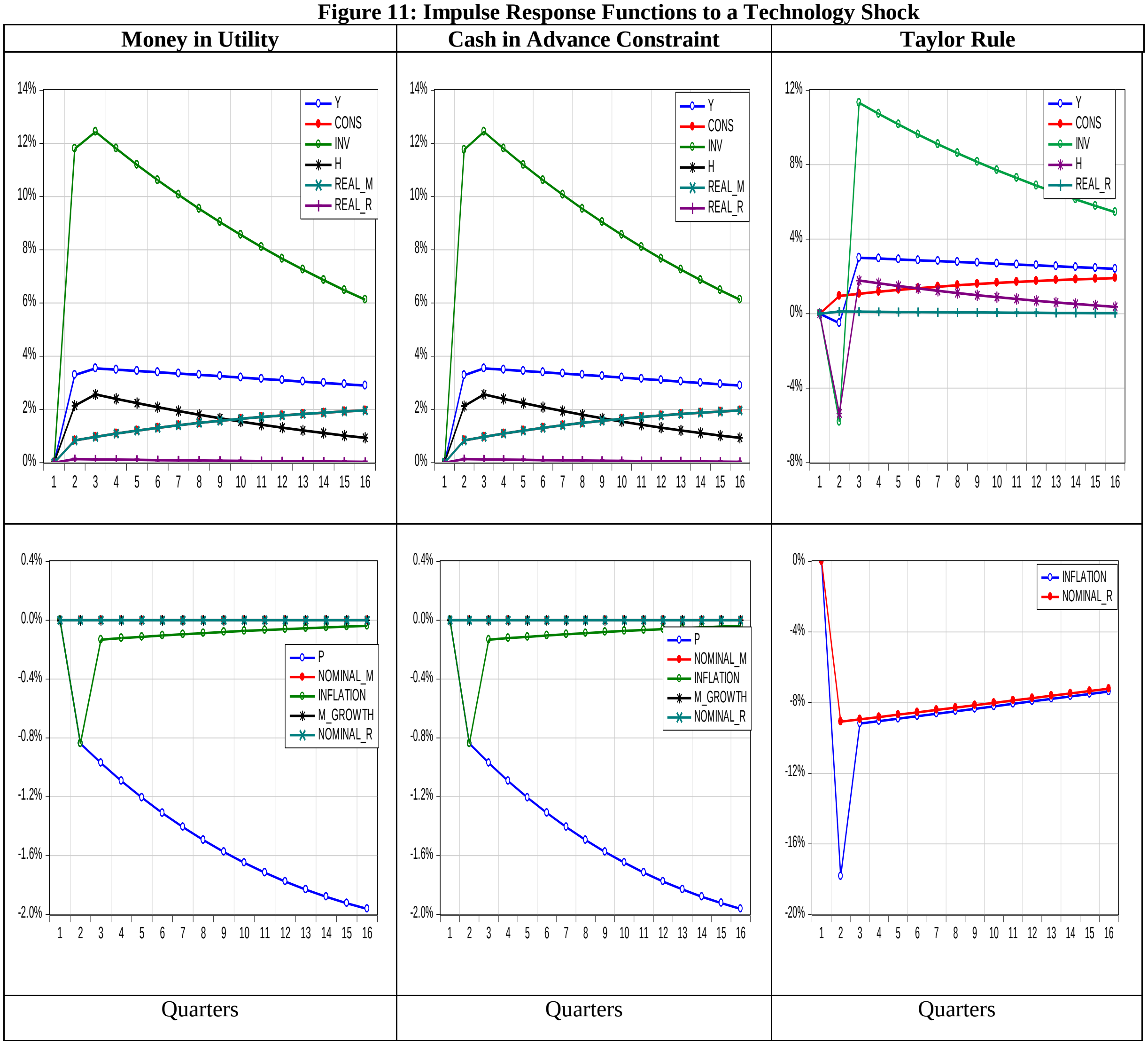


Figure 12: Impulse Response Functions to a Monetary Shock

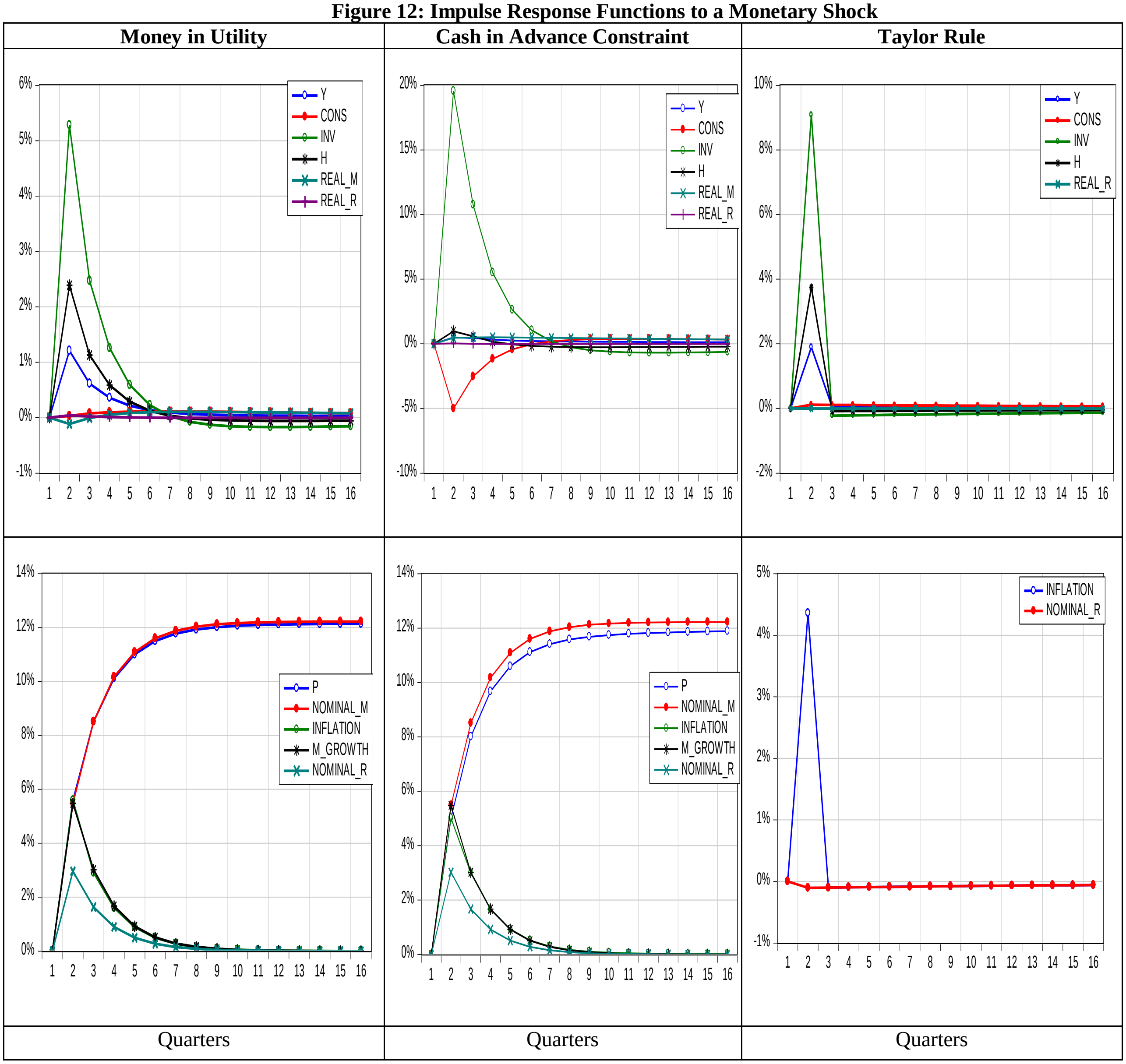
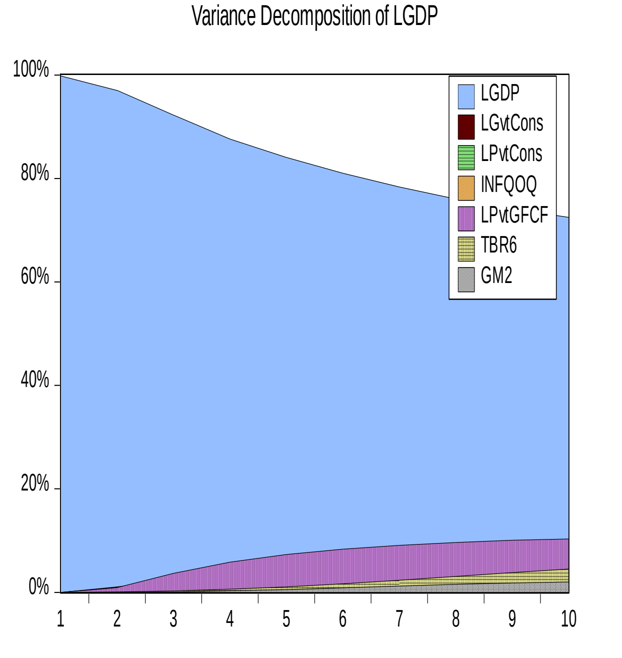
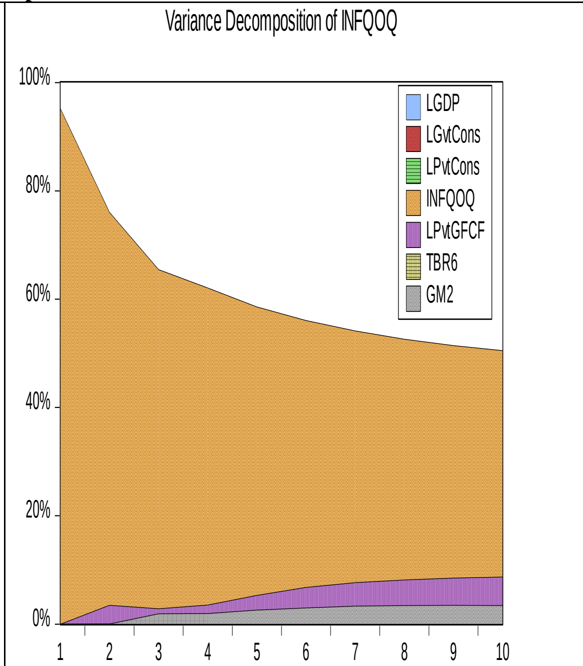


Figure 13: Variance Decompositions from VAR Models



Quarters



Quarters

Table A1: Contemporaneous Correlations.

	GDP		LSM		INFLATION	
	Annual	**Quarterly**	**Annual**	**Quarterly**	**Annual**	**Quarterly**
HPLM0	0.63***	0.35***	0.40*	0.10	0.54**	0.31***
HPLM1	0.53***	0.46***	0.65***	0.24*	0.27	0.09
HPLM2	0.78***	0.27***	0.40*	0.41***	0.51**	0.26**
HPLR M0	0.46**	0.35***	0.73***	0.23**	0.13	0.22**
HPLRM1	0.36*	0.39***	0.73***	0.29***	0.02	0.05
HPLRM2	0.68***	0.27***	0.83***	0.46***	0.18	0.12
GM0	0.20	0.12	0.15	0.06	0.09	0.11
GM1	-0.13	-0.13	0.03	-0.04	-0.32	-0.25**
GM2	0.22	0.01	0.44**	0.11	0.13	0.01
POLR	0.19	0.10	-0.31	-0.05	0.32	0.15
TBR6	0.32	0.11	-0.16	0.02	0.47**	0.29***
MMR	0.28	0.16	-0.16	-0.01	0.46**	0.37***
RPLR	-0.33	-0.15	-0.62***	-0.21**	-0.31	-0.53***
RTBR6	-0.24	-0.16	-0.50**	-0.18*	-0.14	-0.47***
RMMR	-0.26	-0.12	-0.55***	-0.21**	-0.24	-0.46***
Inflation (QoQ)	0.49**	0.03	0.22	-0.02		
GDP					0.48	0.17

