## **Monetary-Exchange Rate Policy and Current Account Dynamics**

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#### Abstract

A dynamic stochastic general equilibrium monetary model with incomplete and imperfect asset markets, monopolistic competition and staggered nominal price rigidities is developed to shed light on the role of exchange rate and its relation with current account dynamics in the formulation of optimal monetary policy. The paper also compares the performance of various monetary-exchange rate policy rules. The paper shows that because of incomplete risk sharing, due to incomplete asset markets, the dynamic relationship between real exchange rate and net foreign assets affect the behaviour of domestic inflation and aggregate output. This, in turn, implies that the optimal monetary policy should entail a response to net foreign asset position or the real exchange rate gap defined as the difference between actual real exchange rate and the value that would prevail with flexible prices and complete asset markets. In comparing the performance of alternative monetary-exchange rate policy rules, an interesting and fairly robust result that stands out is that 'dirty floating' out-performs flexible exchange rate regime with domestic inflation targeting.

#### **JEL Classification**: E52, F41

**Keywords**: optimal monetary policy, incomplete asset markets, net foreign assets, current account dynamics, inflation targeting, exchange rate policy.

## **1 - Introduction**

The debate over the role of exchange rate in the formulation of monetary policy is far from being settled. Moreover, the implications of current account dynamics for monetary policy are either not studied or not well understood. The framework developed in the paper is intended to seek answer to two questions. First, how would the overall design of monetary policy change when we move from closed economy models to open economy models, especially in terms of the appropriate welfare criterion needed to compare alternative monetary-exchange rate policies? Second, what is the role of exchange rate and its dynamic relation with the current account in the conduct of monetary policy in an open economy setting?

Traditionally, the effects of monetary policy on aggregate economic activity in an open economy setting have been largely studied within the framework developed by Mundell (1963), Fleming (1962) and Dornbusch (1976). Although the Mundell-Fleming-Dornbusch (MFD) framework has played a dominant role in shaping the literature on open economy macroeconomics (largely due to its empirical success and popularity among policy makers), it has certain important methodological drawbacks. These include lack of microfoundations for the aggregate macroeconomic relationships, inability to provide well-defined welfare criteria by which to evaluate the effectiveness of alternative macroeconomic policies, disregard of the role of the intertemporal budget constraints, which is central in the analysis of the current account and exchange rate dynamics, and failure to provide an explicit account of how monetary policy affects firm's production and price-setting decisions.

Beginning with the seminal contribution by Obstfeld and Rogoff (1995a,1996), considerable amount of research (labelled as 'New Open Economy Macroeconomics' (NOEM)<sup>1</sup>) has been done to overcome these drawbacks. The highlighting features of a typical NOEM model are dynamic general equilibrium framework as the workhorse of analysis, one-period advanced price-setting, and the use of nominal quantity of money as an exogenous instrument of monetary policy. More recent papers such as Clarida, Gali and Gertler (2001), Gali and Monacelli (2002), Monacelli (2000), McCallum and Nelson (2000), Walsh (1999), and many others have introduced staggered price-setting structure, which allows for richer dynamic effects of monetary policy, and make use of the nominal interest rate as the instrument of monetary

<sup>&</sup>lt;sup>1</sup>A number of articles provide detailed and critical survey of this literature. For example, Sarno (2001), Lane (2001), Bowman and Doyle (2003) and VanHoose (2004).

policy. Some of the important and widely accepted results of this literature on optimal monetary policy in an open economy setting are provided by Clarida, Gali and Gertler (2001) and Gali and Monacelli (2002). For instance, they claim that under certain standard conditions, the optimal monetary policy design problem for the small open economy is isomorphic to the problem of closed economy. In other words, domestic inflation targeting is the optimal monetary policy and that the central bank should not respond to exchange rate movements, that is, it should allow the exchange rate to float freely. Moreover, there is no role for current account dynamics and the welfare criterion or the objective function of the central bank just includes targeting domestic inflation and output around the 'flexible-price' optimal value. The contribution of this paper to this literature is that it provides new insights to the questions pertaining to the conduct of monetary policy in an open economy by clarifying the role of exchange rate and its relation with current accounts in a dynamic general equilibrium model with incomplete and imperfect asset markets, monopolistic competition, and staggered nominal price rigidities.

Exchange rates have always been a mystery when it comes to understanding and evaluating their role in the formulation of monetary policy. Numerous issues have been considered in the NOEM literature in this regard. For instance, it depends on the currency in which firms set their prices. If firms set their prices in the seller's currency, known as producer currency pricing (PCP), then a number of researchers such as Obstfeld and Rogoff (1995a), Gali and Monacelli (2002), Clarida, Gali and Gertler (2001), Engel (2002), Corsetti and Pesenti (2001a, 2001b) and Sutherland (2000, 2002) have shown that the monetary authority should only target domestic prices and let the exchange rate float. On the other hand, if firms are assumed to set their prices in the buyer's currency, known as local currency pricing (LCP), then the domestic price level remains completely unaffected by exchange rate movements and therefore to ensure complete risk sharing the monetary authority should keep the exchange rate fixed (Betts and Devereux (2000), Devereux and Engle (2000, 2002)). Other variations in this debate include traded versus non-traded goods (Obstfeld and Rogoff (2000, 2002)), complete versus incomplete exchange rate pass-through (Corsetti and Pesenti (2001b), Sutherland (2002), Smets and Wouters (2002), and domestic versus CPI inflation targeting (Svensson (2000)). It is important to note, however, that most of this literature employs the assumption of complete asset markets. Adding more flavour to this debate, I demonstrate in this paper that with PCP and

incomplete and imperfect asset markets, the welfare-enhancing monetary policy implies a dirty float under domestic inflation targeting.

The interaction of current account dynamics with key macroeconomic variables and their implications for the monetary-exchange rate policy in NOEM models have not been clearly analyzed. Most of the recent literature effectively shuts down the current account channel as a dynamic shock-propagation mechanism to keep the analysis simple. This is accomplished by either incorporating the complete asset markets assumption<sup>2</sup> (e.g., Clarida, Gali and Gertler (2001), Gali and Monacelli (2002), Chari, Kehoe and McGrattan (1998) and Devereux and Engel (2000)) or by imposing a unitary elasticity of substitution between domestic and foreign  $goods^3$ (e.g., Corsetti and Pesenti (2001a, 2001b), Tille (2001)). However, as pointed out by Obstfeld and Rogoff (1995b), the assumption of complete asset markets is not realistic in a model with imperfections and rigidities in goods market because with nominal rigidities monetary policy will affect real variables including the current account. With incomplete asset markets the dynamics of current account do matter for monetary policy because then, besides dealing with the distortions created by monopolistic competition, the central bank need to address the inefficiencies caused by incomplete asset markets. Moreover, in an empirical paper, Lane and Milesi-Ferretti (2002, 2002) link net foreign asset positions to long-run values of real exchange rates and suggest that optimal monetary policy responses may depend on the movements in the current account.

Based on these observations, I incorporate incomplete asset markets by assuming that domestic economic agents have access to a one-period riskless (non-contingent) domestic bond and a foreign bond<sup>4</sup>. Thus, all country specific shocks/risks cannot be fully insured against, so there is a possibility that current account imbalances may occur. For example, after a positive (negative) shock, the domestic agents smooth their consumption by lending to (borrowing from) the foreign country and thus end up running a current account surplus (deficit). It is important to

<sup>&</sup>lt;sup>2</sup> Complete asset markets mean that economic agents are able to trade as many state-contingent assets as there are future states of nature thus insuring themselves against any type of risk or shock that may hit the economy. Thus, complete risk pooling takes place among countries and there will be no gains from intertemporal trade. That is, current account remains unaffected.

 $<sup>^{3}</sup>$  A unit elasticity of substitution implies that expenditures on domestic and foreign goods incurred by the economic agents are constant leading to constant export revenues. This implies that if the current account begins in balance, it remains in balance. The advantage of this approach is that we can solve the model without resorting to log-linearization.

<sup>&</sup>lt;sup>4</sup> An alternative method of introducing incomplete asset markets is to assume that the economic agents have access to state-contingent assets that have nominal rather than real pay-offs. Markets are considered incomplete in the sense that agents can not undo the effects of sticky prices. (see, Devereux and Engel (2001) and Engel (2002)).

note, however, that in all future periods after the shock the consumption differential between countries follows a random walk meaning that there is no well defined endogenously determined unique steady-state. Since there is a possibility of an infinite number of steady-state equilibria, log-linearization of the model is also problematic and can be very inaccurate (Kim and Kose, 2001) because one would be approximating the dynamics of the model around a moving steady state. This is the well-known stationarity problem of open economy models with infinitely-lived economic agents. Avoiding such problems is probably the main reason why most recent papers have chosen to make assumptions that effectively shut down the current account channel by using either of the assumptions discussed above.

In order to explore the implications of current account dynamics for other macroeconomic variables and monetary policy, while maintaining a unique steady-state, I incorporate an endogenous risk premium that depends on the domestic country's net foreign asset position. Examples of this approach include Benigno (2001), and Selaive and Tuesta (2003).<sup>5</sup>

The main result of the paper is that managed exchange rate regime (dirty floating) is superior to flexible exchange rate regime under domestic inflation targeting. Volatility in both output and domestic inflation goes down and so does the volatility in real exchange rate. Put differently, there is no trade-off between stabilizing the real exchange rate and domestic inflation and output gap: welfare improves as the central bank places some weight on stabilizing the real exchange rate and pursues domestic inflation targeting. However, it is important to note that as the central bank tries to stabilize the real exchange rate 'too much', that is, approaches fixed exchange rate case, welfare decreases. Thus, the analysis does not imply that policy should always aim to eliminate exchange rate gaps. Some exchange rate gap may well be necessary to avoid large output gaps. Typically a tight monetary policy, responding to a positive output gap, will lead to an appreciation, which is a helpful element in the transmission mechanism. Where the exchange rate gap term plays an essential role is if a positive demand shock is combined with

<sup>&</sup>lt;sup>5</sup> Other approaches include Mendoza (1991), which assume that the rate of time preference is a decreasing function of consumption; Schmitt-Grohe and Uribe (2001a), Kollmann (2001), and Bergin (2002), which assume an exogenous risk premium term; Smets and Wouters (2002), which incorporate Blanchard's (1985) overlapping generations model in which domestic agents face a non-zero probability of death at each point in time; Ghironi (2001) and Cavallo and Ghironi (2002), which uses Weil's (1989) specification of an overlaepping generation set-up (where agents are born on different dates with no assets) to attain determinacy. The choice among different stationarity-generating approaches is quite adhoc and difficult to distinguish quantitatively as shown by Schmitt-Grohe and Uribe (2001b).

an excess appreciation as a result of some distortion to international risk sharing or uncovered interest parity (such as the net foreign asset position). In this case the appreciation may eliminate the impact of the shock on the output gap, but a consumption gap would remain, and so a cut in interest rates to moderate the appreciation will enhance welfare. This result is quite robust and holds regardless of the welfare criterion used: whether it includes real exchange rate movements, or focuses only on output gap and inflation movements. Also, the result remains unchanged whether the central bank adopts flexible inflation targeting or strict inflation targeting.

The key reason behind this unconventional yet important result is the presence of current account dynamics affecting not only the real exchange rate behaviour via imperfect risk sharing due to incomplete asset markets, but also the output gap via the risk premium term in the interest parity relationship. The intuition is as follows. Suppose the economy experiences a positive taste shock that tends to push up both output gap and domestic inflation and causes some appreciation of the real exchange rate. A typical response would be to increase the nominal interest rate, which leads to further appreciation that helps the transmission mechanism. In a model with complete asset markets and thus without current account dynamics (zero net foreign assets), the analysis would stop here and predict that the central bank can completely stabilize shocks that push up output gap and domestic inflation in the same direction implying flexible domestic inflation targeting with completely flexible exchange rate as the optimal monetary policy (e.g., Clarida, Gali, and Gertler (2001)). On the other hand, in the presence of net foreign assets, there would be 'excess' appreciation due to a taste shock: an appreciation improves the net foreign asset position that in turn causes further appreciation. In this case, increasing the interest rates would exacerbate the excess appreciation. Therefore, placing some positive weight on stabilizing real exchange rate, by lowering the interest rate, eliminates this excess appreciation leading to welfare improvements. Put differently, appreciation (caused by increasing the interest rate) may eliminate the impact of the taste shock on the output gap, but a consumption gap would remain due to incomplete risk sharing, and so a less aggressive response by the policy authority (a slight cut in interest rates to moderate the appreciation) will enhance welfare. However, the policy should not try to eliminate the real exchange rate gap completely by lowering the interest rates too much as it may lead to large output gaps; some exchange rate gap may well be necessary to avoid these gaps and improve welfare.

The rest of the paper is organized as follows. Section 2 develops the dynamic general equilibrium model elaborating the behaviour of households and firms and incorporates incomplete asset markets and staggered nominal price rigidities. Section 3 linearizes the optimality conditions around the steady state and expresses the dynamics of key macroeconomic variables such as output, domestic inflation, real exchange rate, and current account. Section 4 studies the behaviour of the monetary authority and discusses the welfare criterion --- the optimal monetary policy --- in addition to alternative monetary-exchange rate policy rules. Section 5 calibrates the model and analyzes the performance of domestic and CPI inflation targeting (both flexible and strict) with varying degree of exchange rate flexibility under taste, foreign output, and technology shocks. Section 6 summarizes and provides concluding remarks.

## 2 - The Model

There are three types of economic agents in the economy: households, firms, and the monetary authority. Given their preferences, households decide how much to consume (both domestically produced goods and imported goods) and how to allocate time between leisure and work. The firms, operating in a monopolistically competitive environment, take two decisions: how much to produce using the labour services of the households and how to set the price for their output. The monetary authority issues money and employs nominal interest rates as an instrument of monetary policy to achieve certain well-specified goals.

#### 2.1 – Households

The economy consists of a continuum of identical households. The model is described in terms of a representative household making decisions in the presence of uncertainties about the future with preferences defined over a composite consumption good  $C_t$ , a taste shock  $u_t$ , leisure  $1 - N_t$ , and real money balances  $M_t/P_t$ , with  $M_t$  being nominal quantity of money, and  $P_t$  being the overall consumer price index (CPI). This representative household seeks to maximize the expected present discounted value of utility:

$$E_{t}\sum_{k=0}^{\infty}\beta^{k}\left[\frac{u_{t}C_{t+k}^{1-\sigma}}{1-\sigma}-\psi\frac{N_{t+k}^{1+\phi}}{1+\phi}+\frac{\chi}{1-\xi}\left(\frac{M_{t+k}}{P_{t+k}}\right)^{1-\xi}\right]$$
(1)

where ' $\beta$ ' captures rate of time preference, ' $\sigma$ ' represents the intertemporal elasticity of substitution, ' $\phi$ ' is the elasticity of substitution between consumption and leisure, and thus measures the elasticity of labour supply, and ' $\xi$ ' is the interest rate elasticity of money demand. All parameters are assumed to be positive.

The composite consumption index is a function of domestic goods and foreign goods, and is defined as:

$$C_{t} = \left[ \left(1 - a\right)^{\frac{1}{\eta}} C_{H,t}^{\frac{\eta - 1}{\eta}} + a^{\frac{1}{\eta}} C_{F,t}^{\frac{\eta - 1}{\eta}} \right]^{\frac{\eta}{\eta - 1}}$$
(2)

where  $C_{H,t}$  denotes the index of domestic consumption goods and  $C_{F,t}$  the index of foreign consumption goods (imports). ' $\eta$ ' is a measure of the elasticity of substitution between domestic and foreign goods and is assumed to be positive.<sup>6</sup> Parameter '*a*' represents the share of foreign (imported) goods in the consumption index. I assume that consumption is differentiated at the individual goods level. Thus, the domestic and foreign goods consumption indices can be written as CES aggregators of the quantities consumed of each type of good.

$$C_{H,t} = \left(\int_{0}^{1} C_{H,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$$
(3)

$$C_{F,t} = \left(\int_{0}^{1} C_{F,t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$$
(4)

where ' $\epsilon$ ' is the elasticity of substitution within each category and is assumed to be greater than one, i.e.,  $\epsilon > 0^7$ .

$$C_{H,i}(j) = \left(\frac{P_{H,i}(j)}{P_{H,i}}\right)^{-\epsilon} C_{H,i} \quad \text{and} \quad C_{F,i}(j) = \left(\frac{P_{F,i}(j)}{P_{F,i}}\right)^{-\epsilon} C_{F,i}$$

<sup>&</sup>lt;sup>6</sup> Note that assuming  $\eta = 1$  yields a Cobb-Douglas form for the consumption index as analyzed by Corsetti and Pesenti (2001a).

<sup>&</sup>lt;sup>7</sup> The demand functions for goods within each category can be determined by maximizing equation (3) and (4), individually, with respect to the total expenditure on the respective category of good, given as  $Z_{H,i} = \int_{0}^{1} P_{H,i}(j)C_{H,i}(j)dj$ and  $Z_{F,i} = \int_{0}^{1} P_{F,i}(j)C_{F,i}(j)dj$ .  $P_{H,i}(j)$  and  $P_{F,i}(j)$  are the prices of the consumption goods  $C_{H,i}(j)$  and  $C_{F,i}(j)$  respectively. The demand functions that emerge from this maximization exercise are given as:

By maximizing equation (2) subject to the total expenditure on home and foreign goods, I can derive the demand functions for home and foreign consumption goods. The total expenditure equation can be written as:  $Z_t = P_{H,t}C_{H,t} + P_{F,t}C_{F,t}$ . The optimality condition yields the following equations:

$$C_{H,t} = (1-a) \left(\frac{P_{H,t}}{P_t}\right)^{-\eta} C_t$$
(5)

$$C_{F,t} = a \left(\frac{P_{F,t}}{P_t}\right)^{-\eta} C_t \tag{6}$$

where  $P_t$  is the overall price index and is given as:

$$P_{t} = \left[ (1-a)P_{H,t}^{1-\eta} + aP_{F,t}^{1-\eta} \right]^{\frac{1}{1-\eta}}$$
(7)

In the rest of the world a representative household faces a problem identical to the domestic household's problem. It is assumed that a foreign household's utility function and aggregate consumption good is analogous to that of a domestic household with the exception that foreign households do not face any taste shocks. Thus, relationships similar to equation (5) and (6) for the foreign country can be written as:

$$C_{F,t}^{*} = (1 - a^{*}) \left(\frac{P_{F,t}^{*}}{P_{t}^{*}}\right)^{-\eta} C_{t}^{*}$$
(8)

$$C_{H,t}^{*} = a^{*} \left(\frac{P_{H,t}^{*}}{P_{t}^{*}}\right)^{-\eta} C_{t}^{*}$$
(9)

where ' $a^*$ ' is the share of foreign goods (domestic economy's exports) in the overall consumption of the foreign country.  $P_t^*$  is the overall price index in the foreign country and is given as:

$$P_t^* = \left[a^* P_{H,t}^{*^{1-\eta}} + (1-a^*) P_{F,t}^{*^{1-\eta}}\right]^{\frac{1}{1-\eta}}$$
(10)

where  $P_{H,i}$  and  $P_{F,i}$  are the price indices for domestic and foreign (imported) goods respectively that satisfy the expenditure equations that can be expressed as  $Z_{H,i} = P_{H,i}C_{H,i}$  and  $Z_{F,i} = P_{F,i}C_{F,i}$ . The expressions for  $P_{H,i}$  and  $P_{F,i}$  are given as:

$$P_{H,t} = \left(\int_{0}^{1} P_{H,t}(j)^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}} \quad \text{and} \quad P_{F,t} = \left(\int_{0}^{1} P_{F,t}(j)^{1-\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}}$$

Following Gali and Gertler (2002), I assume that the foreign country is very large relative to the domestic economy. One way to think of it is to consider the foreign country as the rest of the world. This assumption implies that the share of domestic goods consumed by the rest of the world is negligible; so  $P_{F,t}^* = P_t^*$  as  $a^* \to 0$ . Thus, I can write the relationship linking the terms of trade and the real exchange rate as:

$$Q_t = ToT_t \frac{P_{H,t}}{P_t} \tag{11}$$

where the real exchange rate is defined as:  $Q_t = \frac{S_t P_t^*}{P_t}$  and the terms of trade (the price of

imported goods relative to domestic goods) is defined as:  $ToT_t = \frac{P_{F,t}}{P_{H,t}} = \frac{S_t P_{F,t}^*}{P_{H,t}}$ .  $S_t$  is the

nominal exchange rate.