*** p<0.01, ** p<0.05 and *p<0.1.

Table A2: Granger Causality Results (Annual Data)*

	Causal Variables															
Dependent variable	**GDP**	**LSM**	**Inflation**	**M0**	**M1**	**M2**	**RM0**	**RM1**	**RM2**	**GM0**	**GM1**	**GM2**	**MMR**	**POLR**	**RMMR**	**RPOLR**
HPLGDP	…	Yes**	No	No	Yes**	No	No	Yes**	Yes***	No	No	No	Yes*	Yes*	No	No
HPLLSM	No	…	No	No	Yes*	No	Yes**	Yes***	Yes*	No	No	No	Yes**	Yes**	No	No
Inflation	No	Yes**	…	No	Yes**	No	Yes**	Yes***	Yes***	No	No	Yes*	No	No	No	No
HPLM0	Yes*	Yes**	No	…	Yes***	No	Yes**	Yes***	Yes***	No	No	No	No	Yes*	No	Yes*
HPLM1	No	No	No	No	…	No	No	Yes***	Yes**	No	Yes**	Yes**	Yes*	Yes**	No	No
HPLM2	No	Yes**	No	No	No	…	Yes*	Yes*	Yes**	No	No	Yes*	Yes**	Yes**	Yes**	Yes*
HPLRM0	No	No	Yes**	Yes***	No	Yes**	…	Yes***	No	No	No	No	Yes**	Yes***	No	No
HPLRM1	Yes**	No	Yes**	Yes*	Yes***	No	No	…	No	No	No	No	Yes**	Yes***	No	No
HPLRM2	Yes**	No	No	No	No	Yes***	No	No	…	No	No	No	Yes***	Yes**	No	No
GM0	No	No	No	Yes***	No	No	No	No	No	…	Yes**	No	No	Yes**	No	No
GM1	Yes*	No	No	Yes*	Yes***	No	No	No	No	No	…	No	No	No	No	No
GM2	Yes*	No	No	Yes*	No	Yes**	No	No	No	No	No	…	Yes**	Yes**	No	No
MMR	Yes***	No	Yes**	Yes**	Yes**	Yes***	No	No	Yes*	No	No	No	…	No	Yes**	No
POLR	Yes**	No	Yes**	Yes**	Yes***	Yes***	Yes*	Yes**	Yes**	No	No	No	No	…	No	Yes**
RMMR	No	No	No	No	No	No	Yes*	Yes***	No	No	No	No	No	Yes**	…	No
RPOLR	No	No	Yes**	No	No	No	*	Yes**	Yes*	No	No	Yes*	No	Yes**	No	…

* Null hypothesis: Causal variable does not Granger causes dependent variable. *** p<0.01, ** p<0.05 and *p<0.1. 'No' means p>0.1 and null hypothesis is not rejected indicating lack of Granger causality. For the annual data all the variables have been tested for 1 lag or L=1.

Table A3: Granger Causality Results (Quarterly Data)*

	Causal Variables																
Dependent variable	**GDP**	**LSM**	**Inflation**	**M0**	**M1**	**M2**	**RM0**	**RM1**	**RM2**	**GM0**	**GM1**	**GM2**	**MMR**	**POLR**	**RMMR**	**RPOLR**	**Dependent Variable**
HPLGDP	…	No	No	No	Yes (L=4)**	Yes (L=1)**	No	Yes (L=4)***	Yes (L=4)**	No	Yes (L=4)**	Yes (L=1)*	No	No	Yes (L=4)*	No	**HPLGDP**
HPLLSM	Yes (L=2)**	…	No	Yes (L=3)*	No	Yes (L=2)*	Yes (L=4)*	Yes (L=3)**	Yes (L=2)***	Yes (L=3)*	No	No	No	No	No	No	**HPLLSM**
Inflation	No	Yes (L=2)*	…	No	No	No	Yes (L=1)*	Yes (L=4)*	Yes (L=4)**	No	No	No	No	No	No	No	**Inflation**
HPLM0	No	No	Yes (L=2)*	…	Yes (L=4)**	No	Yes (L=4)**	Yes (L=4)***	Yes (L=1)**	No	Yes (L=4)**	Yes (L=1)*	No	No	Yes (L=3)*	Yes (L=2)*	**HPLM0**
HPLM1	Yes (L=1)***	No	No	Yes (L=1)**	…	No	Yes (L=1)***	No	No	No	Yes (L=3)*	Yes (L=3)**	No	No	No	No	**HPLM1**
HPLM2	Yes (L=1)***	Yes (L=4)*	No	No	Yes (L=1)**	…	No	No	No	No	Yes (L=1)***	Yes (L=3)**	No	No	No	No	**HPLM2**
HPLRM0	No	No	Yes (L=3)**	Yes (L=4)***	Yes (L=4)**	No	…	Yes (L=4)***	Yes (L=4)*	Yes (L=4)***	Yes (L=4)**	Yes (L=1)*	No	Yes (L=1)*	Yes (L=2)*	No	**HPLRM0**
HPLRM1	Yes (L=1)***	No	Yes (L=2)*	Yes (L=1)*	Yes (L=4)**	Yes (L=2)**	Yes (L=1)***	…	No	No	Yes (L=3)**	Yes (L=4)*	Yes (L=1)*	Yes (L=1)*	No	No	**HPLRM1**
HPLRM2	Yes (L=1)**	Yes (L=4)*	Yes (L=4)*	No	Yes (L=2)*	Yes (L=4)***	No	No	…	No	Yes (L=1)***	Yes (L=4)***	No	No	No	Yes (L=2)*	**HPLRM2**
GM0	No	No	Yes (L=2)**	Yes (L=4)***	Yes (L=4)**	No	Yes (L=3)**	Yes (L=4)**	No	…	Yes (L=4)**	No	No	No	Yes (L=4)*	Yes (L=3)*	**GM0**
GM1	No	No	No	No	Yes (L=4)***	No	No	Yes (L=4)**	No	No	…	Yes (L=1)*	No	No	No	No	**GM1**
GM2	No	No	No	No	Yes (L=2)*	Yes (L=4)**	No	No	Yes (L=3)**	No	No	…	No	No	No	No	**GM2**
MMR	Yes (L=4)***	No	Yes (L=4)***	Yes (L=1)*	Yes (L=4)***	Yes (L=2)*	Yes (L=4)**	Yes (L=4)***	No	No	No	No	…	Yes (L=3)*	Yes (L=4)**	Yes (L=4)*	**MMR**
POLR	Yes (L=3)*	No	Yes (L=4)**	No	No	Yes (L=1)*	Yes (L=4)**	Yes (L=4)**	Yes (L=1)*	No	No	No	No	…	Yes (L=4)*	Yes (L=4)**	**POLR**
RMMR	Yes (L=1)**	No	No	No	No	No	Yes (L=2)*	No	Yes (L=2)***	No	No	No	No	Yes (L=4)*	…	Yes (L=1)*	**RMMR**
RPOLR	No	Yes (L=2)*	Yes (L=4)*	No	No	No	Yes (L=1)*	Yes (L=4)**	Yes (L=4)*	No	No	No	Yes (L=1)*	No	No	…	**RPOLR**