In nominal terms the representative household 'h's intertemporal budget constraint is given as:

$$W_{t}^{h}N_{t}^{h} + P_{t}TR_{t}^{h} + P_{t}\Pi_{t}^{h} + M_{t-1}^{h} + B_{H,t-1}^{h} + S_{t}B_{F,t-1}^{h} = P_{t}C_{t}^{h} + M_{t}^{h} + \frac{B_{H,t}^{h}}{(1+i_{t})} + \frac{S_{t}B_{F,t}^{h}}{(1+i_{t}^{*})\phi(S_{t}B_{F,t}/P_{t})}$$
(12)

The left hand side represents the resources the consumer has at his disposal at the beginning of period t. These consist of wage earnings  $W_t^h N_t^h$ , obtained by supplying labour services to the firm, transfers  $P_t T R_t^h$  from the monetary authority, share of profits  $P_t \Pi_t^h$  from firms', amount of money  $M_{t-1}^h$  held, the amount of one-period risk-free domestic bonds  $B_{H,t-1}^h$  that earns an interest rate  $i_t$ , and the amount of one-period risk-free foreign currency denominated bonds  $B_{F,t-1}^h$  purchased. The right hand side corresponds to the uses of these resources. The household can use these to consume goods, acquire new money balances or purchase new bonds. The important point to note is that the price of foreign currency denominated bond is proportional to its gross nominal interest rate,  $(1 + i_t^*)$ . Following Benigno (2001) and Selaive and Tuesta (2003), I assume that the factor of proportionality function,  $\phi(.)$ , depends on the real holdings of the foreign assets in the domestic economy.  $\phi(.)$  is the risk premium term representing the cost of participating in the international assets market and allows

us to obtain a well-defined steady state. In particular,  $\phi(.)$  is assumed to be a decreasing function of the economy's stock of real foreign assets,  $b_t = S_t B_{F,t} / P_t$ , and is given as:

$$\phi(b_t) = \kappa(e^{\bar{b}-b_t} - 1) \tag{13}$$

where,  $\kappa$  is some constant and  $\overline{b}$  is the steady state level of real foreign assets.

I further assume that the initial wealth is the same across all the domestic economy's households and that they all work for all the firms sharing the profits in equal proportion. This set of assumptions implies that all the households in the domestic economy face the same budget constraint and that in their consumption decision will choose the same consumption path. Thus, I can drop the index h and consider a representative household's behaviour. The following optimality conditions (derived by maximizing equation (1) subject to equation (12)) must hold for this household in equilibrium:

$$\beta E_t \left(\frac{u_{t+1}}{u_t}\right) \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+1}}\right) = (1+i_t)^{-1}$$
(14)

$$\beta E_t \left(\frac{u_{t+1}}{u_t}\right) \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \left(\frac{P_t}{P_{t+1}}\right) \left(\frac{S_{t+1}}{S_t}\right) = (1+i_t^*)^{-1}$$
(15)

$$\frac{W_t}{P_t} = \psi \left( u_t^{-1} C_t^{\sigma} \right) N_t^{\phi}$$
(16)

$$\frac{M_t}{P_t} = \left(\frac{1}{\chi}C_t^{-\sigma}\frac{i_t}{1+i_t}\right)^{-\frac{1}{\zeta}}$$
(17)

$$\frac{S_t B_{F,t}}{(1+i_t^*) \phi(b_t)} = S_t B_{F,t-1} + P_{H,t} C_{H,t} + S_t P_{H,t}^* C_{H,t}^* - P_t C_t$$
(18)

Equation (14) is the standard Euler equation for the holding of domestic bond. It has the usual interpretation that at a utility maximum, the household cannot gain from feasible shifts of consumption between periods. Similarly, equation (15) is the efficiency condition for the holding of foreign bonds. Equation (16) represents the labour supply decision and equation (17) gives the money demand function. Equation (18) represents the resource constraint of the domestic

economy, which is obtained by aggregating the equilibrium budget constraint of the households with that of the government.<sup>8</sup>

By combining equations (14) and (15), I can derive, after some approximations, the familiar uncovered interest parity condition depicting the optimal portfolio choices of the economic agent:

$$(1+i_t) = (1+i_t^*)\phi(b_t)E_t \frac{S_{t+1}}{S_t}$$
(19)

Analogous to the domestic household's optimization problem, similar optimality conditions holds for the representative household in the foreign country. For example, the counterpart of equation (14) can be written as:

$$\beta E_t \left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma} \left(\frac{P_t^*}{P_{t+1}^*}\right) = (1+i_t^*)^{-1}$$
(20)

Combining equation (20) with (15) and using the definition of real exchange rate yields the following relationship:

$$E_t\left(\frac{u_{t+1}}{u_t}\right)\left(\frac{C_{t+1}}{C_t}\right)^{-\sigma}\phi(b_t) = E_t\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-\sigma}\left(\frac{Q_t}{Q_{t+1}}\right)$$
(21)

This equilibrium condition reflects how the representative households in each country share consumption risk. An important point to note is that both the interest parity relationship (equation (19)) and the international risk-sharing equilibrium condition (equation 21)) are affected by net foreign asset position of domestic households.

#### **2.2- Firms**

This section outlines the mechanics of monopolistic competition in a dynamic general equilibrium setting. To make the presentation clear, I define two types of firms. The first operates in a monopolistically competitive environment and are called intermediate-good-producing firms, and the other operates in competitive markets and is called final-good-producing firms.

<sup>&</sup>lt;sup>8</sup> Assuming no government spending and imposing the condition that domestic bonds are in zero-net supply implies that for the government budget constraint to hold, all the seignorage revenue associated with money creation must be returned to the households in the form of lump-sum transfers in each period, that is,  $M_i - M_{i-1} = P_i T R_i$ . Also, in equilibrium, we have:  $W_i N_i + P_i \Pi_i = P_{i,i} C_{ij,i} + e_i P_{ij,i}^* C_{ij,i}^*$ 

Like every firm operating in a monopolistically competitive market, each intermediategood-producing firm takes two types of decisions --- how much output to produce and at what prices to sell this output that would maximize profits. In doing so, a representative firm 'j' is subject to a number of constraints. First is the specification of the production function. Following McCallum and Nelson (1999), I assume that there is no capital in the economy and so the firm only employs the labour input,  $N_i(j)$ , supplied by households to produce the differentiated good:

$$Y_t(j) = A_t N_t(j) \tag{22}$$

where  $Y_t(j)$  is the intermediate-good produced by firm '*j*'.  $A_t = \exp(z_t)$  where '*z<sub>t</sub>*' represents aggregate technology shock.

The representative firm 'j' then supplies this good to the final good-producers. If the output of the final good, which is produced by using the inputs supplied by a continuum of intermediate-goods-producing firms indexed by  $j \in [0,1]$ , is denoted by  $Y_t$ , then the production function for the final output can be written as:

$$Y_{t} = \left[\int_{0}^{1} Y_{t}(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right]^{\frac{\varepsilon}{\varepsilon-1}}$$
(23)

Profit maximization (max  $P_{H,t}Y_t - \int_0^1 P_{H,t}(j)Y_t(j)dj$ ) by final-goods-producers yields the

following input demand function:

$$Y_t(j) = \left[\frac{P_{H,t}(j)}{P_{H,t}}\right]^{-\varepsilon} Y_t$$
(24)

This input demand function describes the second constraint faced by intermediate-goodsproducing firm 'j'. Note that these goods are consumed both domestically and abroad. The demand function for the domestically produced differentiated good is given in footnote 8. A similar function (the export-demand function) exists for the demand of these goods in the foreign country.

The third constraint introduces price stickiness by assuming that each period some firms are unable to adjust their price. This staggered price adjustment behaviour is based on Calvo (1983). Firms are assumed to face a constant probability  $1-\rho$  in every period to alter their price

in an optimal fashion. This probability is independent of how long their prices have been fixed and the expected duration of price stickiness is  $1/\rho$ . It is easy to verify that with a large number of firms in the economy, the fraction of firms adjusting price optimally in a period is equal to the probability of price adjustment  $1-\rho$ ; the remaining fraction of firms ' $\rho$ ' do not adjust their price. Thus, the parameter ' $\rho$ ' captures the degree of nominal price rigidity.

To facilitate the tractability of the model, I assume initially that all firms are able to adjust their prices every period, that is, the third constraint is not binding yet.<sup>9</sup> Then, the profit function for a representative firm 'j' can be written as:

$$\pi_{H,t}(j) = P_{H,t}(j)Y_t(j) - W_t N_t(j)$$
(25)

The differentiated-good-producing firm chooses  $P_{H,t}(j)$  and  $N_t(j)$  to maximize these profits subject to the conditional demand for their variety of output given by equation (24) and the production function given by equation (22). The expressions for  $P_{H,t}(j)$  and  $N_t(j)$  are given respectively as:

$$P_{H,t}(j) = \frac{\varepsilon}{\varepsilon - 1} M C_t$$
(26)

and

$$\frac{W_t}{P_{H,t}(j)} = \frac{\varepsilon - 1}{\varepsilon} F_N \tag{27}$$

where  $\left(\frac{\varepsilon}{\varepsilon-1}\right) = \upsilon$  is the constant mark-up and  $MC_t$  is the minimized nominal marginal cost.  $F_N$ 

is the marginal product of labour which, given the production function, is simply  $A_t$ .

Equation (26) depicts the relationship between the 'flexible' price chosen by all firms and the minimized marginal cost of production under monopolistic competition; it does not say anything about prices being sticky. Combining equation (26) and (27) I can write:

$$MC_t = \frac{W_t}{A_t} \tag{28}$$

or in real terms as:

<sup>&</sup>lt;sup>9</sup> Put differently, I assume that firm's price-setting decisions are completely independent from their factor demand decisions. One way to interpret this separation of decisions is to think of firms as having two departments; one that decides what price to set each period, and the other that decides how much output to produce taking the prices of the inputs as given.

$$mc_t = \frac{W_t}{P_{H,t}A_t}$$
(28a)

Now, I introduce price stickiness by assuming that price adjustment does not take place simultaneously for all firms. Following Rotemberg (1987), suppose that a representative firm 'j' that is allowed to change its price, set its price to minimize the expected present discounted value of deviations between the price it sets and the minimized nominal marginal cost.

$$\sum_{k=0}^{\infty} \rho^{k} \beta^{k} E_{t} (P_{H,t}(j) - MC_{t+k})^{2}$$
(29)

where  $MC_t$  is the minimized nominal marginal cost. Note that there are two parts to discounting. The first,  $\beta$  represents a conventional discount factor, and the second,  $\rho$  reflects the fact that the firm that has not adjusted its price after 'k' periods, still has the same price in period t + k that she set in period t. The first order condition with respect to  $P_{H,t}(j)$  gives the following optimal value denoted by  $\tilde{P}_{H,t}(j)$ :<sup>10</sup>

$$\widetilde{P}_{H,t} = (1 - \rho\beta)MC_t + \rho\beta E_t \widetilde{P}_{H,t+1}$$
(30)

Thus, the optimally chosen price in period t is a weighted average of nominal marginal cost and expected value of optimal price in the future. However, in period t, only a fraction  $1-\rho$  of firms set their price according to equation (30). The remaining firms are stuck with the prices set in previous periods. Since the fraction of firms that are able to optimally set their price is randomly chosen, the average price of the previous period will be the price of the fraction of firms that are unable to adjust their price this period. Therefore, the overall aggregate price level in period t is a weighted average of current optimally chosen and past prices.

$$P_{H,t} = \left[\rho^{1-\varepsilon} P_{H,t-1} + (1-\rho)^{1-\varepsilon} \widetilde{P}_{H,t}\right]^{\frac{1}{1-\varepsilon}}$$
(31)

where  $\tilde{P}_{H,t}$  is the price chosen by all adjusting domestic firms in period t.

<sup>&</sup>lt;sup>10</sup> It is reasonable to set  $\tilde{P}_{H,t}(j) = \tilde{P}_{H,t}$  because all firms are identical except for the timing of their price adjustment.

## **3-** Log-linearized Model

In this section, the model is log-linearized around the steady state. A variable in lower case represent the log deviation with respect to the steady state. In equilibrium firms are assumed to be symmetric and taking identical decisions. This implies that prices are equal for each variety of good; that is,  $P_{H,t}(j) = P_{H,t}$ ,  $P_{F,t}(j) = P_{F,t}$ .

## 3.1 – Output Dynamics: the new IS-curve

Assuming that the economy's output is can either be consumed domestically or exported to the rest of the world, I can write:  $Y_t = C_{H,t} + C_{H,t}^*$ . Log-linearizing around the steady state gives:

$$y_t = (1-a)c_{H,t} + ac_{H,t}^*$$
(32)

where, assuming balanced trade in steady state 'a', captures the share of the exports in aggregate output.

Similarly, the log-linearized version of equation (2) can be written as:

$$c_t = (1-a)c_{H,t} - ac_{F,t}$$
(33)

Now, consider equation (5) and (6) --- the demand curves for the domestic and foreign goods. The log-linearized version of the two equations are:  $c_{H,t} = c_t - \eta(p_{H,t} - p_t)$  and  $c_{F,t} = c_t - \eta(p_{F,t} - p_t)$ , where the price differentials are given as:  $p_{H,t} - p_t = -a(tot)_t$  and  $p_{F,t} - p_t = (1-a)(tot)_t$  derived by using the log-linear version of equation (7),  $p_t = (1-a)p_{H,t} + ap_{F,t}$ , and the log-linear version of the definition of the terms of trade,  $tot_t = p_{F,t} - p_{H,t}$ . Moreover, the log-linearized version of equation (11) is given as:  $q_t = (1-a)(tot)_t$ . Thus, using these relationships I can derive a simple expression linking the two demand curves:

$$c_{H,t} - c_{F,t} = \left(\frac{\eta}{1-a}\right) q_t \tag{34}$$

Similarly, using equation (8) and (9) and noting that as  $a^* \to 0$ ,  $c_{F,t}^* = c_t^* = y_t^*$ , an expression analogous to equation (34) can be derived:

$$c_{H,t}^* - y_t^* = \left(\frac{\eta}{1-a}\right) q_t$$
 (35)

Combining equations (33) - (35), I can write:

$$y_{t} = (1-a)c_{t} + ay_{t}^{*} + \left(\frac{a\eta(2-a)}{1-a}\right)q_{t}$$
(36)

Thus, domestic output is a weighted average of domestic and foreign expenditures, plus an 'expenditure-switching factor' which is proportional to the real exchange rate.

In order to derive an IS-type relationship that relates output the real interest rate, I need to make use of the Euler equations for domestic consumption (equation (14)), foreign consumption (equation (20)) and the uncovered interest parity condition (equation (19)). The log-linearized versions of these relationships are:

$$c_{t} = E_{t}c_{t+1} - \frac{1}{\sigma} \left[ i_{t} - \left\{ \pi_{H,t+1} + \left( \frac{a}{1-a} \right) E_{t} \Delta q_{t+1} \right\} \right] - \frac{1}{\sigma} \left( E_{t} u_{t+1} - u_{t} \right)$$
(37)

$$y_{t}^{*} = E_{t} y_{t+1}^{*} - \frac{1}{\sigma} \Big[ i_{t}^{*} - E_{t} \pi_{t+1}^{*} \Big]$$
(38)

$$i_{t} - \pi_{H,t+1} = i_{t}^{*} - E_{t}\pi_{t+1}^{*} + \left(\frac{1}{1-a}\right)E_{t}\Delta q_{t+1} - \kappa b_{t}$$
(39)

Note that in deriving equation (37) and (38), I have used the relationship between CPI inflation,  $\pi_t$ , and domestic inflation,  $\pi_H$ , given as:  $\pi_t = \pi_{H,t} + \left(\frac{a}{1-a}\right)\Delta q_t$ .

Thus, after substituting equations (37) - (39) in equation (36), I get a relationship that resembles an IS equation:

$$y_{t} = E_{t} y_{t+1} - \left(\frac{1+w}{\sigma}\right) (i_{t} - E_{t} \pi_{H,t+1}) + \left(\frac{w}{\sigma}\right) (i_{t}^{*} - E_{t} \pi_{t+1}^{*}) - \kappa \left(\frac{w+a}{\sigma}\right) b_{t} - \left(\frac{1-a}{\sigma}\right) (E_{t} u_{t+1} - u_{t})$$
(40)  
where,  $w = a(2-a)(\eta \sigma - 1)$ 

Following Clarida, Gali and Gertler (2001), let  $x_t = y_t - y_t^0$  be defined as the output gap, where  $y_t^0$  is defined as the level of output that arises with perfectly flexible prices. Similarly, let  $r_t^0$  and  $r_t^*$  be the domestic real interest rates for the domestic and foreign economy respectively that arise in the frictionless equilibrium. Also,  $b_t^0$  (which equals zero) corresponds to the net asset holdings in the complete asset market case. Then, I can write equation (40) as:

$$x_{t} = E_{t} x_{t+1} - \left(\frac{1+w}{\sigma}\right) (i_{t} - E_{t} \pi_{H,t+1} - r_{t}^{0}) - \kappa \left(\frac{w+a}{\sigma}\right) b_{t}$$

$$\tag{41}$$

where,

$$r_t^0 = \left(\frac{\sigma}{1+w}\right) E_t(y_{t+1}^0 - y_t^0) + \left(\frac{w}{1+w}\right) r_t^* - \left(\frac{1-a}{1+w}\right) (E_t u_{t+1} - u_t)$$
(42)

The expression for the level of output that arises with perfectly flexible prices,  $y_t^0$  can be calculated by equating log-linear expression for labour demand (equation (27)),  $w_t - p_{H,t} = z_t$  with the log-linear expression for labour supply, (equation (16)),  $w_t - p_t = \sigma c_t + \phi n_t - u_t$ , and using production function (equation (22)),  $y_t = z_t + n_t$ , the relationship,  $p_{H,t} - p_t = -\left(\frac{a}{1-a}\right)q_t$ , and the resource constraint (equation (36)) to eliminate  $c_t$  and  $n_t$ .

$$y_t^0 = \left(\frac{1-a}{\sigma + \phi(1-a)}\right) \left[ (1+\phi)z_t + \left(\frac{a\sigma}{1-a}\right)y_t^* + \left(\frac{w+a}{(1-a)^2}\right)q_t^0 + u_t \right]$$
(43)

 $q_t^0$  is the real exchange rate under flexible prices and complete asset markets, and is derived below.

### 3.2 – Domestic Inflation Dynamics --- the new Phillips curve

The log-linearized version of equation (30) and equation (31) can be combined to produce the following Phillips curve type relationship.

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \theta m c_t \tag{44}$$

where  $\theta = \frac{(1-\rho)(1-\rho\beta)}{\rho}$ 

The expression for  $mc_t$  (real marginal cost) can be had by combining the log-linear expressions for equation (28a),  $mc_t = w_t - p_{H,t} - z_t$ , labour supply equation (equation (16)):  $w_t - p_t = \sigma c_t + \phi n_t - u_t$ , production function (equation (22)),  $y_t = z_t + n_t$ , the relationship,

$$p_{H,t} - p_t = -\left(\frac{a}{1-a}\right)q_t, \text{ and the resource constraint (equation (36)):}$$
$$mc_t = \left(\frac{\sigma}{1-a} + \phi\right)y_t - \left(\frac{a\sigma}{1-a}\right)y_t^* - \left(\frac{w+a}{(1-a)^2}\right)q_t - (1+\phi)z_t - u_t$$
(45)

Note that in the above equation an appreciation (depreciation) of the real exchange rate increases (decreases) the marginal cost<sup>11</sup>. To understand this seemingly puzzling and counterintuitive relationship, need to remember that there are two distinct channels through which real exchange rate can affect real marginal cost: the 'expenditure switching effect' and the 'risk sharing effect'. Only the first one --- the 'expenditure switching effect' --- is captured in equation (45). According to this channel an appreciation increases domestic consumption relative to foreign consumption (and thus real marginal cost) for a given output level and thus increases real marginal cost. The other channel through which real exchange rate affects real marginal cost is captured by the international risk sharing condition (equation (21)). An appreciation of the real exchange rate lowers the domestic consumption relative to foreign consumption leading to a decrease in real marginal cost. This is explained further below while discussing the real exchange rate dynamics.