*Null hypothesis: Causal variable does not Granger causes dependent variable. *** p<0.01, ** p<0.05 and *p<0.1. 'No' means p>0.1 and null hypothesis is not rejected indicating lack of Granger causality. L shows number of lag.

Table A4: Variance Decomposition of Output from DSGE and VAR* Models

	DSGE Models		VAR Model						
	MP Shock	TFP Shock	LGDP	LGvtCons	LPvtCons	INFQoQ	LPvtGFCF	TBR6	GM2
MIU	0.27	99.73	71.57	8.67	0.72	1.42	9.57	5.95	2.10
CIA	0.08	99.92							
Taylor Rule	12.48	87.52							

Table A5: Variance Decomposition of Inflation from DSGE and VAR Models

	DSGE Models		VAR Model						
	MP Shock	TFP Shock	LGDP	LGvtCons	LPvtCons	INFQoQ	LPvtGFCF	TBR6	GM2
MIU	92.09	7.91	11.20	18.13	5.15	51.31	8.35	2.61	3.25
CIA	90.64	9.36							
Taylor Rule	72.53	27.47							

*Variance Decompositions for VAR model are computed by taking average of variance decompositions for 40 quarters horizon.

<u>Appendix A</u>

Steady State

Long run properties of model are studied by applying steady state condition $x_{t-1} = x_t = x_{t+1} = x$ and simultaneous solution of resulting equations by substitution method. Ultimately, all variables are expressed as functions of structural parameters and "great ratios". Since basic framework is same in models, therefore steady states are almost same for all models except slight differences in case of CIA models where steady state consumption, hours and output are less than their counterparts in other models. However, steady state consumption to output and investment to output ratios are still unaffected in these cases.

First of all, let's consider inter-temporal equilibrium condition, equation (22) in steady state form, to pin down value of steady state rental return

$$r^k = \frac{1}{\beta} - 1 + \delta \qquad \text{(A.01)}$$

Profit maximization by intermediate producers implies capital and labour demands are determined through following equations

$$r^k = \theta\left(\frac{k}{h}\right)^{\theta-1} = \theta\frac{y}{k} \text{ and } w = (1-\theta)\left(\frac{k}{h}\right)^{\theta} = (1-\theta)\frac{y}{h} \qquad \text{(A.02)}$$

which implies the following steady state capital-labour ratio

$$\frac{k}{h} = \left(\frac{\theta}{1-\theta}\right)\frac{w}{r^k} \qquad \text{(A.03)}$$

Using A.02,

$$\frac{k}{h} = \left(\frac{r^k}{\theta}\right)^{\frac{1}{\theta-1}} \qquad \text{(A.04)}$$

Having determined $\frac{k}{h}$, we use it back in (A.02) to find steady state wage rate

$$w = (1-\theta)\left(\frac{r^k}{\theta}\right)^{\frac{\theta}{\theta-1}} \qquad \text{(A.05)}$$

c is found using value of w in steady version of intratemporal equilibrium condition (21),

$$c = -\frac{w}{\bar{A}} \qquad \text{(A.06)}$$

For h, again consider intratemporal equilibrium condition (A.06) after substitution of w from (A.02)

$$c = -\frac{(1-\theta)\frac{y}{h}}{\bar{A}}$$

$$\frac{y}{h} = -\frac{\bar{A}c}{(1-\theta)}$$

$$h = -\frac{(1-\theta)y}{\bar{A}c}$$

Using the result from steady state form of aggregate resource constraint

$$h = -\frac{(1-\theta)y}{\bar{A}(y-inv)}$$

$$h=-\frac{(1-\theta)}{\overline{A}\left(1-\frac{inv}{y}\right)} \quad \text{(A.07)}$$

Using the result from steady state form of capital accumulation constraint

$$h=-\frac{(1-\theta)}{\overline{A}\left(1-\delta\frac{k}{y}\right)}$$

$$h=-\frac{(1-\theta)}{\overline{A}\left(1-\delta\frac{\theta}{r^{k}}\right)}$$

In case of CIA models, $c=-\frac{\beta}{\varpi}\frac{w}{\overline{A}}$, therefore resulting expression for h is given by

$$h=-\frac{\beta}{\varpi}\frac{(1-\theta)}{\overline{A}\left(1-\delta\frac{\theta}{r^{k}}\right)} \quad \text{(A.08)}$$

Capital to labour ratio (A.03) is used to pin down value of k, so that

$$k=\frac{\theta}{1-\theta}\frac{w}{r^{k}}h \quad \text{(A.09)}$$

Using values of k and h in production function, we get value of y.