Note that by setting  $mc_t = 0$ , I get the same expression for  $y_t^0$  given in equation (43) above. Thus, subtracting equation (44) from equation (46), I can write:

$$mc_{t} = \left(\frac{\sigma}{1-a} + \phi\right) x_{t} - \theta \left(\frac{w+a}{\left(1-a\right)^{2}}\right) \left(q_{t} - q_{t}^{0}\right)$$

$$\tag{47}$$

Equation (55), therefore, can be written as:

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \theta \left( \frac{\sigma}{1-a} + \phi \right) x_t - \theta \left( \frac{w+a}{(1-a)^2} \right) (q_t - q_t^0)$$
(48)

#### **3.3 – Real Exchange Rate Dynamics**

The equation describing the dynamic behaviour of the real exchange rate derived by combining the log –linearized version of equation (21) with equation (36). The log-linearized version of equation (21) is given as:

$$E_{t}(c_{t+1}-c_{t}) = E_{t}(c_{t+1}^{*}-c_{t}^{*}) + \frac{1}{\sigma}E_{t}(q_{t+1}-q_{t}) - \frac{\kappa}{\sigma}b_{t} + \frac{1}{\sigma}E_{t}(u_{t+1}-u_{t})$$
(49)

<sup>&</sup>lt;sup>11</sup> See King (2000) for more discussion

Write equation (36) one period forward and substitute in equation (49) to get:

$$q_{t} = E_{t}q_{t+1} - \left(\frac{\sigma(1-a)}{1+w}\right)E_{t}\left(y_{t+1} - y_{t}\right) + \left(\frac{\sigma(1-a)}{1+w}\right)E_{t}\left(y_{t+1}^{*} - y_{t}^{*}\right) - \left(\frac{\kappa(1-a)^{2}}{1+w}\right)b_{t} + \left(\frac{(1-a)^{2}}{1+w}\right)E_{t}\left(u_{t+1} - u_{t}\right)$$
(50)

Expressing equation (50) in gap-form provides a dynamic equation for the real exchange rate:

$$q_{t} = E_{t}q_{t+1} - \left(\frac{\sigma(1-a)}{1+w}\right)E_{t}\left(x_{t+1} - x_{t}\right) - \left(\frac{\kappa(1-a)^{2}}{1+w}\right)b_{t} - E_{t}\left(q_{t+1}^{0} - q_{t}^{0}\right)$$
(51)

where  $q_t^0$  is given as:

$$q_t^0 = \left(\frac{\sigma(1-a)}{1+w}\right) \left(y_t^0 - y_t^*\right) - \left(\frac{(1-a)^2}{1+w}\right) u_t$$
(52)

## 3.4 – Net Foreign Assets-Current Account Dynamics

Finally, the dynamic equation for net foreign assets can be had by log-linearizing equation (18). First, it would be convenient to re-write equation (18) in real terms as:

$$\frac{b_t}{(1+i_t^*)\phi(b_t)} = b_{t-1}(1+\pi_t)^{-1} + NX_t$$
(53)

where,  $NX_t$  stands for net exports and equals:

$$NX_{t} = \frac{P_{H,t}}{P_{t}}Y_{t} - C_{t}$$
(54)

The log-linearized version of equations (53), after making use of the relationship,  $\pi_t = \pi_{H,t} + \left(\frac{a}{1-a}\right) \Delta q_t$ , is given as:

$$\beta(1+\kappa)b_{t} = \beta i_{t}^{*} + a_{t-1} - \pi_{H,t} - \left(\frac{a}{1-a}\right)\Delta q_{t} + (\beta - 1)nx_{t}$$
(55)

The log-linearized version of equations (54), after making use of equation (36) to eliminate  $c_t$ , is given as:

$$nx_{t} = \left(\frac{1-2a}{(1-a)^{2}}\right)y_{t} + \left(\frac{a}{1-a}\right)^{2}y_{t}^{*} + \left(\frac{h}{(1-a)^{3}}\right)q_{t}$$
(56)

where,  $h = a^2 \eta (2-a) - a(1-a)$ .

To be consistent with the rest of the model, I express equations (55) and (56) in gap-form to get:

$$b_{t} = nb_{t-1} - n\pi_{H,t} + \left(\frac{n(\beta - 1)}{\beta}\right) \left(nx_{t} - nx_{t}^{0}\right) - \left(\frac{na}{1 - a}\right) \left[\left(q_{t} - q_{t-1}\right) - \left(q_{t}^{0} - q_{t-1}^{0}\right)\right]$$
(57)

where,  $n = \frac{1}{\beta(1+\kappa)}$ 

$$nx_{t} = \left(\frac{1-2a}{(1-a)^{2}}\right)x_{t} + \left(\frac{h}{(1-a)^{3}}\right)(q_{t} - q_{t}^{0}) + nx_{t}^{0}$$
(58)

where,

$$nx_t^0 = \left(\frac{1-2a}{(1-a)^2}\right)y_t^0 + \left(\frac{a}{1-a}\right)^2 y_t^* + \left(\frac{h}{(1-a)^3}\right)q_t^0$$
(59)

# 3.5 – Summary of the Complete Model

The summary of the complete log-linearized model is given as:

$$x_{t} = E_{t} x_{t+1} - \left(\frac{1+w}{\sigma}\right) \left(i_{t} - E_{t} \pi_{H,t+1} - r_{t}^{0}\right) - \kappa \left(\frac{w+a}{\sigma}\right) b_{t}$$

$$\tag{41}$$

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \theta \left(\frac{\sigma}{1-a} + \phi\right) x_t - \theta \left(\frac{w+a}{(1-a)^2}\right) \left(q_t - q_t^0\right)$$
(48)

$$q_{t} = E_{t}q_{t+1} - \left(\frac{\sigma(1-a)}{1+w}\right) \left(E_{t}x_{t+1} - x_{t}\right) - \left(\frac{\kappa(1-a)^{2}}{1+w}\right) b_{t} - E_{t}\left(q_{t+1}^{0} - q_{t}^{0}\right)$$
(51)

$$b_{t} = nb_{t-1} - n\pi_{H,t} + \left(\frac{n(\beta - 1)}{\beta}\right) \left(nx_{t} - nx_{t}^{0}\right) - \left(\frac{na}{1 - a}\right) \left[\left(q_{t} - q_{t-1}\right) - \left(q_{t}^{0} - q_{t-1}^{0}\right)\right]$$
(57)

$$nx_{t} = \left(\frac{1-2a}{(1-a)^{2}}\right)x_{t} + \left(\frac{h}{(1-a)^{3}}\right)(q_{t} - q_{t}^{0}) + nx_{t}^{0}$$
(58)

$$r_t^0 = \left(\frac{\sigma}{1+w}\right) E_t (y_{t+1}^0 - y_t^0) + \left(\frac{w}{1+w}\right) r_t^* - \left(\frac{1-a}{1+w}\right) E_t (u_{t+1} - u_t)$$
(42)

$$y_t^0 = \left(\frac{1-a}{\sigma + \phi(1-a)}\right) \left[ (1+\phi)z_t + \left(\frac{a\sigma}{1-a}\right)y_t^* + \left(\frac{w+a}{(1-a)^2}\right)q_t^0 + u_t \right]$$
(43)

$$q_t^0 = \left(\frac{\sigma(1-a)}{1+w}\right) \left(y_t^0 - y_t^*\right) - \left(\frac{(1-a)^2}{1+w}\right) u_t$$
(52)

$$nx_{t}^{0} = \left(\frac{1-2a}{(1-a)^{2}}\right)y_{t}^{0} + \left(\frac{a}{1-a}\right)^{2}y_{t}^{*} + \left(\frac{h}{(1-a)^{3}}\right)q_{t}^{0}$$
(59)

$$r_t^* = \sigma E_t \left( y_{t+1}^* - y_t^* \right)$$
(38)

where,

$$\theta = \left(\frac{(1-\rho)(1-\rho\beta)}{\rho}\right)$$
$$w = a(2-a)(\sigma\eta - 1)$$
$$h = a^2\eta(2-a) - a(1-a)$$
$$n = \frac{1}{\beta(1+\kappa)}$$

#### **Exogenous processes:**

$$u_{t} = \gamma_{u}u_{t-1} + \varepsilon_{t}^{u} \qquad 0 < \gamma_{u} < 1$$

$$z_{t} = \gamma_{z}z_{t-1} + \varepsilon_{t}^{z} \qquad 0 < \gamma_{z} < 1$$

$$y_{t}^{*} = \gamma_{y^{*}}y_{t-1}^{*} + \varepsilon_{t}^{y^{*}} \qquad 0 < \gamma_{y^{*}} < 1$$

# **4** – The Behaviour of Monetary Authority

To study the implications of the dynamic relationship between current account and real exchange rate for the monetary policy, I define optimal monetary policy in terms of an objective function (also termed as the loss function) for the central bank that explicitly translates the behaviour of the certain target variables into a welfare measure. The objective function thus serves as a guide for the monetary authority to evaluate alternative monetary policies. In particular, the paper compares a variety of alternative (non-optimal) policy rules using this benchmark criterion.

#### **4.1 - Optimal Monetary Policy**

By optimal monetary policy I mean that, given the dynamic general equilibrium structure, the effects of all sources of sub-optimality/economic distortions are fully neutralized and the constrained efficient equilibrium is restored. There are four sources of economic distortions present in the model summarized above with incomplete asset markets and sticky prices. First, the effect of real money balances on household's utility. Ignoring this source of sub-optimality can be justified on the grounds that since the monetary authority uses nominal interest rate as policy instrument the behaviour of real money balances is irrelevant<sup>12</sup>.

Second, market power distortion caused by monopolistically competitive firms that charge a constant mark-up. Following Rotemberg and Woodford (1999) and Gali and Monacelli (2002), I assume that government fully offsets the market power distortion by subsidizing employment, which is financed through lump sum tax on households. This assumption ensures that the central bank has no incentive to increase the economy's output beyond the level corresponding to flexible prices and thus the classic inflation bias problem is assumed away. With the subsidy in place, there is only one effective distortion left in the economy, namely, sticky prices.

Third, nominal rigidity in goods prices introduced in a staggered fashion that causes suboptimal variation in prices across firms. In this case the optimal policy would require that real marginal costs (and thus markups) are stabilized at their steady state level, which in turn implies that domestic prices be fully stabilized. The intuition for that result is straightforward but holds only in a closed economy setting: by stabilizing markups at their "frictionless" level, nominal rigidities cease to be binding, since firms do not feel any desire to adjust prices. By construction, the resulting equilibrium allocation is efficient, and the price level remains constant.

In an open economy, there is an additional factor that distorts the incentives of the monetary authority (beyond the presence of market power and nominal rigidities): the possibility of influencing the terms of trade and thus the real exchange rate in a way beneficial to domestic consumers. This possibility, pointed out by Corsetti and Pesent (2001a), is a consequence of the imperfect substitutability between domestic and foreign goods. The introduction of an employment subsidy that exactly offsets the market power distortion and stabilizes the domestic

<sup>&</sup>lt;sup>12</sup> However, if the weight of real money balances in the utility function is non-negligible, then targeting domestic inflation only would not be the optimal outcome; CPI inflation targeting would be preferred. (Svensson (2000)).

price level may not be sufficient to render the flexible price equilibrium allocation optimal for, at the margin, the monetary authority might have an incentive to deviate from it to improve the real exchange rate. However, Following from Woodford's (2002) derivation of a benevolent monetary policy maker's objective function from agents utility, a number of papers (Aoki (2002), Sutherland (2002), Clarida, Gali and Gertler (2001), Batini, Harrison, and Millard (2003), and De Paoli (2004)) have suggested that policy in an open economy should have the same objectives as in a closed economy, and in particular that the exchange rate should play no role. For example, Aoki (2002) considers a two sector model, where prices in one sector are completely flexible, and shows that it is only inflation in the non-flexible sector that is relevant for welfare. He explicitly suggests that imported goods in an open economy are akin to the flexible price sector, and that therefore the price of imported goods (and by implication the exchange rate) should not appear in the objective function of the central bank representing welfare.