$$y=k^{\theta}h^{1-\theta} \quad \text{(A.10)}$$

We use this value of c to pin down steady state real money demand

$$\frac{1}{c}=\beta\frac{1}{c\pi}+\frac{D}{m}$$

$$\frac{D}{m}=\frac{1}{c}-\beta\frac{1}{c\pi}$$

$$=\frac{\pi-\beta}{c\pi}$$

$$\frac{m}{D}=\frac{c\pi}{\pi-\beta}$$

$$m=\frac{Dc\pi}{\pi-\beta}$$

Since $\pi=\varpi$, we write

$$m=\frac{Dc\varpi}{\varpi-\beta} \quad \text{(A.11)}$$

Financial assets optimization determines nominal interest rate

$$R=\frac{\pi}{\beta} \quad \text{(A.12)}$$

Optimal price equation in steady state becomes,

$$P^* = \frac{\varepsilon_p}{(\varepsilon_p - 1)} Pmc$$

$$P^* = \frac{\varepsilon_p}{(\varepsilon_p - 1)} MC \qquad \text{(A.13)}$$

Calibration of β as a function of $\frac{inv}{y}$

In order to utilize steady state investment-output ration for calibration of β, we need to express β as a function of $\frac{i}{y}$. To this end, we substitute of h from (A.07) in capital-labour ratio (A.03)

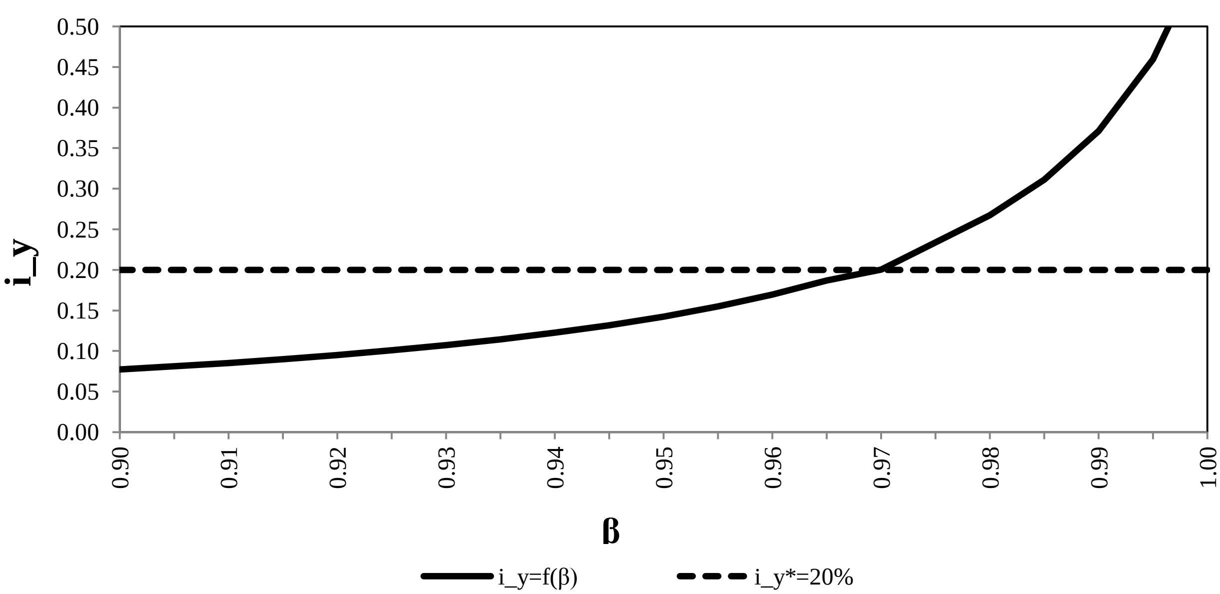
$$\frac{k}{-\frac{(1-\theta)}{\bar{A}\left(1-\frac{inv}{y}\right)}} = \left(\frac{\theta}{1-\theta}\right)\frac{w}{r^k}$$

$$\left(1-\frac{inv}{y}\right) = \frac{-\theta w}{\bar{A} r^k k}$$

$$\frac{inv}{y} = f(\beta) = 1 + \frac{\theta w}{\bar{A} r^k k} \qquad \text{(A.14)}$$

Since w, r^k and k have already been found as functions of β along with other structural parameters. Therefore, (A.14) expresses $\frac{inv}{y}$ as a function of β.

Figure A1: The relationship between β and Steady State investment to output ratio



Appendix B

Log-Linearization of Calvo

All model equations are log-linearized by first taking log and than taking total differential around steady state.

The derivation of the New Keynesian Phillips curve; which is slightly involved is presented here. We start from the re-optimized price equation

$$\hat{P}_t = \frac{\varepsilon_p}{(\varepsilon_p - 1)} \frac{E_t \sum_{k=0}^{\infty} (\beta\varepsilon_p)^k P_{t+k} y_{t+k} mc_{t+k}}{E_t \sum_{k=0}^{\infty} (\beta\varepsilon_p)^k y_{t+k}^j}$$

$$\hat{P}_t E_t \sum_{k=0}^{\infty} (\beta\varepsilon_p)^k y_{t+k}^j = \frac{\varepsilon_p}{(\varepsilon_p - 1)} E_t \sum_{k=0}^{\infty} (\beta\varepsilon_p)^k P_{t+k} y_{t+k}^j mc_{t+k}$$

Applying Uhlig's rule

$$P^*(m)Y(m)E_t \sum_{k=0}^{\infty} (\beta\varepsilon_p)^k e^{\tilde{y}_{t+k} + \tilde{P}_t^*} = \frac{\varepsilon_p}{(\varepsilon_p - 1)} PY(m)mcE_t \sum_{k=0}^{\infty} (\beta\varepsilon_p)^k e^{\tilde{P}_{t+k} + \tilde{y}_{t+k} + m\tilde{c}_{t+k}}$$

$$E_t \sum_{k=0}^{\infty} (\beta\varepsilon_p)^k (1 + \tilde{y}_{t+k} + \tilde{P}_t^*) = E_t \sum_{k=0}^{\infty} (\beta\varepsilon_p)^k \left(1 + \tilde{P}_{t+k} + \tilde{y}_{t+k} + m\tilde{c}_{t+k} \right)$$

$$\tilde{P}_t^* \sum_{k=0}^{\infty} (\beta\varepsilon_p)^k = E_t \sum_{k=0}^{\infty} (\beta\varepsilon_p)^k (\tilde{P}_{t+k} + m\tilde{c}_{t+k})$$

$$\tilde{P}_t^* = \frac{E_t \sum_{k=0}^{\infty} (\beta\varepsilon_p)^k (\tilde{P}_{t+k} + m\tilde{c}_{t+k})}{E_t \sum_{k=0}^{\infty} (\beta\varepsilon_p)^k}$$

$$\tilde{P}_t^* = (1 - \beta\varepsilon_p) E_t \sum_{k=0}^{\infty} (\beta\varepsilon_p)^k (\tilde{P}_{t+k} + m\tilde{c}_{t+k})$$