But, all these papers invoke the assumption of complete asset markets. What would happen in case of incomplete and imperfect international asset markets? This is the fourth source of sub-optimality in my paper. Presence of incomplete asset markets causes imperfect risk-sharing and may lead to shifts in wealth across countries. Kirsonova, Leith, and Wren-Lewis (2004) derive the objective function of the central bank from the utility function of the households and show that when there are shocks to international risk sharing, the exchange rate appears alongside output and inflation in the social welfare function. Benigno, P. (2001) reached a similar conclusion in a model with incomplete asset markets and argued that since there are trade-offs among several distortions, it is optimal to distribute the losses across different uses.

To understand the role of exchange rate in the objective function of the central bank, consider equations (36) and (49) that are combined to derive the exchange rate dynamics:

$$y_{t} = (1-a)c_{t} + ay_{t}^{*} + \left(\frac{a\eta(2-a)}{1-a}\right)q_{t}$$

$$E_{t}(c_{t+1} - c_{t}) = E_{t}(c_{t+1}^{*} - c_{t}^{*}) + \frac{1}{\sigma}E_{t}(q_{t+1} - q_{t}) - \frac{\kappa}{\sigma}b_{t} + \frac{1}{\sigma}E_{t}(u_{t+1} - u_{t})$$

Given expectation, these equations represent a two equation system involving three endogenous variables (output, consumption and the real exchange rate), one exogenous variable (world consumption), taste shock, and the net foreign asset depicting the imperfect international risk

sharing distortion. Since the presence of net foreign assets captures the imperfect risk-sharing distortion, it is appropriate to define the optimal level of a variable as the level that would occur in the absence of this distortion as well as nominal price stickiness. This allows us to express the two equations (and more generally all equations in the model) in terms of 'gaps' i.e. the difference between actual and the level that would prevail in the absence of these distortions (represented by the superscript '0' in the model).

If, there is no taste shock and net foreign assets, as in Clarida, Gali, and Gertler (2001), then these two equations allow us to express both the consumption and real exchange rate gaps as proportionate to the output gap. We only need to know what the output gap is to know exactly what the other two gaps will be, so if these other gaps appear in any expression for social welfare, we can always replace them by terms in the output gap. However, once we allow for an international risk sharing distortion captured by the presence of net foreign asset term, this will no longer be the case. Such a distortion may generate a real exchange rate or consumption gap, yet the output gap could be zero. As a result, in an open economy, welfare cannot be expressed in terms of output gap variable alone. The minimum value for welfare will not in general be the point in which output gap is zero.

#### 4.1 – Welfare Criterion

Based on the above discussion, I can define the social welfare function that can be used to evaluate the welfare implications of alternative monetary policy rules and exchange rate regimes discussed below. This welfare criterion (SW), defined in terms of the central bank's loss or objective function, can be derived as an approximation of the representative households utility function:

$$SW = \max -\frac{1}{2} E_t \left[ \sum_{k=0}^{\infty} \beta^k L_{t+k} \right]$$
(60)

where  $L_t$  stands for the period 't' loss function of the central bank that takes the output gap  $x_t$ , domestic inflation  $\pi_{H,t}$ , and the real exchange rate gap,  $q_t - q_t^0$  as the target variables:

$$L_{t} = \left(\alpha_{x}x_{t}^{2} + \alpha_{\pi_{H}}\pi_{H,t}^{2} + \alpha_{q}(q_{t} - q_{t}^{0})^{2}\right)$$
(61)

where  $\alpha_x$ ,  $\alpha_{\pi_H}$  and  $\alpha_q$ ' is the weight that the policy authority places on output, domestic inflation, and real exchange rate deviation from their respective level under flexible prices and complete asset markets.

After taking unconditional expectations, the loss function becomes:

$$E(L_t) = \left(\alpha_x \operatorname{var}(x_t) + \alpha_{\pi_H} \operatorname{var}(\pi_{H,t}) + \alpha_q \operatorname{var}(q_t - q_t^0)\right)$$
(62)

where  $var(x_t)$ ,  $var(\pi_{H_t})$  and  $var(q_t - q_t^0)$  are the unconditional variances of domestic inflation, the output gap, and the real exchange rate gap, respectively.

An important point to note about the real exchange rate term is that, like output, it is in the form of deviation from the level that would occur with no nominal price rigidities and international risk sharing shocks like net foreign assets. A number of studies have experimented with simple feedback rules which include some form of exchange rate targeting, but generally not in terms of deviations from its 'natural' level. For example, Kollmann (2002) finds that adding a quadratic term in the change in exchange rate to a feedback rule (with optimized parameters) that already includes domestic price inflation and output disequilibrium terms adds virtually nothing to welfare. This result is interesting because CPI inflation targeting can be roughly 'recovered' from separate terms on domestic price inflation and the change in the exchange rate. However, the analysis presented in discussion above suggests terms in exchange rate 'gap': the difference between actual exchange rate disequilibrium and the disequilibrium that would occur with no distortions. Not only is the dimension of this expression different from the change in the exchange rate, but a change in exchange rate term makes no attempt to allow for 'warranted' exchange rate movements i.e. natural disequilibrium.

It is important, however, to recognize that my analysis does not imply that policy should always aim to eliminate exchange rate gaps. Some exchange rate gap may well be necessary to avoid large output gaps. Typically a tight monetary policy, responding to a positive output gap, will lead to an appreciation, which is a helpful element in the transmission mechanism. Where the exchange rate gap term plays an essential role is if a positive demand shock is combined with an excess appreciation as a result of some distortion to international risk sharing or uncovered interest parity. In this case the appreciation may eliminate the impact of the taste shock on the output gap, but a consumption gap would remain, and so a cut in interest rates to moderate the appreciation will enhance welfare.

#### **4.3 - Monetary Policy Rules**

The monetary authority uses the short-term nominal interest rate  $i_t$  as its policy instrument to affect the behaviour of output gap, inflation and the real exchange rate to maximize the social welfare subject to the constraints on their behaviour implied by the system of equations summarized earlier. The central bank manages this interest rate according to an open economy variant of the Taylor-type feedback rule.<sup>13</sup> In particular, I analyze the macroeconomic implications of two alternative monetary policy regimes --- domestic inflation targeting and CPI inflation targeting ---- considering both flexible and dirty floating regimes. The analysis also contrasts differences between strict and flexible inflation targeting ---- both domestic and CPI inflation. The general form of the open economy Taylor-rule is given as:

$$i_t = \lambda_x x_t + \lambda_\pi \pi_t^j + \lambda_q \left( q_t - q_t^0 \right)$$
(63)

where  $\pi_t^j$  could be domestic inflation or CPI inflation depending on the targeting regime considered.  $\lambda_x, \lambda_\pi$  and  $\lambda_q$  are the weights associated with stabilizing output gap, inflation rate (around zero), and the real exchange rate around the flexible price/complete asset market real exchange rate level respectively.

The value of parameter  $\lambda_q$  implies the type of regime that the monetary authority chooses. For example,  $\lambda_q = 0$  means that the central bank does not care about deviations of the real exchange rate from the target, i.e., the economy has a flexible exchange rate. On the other hand,  $\lambda_q > 0$  means that the central bank responds by changing the interest rate if there is some deviation of the real exchange rate from its target value. Thus, this case corresponds to a managed exchange rate, and in the extreme to a fixed exchange rate.

# 5 – A Numerical Analysis of Alternative Monetary-Exchange Rate Policies

In this section, I present some quantitative results based on a calibrated version of the model economy<sup>14</sup>. In particular, I report the variances for key variables and the expected loss of

<sup>&</sup>lt;sup>13</sup> For similar work, see Guender (2001), Leitmo and Soderstorm (2001), and Taylor (2001).

<sup>&</sup>lt;sup>14</sup> The model is calibrated and simulated by using the technique provided by Soderlind (1999). The software used for this purpose is MATLAB.

the central bank under alternative monetary-exchange rate regimes. These experiments allows us to compare the effects of alternative targeting regimes (by varying the value of the parameter ' $\lambda$ ' in equation (63)) on key macroeconomic variables (output gap, inflation and real exchange rate) within the dynamic general equilibrium framework developed in the paper.

#### **5.1-** Calibration

For parameter values, standard baseline values that appear in the traditional related literature are chosen. The value for  $\beta = 0.99$  is standard which implies a risk-less annual return of about 4 percent in the steady state.  $\sigma = 1$  is the elasticity of intertemporal substitution which corresponds to log utility.  $\phi = 3$ , which implies a labor supply elasticity of 1/3. The elasticity of substitution between domestic and foreign goods,  $\eta$ , equals 1.5. The baseline value for '*a*'(or degree of openness) is assumed to be 0.4, which roughly corresponds to the import/GDP ratio in a typical small open economy. Parameter  $\rho$  is set equal to 0.75, a value consistent with an average period of one year between price adjustments. ' $\kappa$ .' is assumed to equal 0.0007. In general, the main conclusions do not differ with alternative reasonable parameter values.

The variances for the white noise taste, technology, and foreign output shocks are taken to be 0.000175 with a persistence parameter of  $\rho_u = 0.5$ ,  $\rho_z = 0.8$  and  $\rho_{y^*} = 0.65$ . Taken together these numbers imply an annualized standard deviation of around 6% for the model economy.<sup>15</sup> The values chosen for the variances of the shock have a direct effect on the absolute magnitude of expected losses, but do not influence the relative magnitudes of the losses with and without the cost channel of monetary policy. It is important to note that it is the relative losses that are relevant for comparison.

#### **5.2 – Analysis: Discussion of Results**

Three types of aggregate shocks are considered: taste shocks, foreign output shocks, and technology shocks.

<sup>&</sup>lt;sup>15</sup> For example, standard deviation of the taste shock =  $\sqrt{\operatorname{var}(u_t)} = \sqrt{(1/(1-(0.5)^2))^* 0.000175)} = 0.015$ . Multiplying this number by 4 gives the annualized standard deviation of 6%

#### 5.2.1 – Taste Shock

Table 1 reports the results for domestic inflation targeting (DIT) with alternative exchange rate policies in response to taste shocks. The numbers in blue correspond to flexible domestic inflation targeting (FDIT) while the numbers in red are for strict domestic inflation targeting (SDIT). A number of very interesting results can be observed.