General price equation in log-linearized form is given as

$$\tilde{P}_t = \varepsilon_p \tilde{P}_{t-1} + (1 - \varepsilon_p) \tilde{P}_t^*$$

Now putting the value of $\tilde{P}_t^*$ log-linearized equation of general price level

$$\tilde{P}_t = \left[\varepsilon_p \tilde{P}_{t-1} + (1 - \varepsilon_p)(1 - \beta\varepsilon_p) E_t \sum_{k=0}^{\infty} (\beta\varepsilon_p)^k (\tilde{P}_{t+k} + m\tilde{c}_{t+k}) \right]$$

We use quasi differencing approach to eliminate infinite sums appearing in the above equation. Using the definition $\pi_t = \ln \frac{P_t}{P_{t-1}}$ and rearranging, we get the final expression:

$$\tilde{\pi}_t - \beta E_t \tilde{\pi}_{t+1} = \frac{(1 - \varepsilon_p)(1 - \beta\varepsilon_p)}{\pi\varepsilon_p} \tilde{mc}_t$$

Appendix C

Structural Differences

Table C1: Financial Access Indicators

Country	Loan Accounts (per 1000 adults) 2009	Deposit Accounts (per 1000 adults) 2009	Bank Branches (per 1,000,000 adults) 2012
Bangladesh	42	319	81
Pakistan	47	226	90
Indonesia	181	484	96
India	124	680	114
Peru	367	716	697
Sri Lanka	487	1652	175
Turkey	315	1851	187
Malaysia	973	2227	199

Table C2: Currency in Circulation

Country	Currency in Circulation/GDP (2006-2012 Average)
India	0.12
China	0.11
Pakistan	0.10
Malaysia	0.06
Bangladesh	0.06
Peru	0.05
Indonesia	0.04
Sri Lanka	0.04
Turkey	0.03

Source: Haver Analytics and authors' calculations

Figure C1: Informal Sector and Currency in Circulation

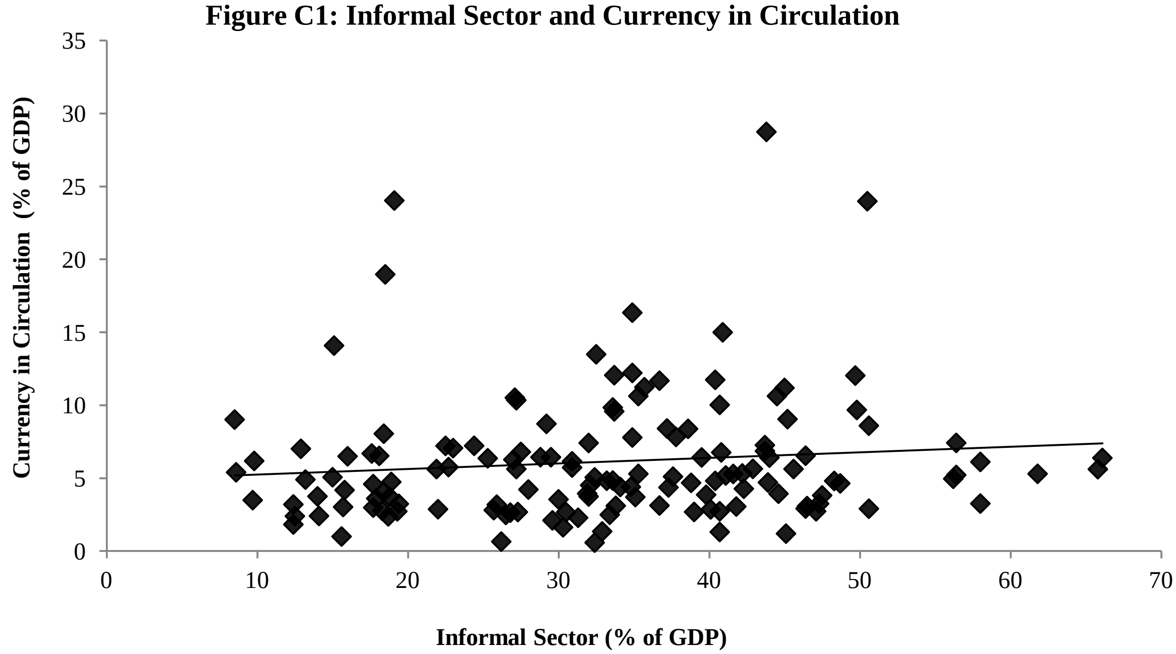


Table C3: Size of the Informal Sector

Country	Informal Economy as Percentage of Official Economy (1999-2007 Average)
Peru	58.0
Sri Lanka	43.9
Pakistan	35.7
Bangladesh	35.3
Turkey	31.3
Malaysia	30.9
India	22.2
Indonesia	18.9
Iran	18.3
China	12.7

Source: Haver Analytics, Schneider et al. (2010) and authors' calculations

Appendix D

Taylor Rule Estimation

Keeping in view the importance of Taylor rule parameters, we have estimated the Taylor rule using different specifications following Ireland (2000).

1) We have estimated Taylor rule using two proxies of interest rate: 6-month T-bill rate and money market rate as Figure D1 shows that money market rate and T-bill rate are not co-moving during first half of the sample period.

2) Moreover, we have tried contemporaneous response and backwards looking monetary policy by trying current and lagged deviations of inflation and output from steady states. Results are summarized in the Table D1.