	$\operatorname{var}(x_t)$		var(;	$(\pi_{H,t})$	$\operatorname{var}(q_t)$		Lo	Loss1 Loss2		2
	FDIT	SDIT	FDIT	SDIT	FDIT	SDIT	FDIT	SDIT	FDIT	SDIT
$\lambda_q = 0$	3.60	5.09	0.150	0.070	431.18	418.89	215.99	212.10	0.405	2.65
$\lambda_q = 0.5$	0.052	2.31	0.015	0.192	344.02	273.03	172.06	137.96	0.049	1.44
$\lambda_q = 1.0$	0.750	31.93	0.173	2.52	283.44	207.80	124.35	123.64	0.634	19.75
$\lambda_q = 1.5$	5.292	83.54	1.276	8.12	239.96	170.64	142.54	139.27	4.561	53.95

**Table 1: Taste Shock --- Domestic Inflation Targeting (DIT)** 

 $\lambda_q = 0$  corresponds to the **flexible exchange rate regime**, while  $\lambda_q = 0.5, 1.0$  and 1.5 capture alternative degrees of **dirty floating** or managed exchange rate regime. FDIT stands for **flexible DIT** with  $\lambda_x = 0.5$  and  $\lambda_{\pi_H} = 1.5$  while SDIT stands for **strict DIT** with  $\lambda_x = 0$  and  $\lambda_{\pi_H} = 1.5$ . Loss1 corresponds to the value of social welfare function with  $\alpha_x = 0.5, \alpha_{\pi_H} = 1.5$  and  $\alpha_q = 0.5$ , whereas Loss2 corresponds to the case when  $\alpha_x = 0.5, \alpha_{\pi_H} = 1.5$  and  $\alpha_q = 0.5$ .

First, managed exchange rate regime is superior to flexible exchange rate regime under domestic inflation targeting --- the expected loss goes down as the central bank places some weight on stabilizing the real exchange rate. However, it is important to note that as the central bank tries to stabilize the real exchange rate 'too much', that is, approaches fixed exchange rate case, loss increases. This result is quite robust and holds regardless of the welfare criterion used: loss1, which includes real exchange rate movements, or loss2, which focuses only on output gap and inflation movements. Also, the result remains unchanged whether the central bank adopts flexible inflation targeting or strict inflation targeting. For example, under both flexible and strict inflation targeting, as the parameter ' $\lambda_q$ ' changes from 0 to 1.0 loss1 decreases, but, as ' $\lambda_q$ ' approaches 1.5 or higher, loss1 increases. In case where welfare criterion, loss2, is considered, increasing ' $\lambda_q$ ' from 0 to 0.5 decrease it, but as ' $\lambda_q$ ' approaches 1.0 or higher, it increases. In other words, whether central bank cares about real exchange rate movements or not, placing some positive weight on stabilizing it pays off as it lowers the volatility in output gap and domestic inflation. At the same time, stabilizing real exchange rate too much increases their volatility.

The key reason behind this unique and powerful result is the presence of current account dynamics affecting not only the real exchange rate behaviour via imperfect risk sharing due to incomplete asset markets, but also the output gap via the risk premium term in the interest parity relationship. The intuition is as follows. Suppose the economy experiences a positive taste shock that tends to push up both output gap and domestic inflation and causes some appreciation of the real exchange rate. A typical response would be to increase the nominal interest rate, which leads to further appreciation that helps the transmission mechanism. In a model without current account dynamics (zero net foreign assets) the analysis would stop here and predict that the central bank can completely stabilize shocks that push up output gap and domestic inflation in the same direction implying flexible domestic inflation targeting with completely flexible exchange rate as the optimal monetary policy. (e.g., Clarida, Gali, and Gertler (2001). On the other hand, in the presence of net foreign assets, there would be 'excess' appreciation due to a taste shock: an appreciation improves the net foreign asset position that in turn causes further appreciation (see, equation (51) and (57)). In this case, increasing the interest rates would exacerbate the excess appreciation. Therefore, placing some positive weight on stabilizing real exchange rate, by lowering the interest rate, eliminates this excess appreciation leading to welfare improvements. Put differently, appreciation (caused by increasing the interest rate) may eliminate the impact of the taste shock on the output gap, but a consumption gap would remain due to incomplete risk sharing, and so a less aggressive response by the policy authority (a slight cut in interest rates to moderate the appreciation) will enhance welfare. However, the policy should not try to eliminate the real exchange rate gap completely by lowering the interest rates too much as it may lead to large output gaps; some exchange rate gap may well be necessary to avoid these gaps and improve welfare.

This result also challenges the conventional wisdom --- the famous insulation property of flexible exchange rate regime --- that flexible exchange rate is better compared to 'targeted' exchange rate in case of real shocks such as taste shocks. Some exchange rate targeting --- a dirty float --- turns out to be a superior outcome.

Another stark result reported in table 2 is that strict domestic inflation targeting is slightly better than flexible domestic inflation targeting or at least the difference is very small compared to what is usually reported in the literature. The intuition for this result is simple. Strict domestic inflation targeting means that the central bank does not care about output gap movements. Thus, the implications of an 'excess' appreciation for the output gap, as discussed above, is not binding, which induces the central bank to stabilize the real exchange rate gap more. This implies lower losses if the welfare criterion used is loss1. If the welfare criterion used is loss2, as in Clarida, Gali and Gertler (2001), however, the opposite result would hold not so surprisingly.

Apart from comparing social welfare across alternative monetary-exchange rate policies, a careful inspection of output volatility and domestic inflation volatility also reveal some unconventional results. For example, a conventional model predicts that in the presence of nominal price rigidities, flexibility of exchange rates (the famous over-shooting result) ensures lower output volatility. However, the volatility of output decreases, in my model, as the central bank moves to dirty floating. The reason is as follows. A positive taste shock leads to excess appreciation via dynamic interaction between real exchange rates and net foreign assets, which causes output volatility. Eliminating this excess appreciation, by stabilizing the real exchange rate slightly, therefore, would reduce output volatility. Similarly, the predictions of the model regarding inflation volatility are also unconventional. A standard model suggests that inflation volatility goes down as the economies move towards fixed exchange rates. Indeed, a famous argument in favour of fixed exchange rate regime is that it pins down the inflation expectations leading to lower inflation volatility. This is not the case in my model that boasts rich dynamic interactions among net foreign assets and inflation, real exchange rates and output gap (see last row in table 1).

Table 2 reports the results for CPI inflation targeting with alternative exchange rate policies in response to a taste shock. Before discussing any result, an important point need to be made regarding the difference in CPI inflation targeting and domestic inflation targeting with managed exchange rates. In line with conventional wisdom, Parrado (2004) argues that if an

economy has a managed exchange rate there is no difference in CPI and domestic inflation targeting as the volatility in key macroeconomic variables is the same across these regimes. The reason, pointed out by Parrado (2004), is that targeting the CPI inflation is the same thing as targeting both domestic inflation and the exchange rate, which is equivalent to targeting domestic inflation with managed exchange rates.

	$\operatorname{var}(x_t)$		var()	$(\pi_t)$	$\operatorname{var}(q_t)$		Loss1		Loss2	
	FCPI	SCPI	FCPI	SCPI	FCPI	SCPI	FCPI	SCPI	FCPI	SCPI
$\lambda_q = 0$	9.83	84.95	17.30	11.66	253.75	205.72	157.74	162.82	30.87	59.96
$\lambda_q = 0.5$	19.11	124.13	15.50	13.19	222.26	175.87	143.92	169.78	32.80	81.85
$\lambda_q = 1.0$	32.30	169.78	15.77	17.04	197.16	153.73	138.38	187.31	39.80	110.45
$\lambda_q = 1.5$	49.34	220.08	17.65	23.06	176.80	136.61	139.54	212.93	51.14	144.63

Table 2: Taste Shock --- CPI Inflation Targeting

 $\lambda_q = 0$  corresponds to the flexible exchange rate regime, while  $\lambda_q = 0.5, 1.0$  and 1.5 capture alternative degrees of dirty floating or managed exchange rate regime. FCIT stands for flexible CPI inflation targeting with  $\lambda_x = 0.5$  and  $\lambda_{\pi} = 1.5$  while SCPI stands for strict CPI inflation targeting with  $\lambda_x = 0$  and  $\lambda_{\pi} = 1.5$ . Loss1 corresponds to the value of social welfare function with  $\alpha_x = 0.5, \alpha_{\pi_H} = 1.5$  and  $\alpha_q = 0.5$ , whereas Loss2 corresponds to the case when  $\alpha_x = 0.5, \alpha_{\pi_H} = 1.5$  and  $\alpha_q = 0.5$ .

However, an important point need to be noted about the real exchange rate term as it appears in the policy rule in this paper. The point is that, like output, it is in the form of deviation from the level that would occur with no nominal price rigidities and international risk sharing shocks like net foreign assets, and not in change form. That is, unlike the traditional models that study CPI inflation targeting and role of exchange rates in policy rules, the analysis presented in the paper suggests terms in exchange rate 'gap': the difference between actual exchange rate disequilibrium and the disequilibrium that would occur with no distortions. Not only is the dimension of this expression different from the change in the exchange rate, but a change in exchange rate term makes no attempt to allow for 'warranted' exchange rate movements i.e. natural disequilibrium. This distinction is important in understanding the results reported in Table 2.

The first result is that flexible CPI inflation targeting with managed exchange rates is superior to flexible CPI inflation targeting with completely floating exchange rates. That is, responding to real exchange rate gap in addition to CPI inflation (that implicitly incorporates response to exchange rate changes) is welfare improving. Put differently, responding to exchange rate changes alone (as embedded in the response to CPI inflation) is not enough to improve welfare. An additional response to exchange rate gap leads to better outcomes. However, this result does not hold when either an alternative welfare criterion, loss2, is used or strict CPI inflation targeting is pursued. Thus, the case for dirty floating is not that strong as was the case with domestic inflation targeting, stabilizing real exchange rate eliminates the excess appreciation that follows due to a taste shock, and thus improves welfare. On the hand, with CPI inflation targeting, response to real exchange rate is already included in the regime; responding to real exchange rate gap on top of this would be harmful as it leads to excess output volatility.

Another result is that, unlike the domestic inflation targeting case, flexible CPI inflation targeting is always superior to strict CPI inflation targeting regardless of the welfare criterion used because strict inflation targeting dramatically increases the output volatility. Similarly, in line with conventional wisdom, output volatility indeed goes up as the economy moves towards more managed exchanged rates. However, a point worth noting is that no noticeable gain is made on the volatility of CPI inflation.