Figure D1: Behaviour of Different Interest Rate Indicators (1990Q1-2012Q4)

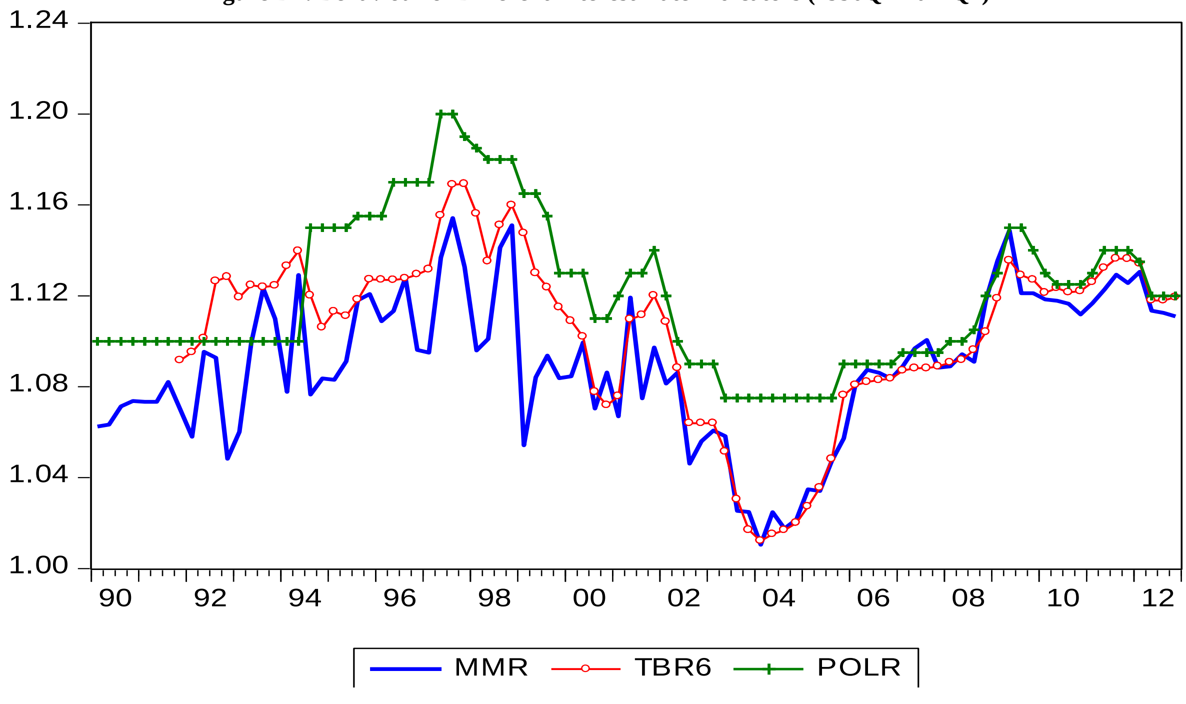


Table D1: Estimation of Different Specifications of Taylor Rule[28]

	MMR	MMR	6M T-bill rate	6M T-bill rate
Constant	0.02 [0.000]	0.03 [0.000]	0.01 [0.02]	0.01 [0.003]
Interest Rate Smoothing	0.71 [0.000]	0.67 [0.000]	0.93 [0.000]	0.91 [0.000]
Current Inflation Deviation	1.28 [0.053]		3.55 [0.064]	
Current Output Deviation	0.27 [0.28]		1.04 [0.116]	
Lagged Inflation Deviation		0.887 [0.130]		2.90 [0.025]
Lagged output Deviation		0.39 [0.103]		0.92 [0.052]
Adjusted R-Square	0.61	0.60	0.93	0.94
Sample	1990Q1-2012Q4	1990Q1-2012Q4	1992Q1-2012Q4	1992Q1-2012Q4

These estimations were checked for potential autocorrelation using Breusch-Godfrey serial correlation LM test for different lag lengths. Results showed minimal or no autocorrelation; as expected owing to lagged interest rate term.

It is interesting to note that normalized weight of inflation and output deviations are almost same for all specifications of Taylor rule except lagged money market rate.

Table D2: Normalized Weight of Inflation and Output in Taylor Rule[29]

	Inflation Deviation	Output Deviation
Money Market Rate (current)	0.82	0.18
Money Market Rate (lagged)	0.69	0.31
6 Month T-bill Rate (current)	0.77	0.23
6 Month T-bill Rate (lagged)	0.76	0.24

When the Taylor rule equation is written in unconstrained form, the parameters for all four specifications do not seem much different. Please see Table D3

Table D3: Unconstrained Taylor rule Specifications

	Smoothing	Inflation	Output
Money Market Rate (current)	0.71	0.37	0.08
Money Market Rate (lagged)	0.67	0.29	0.13
6 Month T-bill Rate (current)	0.93	0.25	0.07
6 Month T-bill Rate (lagged)	0.91	0.26	0.08

28 p-values are given in square brackets.
29 Normalized Weight for Inflation = Inflation Coefficient / (Inflation Coefficient + Output Coefficient.)

Appendix E

Table E1: Common Equations

Levels Form	Log-linearized Form
$y_t = \exp^{A_t} h_t^{1-\theta} k_t^{\theta}$	$\tilde{y}_t = \tilde{A}_t + (1-\theta)\tilde{h}_t + \theta \tilde{k}_t$
$\frac{k_t}{h_t} = \frac{\theta}{1-\theta}\frac{w_t}{r_t^k}$	$\tilde{k}_t - \tilde{h}_t = \tilde{w}_t - \tilde{r}_t^k$
$mc_t = \frac{1}{A_t}\theta^{-\theta}(1-\theta)^{-(1-\theta)} w_t^{1-\theta} r_t^{k\theta}$	$m\tilde{c}_t = (1-\theta)\tilde{w}_t + \theta \tilde{r}_t^k - \tilde{A}_t$
$P_t^*(m) = \frac{\varepsilon_p}{(\varepsilon_p - 1)} \frac{E_t \sum_{i=0}^{\infty} (\beta\varepsilon_p)^i P_{t+i} y_{t+i}(m) mc_t}{E_t \sum_{i=0}^{\infty} (\beta\varepsilon_p)^i y_{t+i}(m)}$	$\tilde{\pi}_t - \beta\tilde{\pi}_{t+1} = \frac{(1-\varepsilon_p)(1-\beta\varepsilon_p)}{\pi\varepsilon_p} m\tilde{c}_t$
$P_t = \left[\varepsilon_p P_{t-1}^{1-\varepsilon_p} + (1-\varepsilon_p)\hat{P}_t^{1-\varepsilon_p}\right]^{\frac{1}{1-\varepsilon_p}}$	$\tilde{P}_t = \varepsilon_p \tilde{P}_{t-1} + (1-\varepsilon_p)\tilde{P}_t^*$
$r_t = \frac{R_t}{E_t \pi_{t+1}}$	$\tilde{r}_t = \tilde{R}_t - E_t \tilde{\pi}_{t+1}$
$k_{t+1} = inv_t + (1-\delta)k_t$	$\tilde{k}_{t+1} = \delta\, \widetilde{inv}_t + (1-\delta)\tilde{k}_t$
$A_t = \exp\left[\rho_A A_{t-1} + (1-\rho_A)\overline{A} + \varepsilon_t^A\right]$	$\tilde{A}_t = \rho_A \tilde{A}_{t-1} + \varepsilon_t^A$

Table E2: Model specific equations

MIU	CIA	Taylor Rule
$\frac{1}{c_t} = -\frac{\bar{A}}{w_t}$	$\frac{1}{c_t} = -\frac{\bar{A}}{w_t}$	$\frac{1}{c_t} = -\frac{\bar{A}}{w_t}$
$E_t \frac{c_{t+1}}{c_t} = \beta E_t (1 + r_{t+1}^k - \delta)$	$E_t \frac{w_{t+1}}{w_t} = \beta E_t (1 + r_{t+1}^k - \delta)$	$E_t \frac{c_{t+1}}{c_t} = \beta E_t (1 + r_{t+1}^k - \delta)$
$\frac{1}{c_t} = \beta \frac{1}{c_{t+1}\pi_{t+1}} + \frac{D}{m_t}$	$\frac{\bar{A}}{w_t} = -\beta \frac{1}{\pi_{t+1}c_{t+1}}$	
$\frac{c_{t+1}}{c_t} = \beta \frac{R_t}{\pi_{t+1}}$	$\frac{w_{t+1}}{w_t} = \beta \frac{R_t}{\pi_{t+1}}$	$\frac{c_{t+1}}{c_t} = \beta \frac{R_t}{\pi_{t+1}}$
	$\pi_t c_t = m_{t-1}$	
$m_t = \frac{\varpi_t}{\pi_t} m_{t-1}$	$m_t = \frac{\varpi_t}{\pi_t} m_{t-1}$	
$c_t + inv_t = y_t$	$c_t + inv_t = y_t$	$c_t + inv_t = y_t$
$\varpi_t = \exp\left[\rho_{\varpi}\varpi_{t-1} + \varepsilon_t^M\right]$	$\varpi_t = \exp\left[\rho_{\varpi}\varpi_{t-1} + \varepsilon_t^M\right]$	
		$R_t = \left(R_{t-1}\right)^{\rho_R} \left[R\left(\frac{\pi_t}{\pi}\right)^{\psi_\pi}\left(\frac{y_t}{y}\right)^{\psi_y}\right]^{(1-\rho_R)} \varepsilon_t^R$

Table E3: Model specific equations (log-linearized)

MIU	CIA	Taylor Rule
$\tilde{c}_t = \tilde{w}_t$	$\tilde{c}_t = \tilde{w}_t$	$\tilde{c}_t = \tilde{w}_t$
$E_t\tilde{c}_{t+1} - \tilde{c}_t = \beta r^k E_t \tilde{r}_{t+1}^k$	$E_t\tilde{w}_{t+1} - \tilde{w}_t = \beta r^k E_t \tilde{r}_{t+1}^k$	$E_t\tilde{c}_{t+1} - \tilde{c}_t = \beta r^k E_t \tilde{r}_{t+1}^k$
$\frac{\beta m}{\beta m + Dc\pi} E_t\left(\tilde{c}_{t+1} + \tilde{\pi}_{t+1}\right) + \frac{Dc\pi}{\beta m + Dc\pi}\tilde{m}_t = \tilde{c}_t$	$E_t\tilde{\pi}_{t+1} + \tilde{c}_{t+1} = \tilde{w}_t$	
	$\tilde{\pi}_t + \tilde{c}_t = \tilde{m}_{t-1}$	
$E_t\tilde{c}_{t+1} - \tilde{c}_t = \tilde{r}_t$	$E_t\tilde{w}_{t+1} - \tilde{w}_t = \tilde{r}_t$	$E_t\tilde{c}_{t+1} - \tilde{c}_t = \tilde{r}_t$
$\tilde{m}_t = \tilde{\varpi}_t - \tilde{\pi}_t + \tilde{m}_{t-1}$	$\tilde{m}_t = \tilde{\varpi}_t - \tilde{\pi}_t + \tilde{m}_{t-1}$	
$\frac{c}{y}\tilde{c}_t + \frac{inv}{y}\widetilde{invt} = \tilde{y}_t$	$\frac{c}{y}\tilde{c}_t + \frac{inv}{y}\widetilde{invt} = \tilde{y}_t$	$\frac{c}{y}\tilde{c}_t + \frac{inv}{y}\widetilde{invt} = \tilde{y}_t$
$\varpi_t = \rho_M \varpi_{t-1} + \varepsilon_t^M$	$\varpi_t = \rho_M \varpi_{t-1} + \varepsilon_t^M$	$\tilde{R}_t = \rho_R \tilde{R}_{t-1} + \left(1-\rho_R\right)\psi_\pi \tilde{\pi}_t + + \left(1-\rho_R\right)\psi_y \tilde{y}_t + \varepsilon_t^R$