Note that a direct comparison between domestic inflation targeting and CPI inflation targeting can not be made because their respective loss functions involve different arguments (domestic inflation in one and CPI inflation in other).

	$\operatorname{var}(x_t)$		var(;	$\pi_{H,t})$	$\operatorname{var}(q_t)$		Loss1		Loss2	
	FDIT	SDIT	FDIT	SDIT	FDIT	SDIT	FDIT	SDIT	FDIT	SDIT
$\lambda_q = 0$	0	0	0	0	17.721	17.56	8.861	8.778	0	0
$\lambda_q = 0.5$	0.001	0.038	0.002	0.009	15.251	13.891	7.629	6.978	0.003	0.033
$\lambda_q = 1.0$	0.020	0.335	0.020	0.076	13.417	11.642	6.749	6.103	0.040	0.282
$\lambda_q = 1.5$	0.091	0.903	0.075	0.241	11.976	10.185	6.147	5.905	0.160	0.812

Table 3: Foreign Output Shock --- Domestic Inflation Targeting (DIT)

 $\lambda_q = 0$  corresponds to the **flexible exchange rate regime**, while  $\lambda_q = 0.5, 1.0$  and 1.5 capture alternative degrees of **dirty floating** or managed exchange rate regime. FDIT stands for **flexible DIT** with  $\lambda_x = 0.5$  and  $\lambda_{\pi_H} = 1.5$  while SDIT stands for **strict DIT** with  $\lambda_x = 0$  and  $\lambda_{\pi_H} = 1.5$ . Loss1 corresponds to the value of social welfare function with  $\alpha_x = 0.5, \alpha_{\pi_H} = 1.5$  and  $\alpha_q = 0.5$ , whereas Loss2 corresponds to the case when  $\alpha_x = 0.5, \alpha_{\pi_H} = 1.5$  and  $\alpha_q = 0.5$ .

Table 3 reports the results for foreign output shock with domestic inflation targeting. A positive foreign out put shock, by decreasing the flexible price real interest rate leads to a negative output gap and thus lower domestic inflation. Also, it causes real exchange rate appreciation. The central bank responds by lowering the nominal interest rate that leads to real depreciation. This, in turn, pushes up the output gap and domestic inflation to their original level. Thus, both output and domestic inflation are completely stabilized under flexible exchange rates. Unlike the response to taste shocks, this policy does not lead to 'excess' appreciation due to dynamics of real exchange rate and net foreign assets because a cut in the interest rate dampens the real appreciation rather than exacerbating it. Therefore, as the central bank moves towards the managed exchange rate regime volatility of exchange rate decreases, but at the same time output and domestic inflation volatility increases. Thus, the difference between loss1 across alternative exchange rate regimes is very insignificant. Obviously, if loss2 is used as a welfare criterion then flexible exchange rate regime would be superior.

Comparing flexible DIT with strict DIT under managed exchange rates reveal a rather surprising result. Volatility of domestic inflation increases under the strict case. With lower real exchange rate volatility and higher output volatility, loss1 is actually slightly lower under strict DIT compared to flexible DIT. This is the same result as observed in case of taste shocks. Essentially, central bank trades-off some inflation volatility for lower real exchange rate volatility in case the welfare criterion includes real exchange rate gap terms, such as loss1. Needless to say, flexible DIT would be superior if welfare criterion, loss2 is used.

	$\operatorname{var}(x_t)$		$\operatorname{var}(\pi_t)$		$\operatorname{var}(q_t)$		Loss1		Loss2	
	FCPI	SCPI	FCPI	SCPI	FCPI	SCPI	FCPI	SCPI	FCPI	SCPT
$\lambda_q = 0$	0.16	1.168	0.633	0.507	12.843	11.667	7.451	7.177	1.03	1.344
$\lambda_q = 0.5$	0.293	1.655	0.610	0.564	11.678	10.428	6.901	6.887	1.062	1.673
$\lambda_q = 1.0$	0.489	1.886	0.658	0.612	10.709	10.013	6.586	6.868	1.231	1.861
$\lambda_q = 1.5$	0.703	0.197	0.745	0.689	10.013	9.848	6.476	6.852	1.470	1.88

Table 4: Foreign Output Shock --- CPI Inflation Targeting

 $\lambda_q = 0$  corresponds to the **flexible exchange rate regime**, while  $\lambda_q = 0.5, 1.0$  and 1.5 capture alternative degrees of **dirty floating** or managed exchange rate regime. FCIT stands for **flexible CPI inflation targeting** with  $\lambda_x = 0.5$  and  $\lambda_{\pi} = 1.5$  while SCPI stands for **strict CPI inflation targeting** with  $\lambda_x = 0$  and  $\lambda_{\pi} = 1.5$ . Loss1 corresponds to the value of social welfare function with  $\alpha_x = 0.5, \alpha_{\pi_H} = 1.5$  and  $\alpha_q = 0.5$ , whereas Loss2 corresponds to the case when  $\alpha_x = 0.5, \alpha_{\pi_H} = 1.5$  and  $\alpha_q = 0$ .

Similar results hold in case of CPI inflation targeting except that output and inflation are not completely stabilized and inflation volatility does not increase in case of strict CPI targeting. As before dirty floating is slightly better than flexible exchange rates but the result is not quite robust if welfare criterion, loss2, is used.

	$\operatorname{var}(x_t)$		var(2	$\operatorname{var}(\pi_{H,t})$		$\operatorname{var}(q_t)$		Loss1		2
	FDIT	SDIT	FDIT	SDIT	FDIT	SDIT	FDIT	SDIT	FDIT	SDIT
$\lambda_q = 0$	0	0	0	0	43.842	38.638	21.921	19.319	0	0
$\lambda_q = 0.5$	0	0.002	0.002	0.005	41.171	40.480	20.588	20.249	0.003	0.009
$\lambda_q = 1.0$	0.004	0.047	0.021	0.054	38.771	37.352	19.419	18.781	0.033	0.105
$\lambda_q = 1.5$	0.023	0.167	0.084	0.194	36.744	35.061	18.509	17.904	0.137	0.375

Table 5: Technology Shock --- Domestic Inflation Targeting (DIT)

 $\lambda_q = 0$  corresponds to the flexible exchange rate regime, while  $\lambda_q = 0.5, 1.0$  and 1.5 capture alternative degrees of dirty floating or managed exchange rate regime. FDIT stands for flexible DIT with  $\lambda_x = 0.5$  and  $\lambda_{\pi_H} = 1.5$  while SDIT stands for strict DIT with  $\lambda_x = 0$  and  $\lambda_{\pi_H} = 1.5$ . Loss1 corresponds to the value of social welfare function with  $\alpha_x = 0.5, \alpha_{\pi_H} = 1.5$  and  $\alpha_q = 0.5$ , whereas Loss2 corresponds to the case when  $\alpha_x = 0.5, \alpha_{\pi_H} = 1.5$  and  $\alpha_q = 0.5$ .

Table 5 and 6 report the results for a technology shock with domestic inflation targeting and CPI inflation targeting respectively under alternative exchange rate regimes and parallel the results for foreign output shocks.

	$\operatorname{var}(x_t)$		var(	$(\pi_t)$	$\operatorname{var}(q_t)$		Loss1		Loss2	
	FCPI	SCPI	FCPI	SCPI	FCPI	SCPI	FCPI	SCPI	FCPI	SCPI
$\lambda_q = 0$	0.062	0.442	0.781	0.694	39.005	38.608	20.705	20.304	1.202	1.262
$\lambda_q = 0.5$	0.106	0.565	0.795	0.762	37.094	35.921	19.793	19.387	1.245	1.426
$\lambda_q = 1.0$	0.174	0.746	0.897	0.945	35.411	34.099	19.138	18.839	1.433	1.790
$\lambda_q = 1.5$	0.269	0.980	1.087	1.245	33.913	32.519	18.721	18.616	1.765	2.357

Table 6: Technology Shock --- CPI Inflation Targeting

 $\lambda_q = 0$  corresponds to the flexible exchange rate regime, while  $\lambda_q = 0.5, 1.0$  and 1.5 capture alternative degrees of dirty floating or managed exchange rate regime. FCIT stands for flexible CPI inflation targeting with  $\lambda_x = 0.5$  and  $\lambda_{\pi} = 1.5$  while SCPI stands for strict CPI inflation targeting with  $\lambda_x = 0$  and  $\lambda_{\pi} = 1.5$ . Loss1 corresponds to the value of social welfare function with  $\alpha_x = 0.5, \alpha_{\pi_H} = 1.5$  and  $\alpha_q = 0.5$ , whereas Loss2 corresponds to the case when  $\alpha_x = 0.5, \alpha_{\pi_H} = 1.5$  and  $\alpha_q = 0.5$ .

## **<u>6 - Concluding Remarks</u>**

The present paper has developed and analyzed a dynamic general equilibrium model with staggered price rigidities and incomplete and imperfect international asset markets. The key contribution of the paper is that it allows for the dynamic relationship between real exchange rate and net foreign assets to affect the dynamics of domestic inflation and output gap. Thus, unlike other similar open economy models, for example, Gali and Monacelli (2002) and Clarida Gali and Gertler (2001), this paper has shown that the dynamics of domestic inflation and output gap does not have a canonical representation analogous to closed economy models, and therefore the optimal monetary policy design problem for an open economy is not 'isomorphic' to a closed economy. Furthermore, relying on the recent literature that formally derives the welfare criterion or the loss function for an open economy (e.g, Kirsonova, Leith, and Wren-Lewis (2004), Benigno and Woodford (2004), and Benigno (2001)) as an approximation to the representative agents' utility function, the paper has also shown that this loss function is not analogous to the

one applying to the corresponding closed economy. In particular, due to the current account dynamics, the loss function also includes the real exchange rate gap term in addition to domestic inflation and output gap. This implies that in general targeting domestic inflation with flexible exchange rate would not be the welfare maximizing optimal monetary policy.

The framework is then used to study various monetary-exchange rate policies using Taylor-type interest rate based rules. In particular, I compare the performance of domestic inflation targeting and CPI inflation targeting with flexible and managed exchange rate regimes. Moreover, I also study flexible and strict inflation targeting considering both inflation indices. The main results of the paper are: (1) - Managed exchange rate regime (dirty floating) is superior to flexible exchange rate regime under domestic inflation targeting. Volatility in both output and domestic inflation goes down and so does the volatility in real exchange rate. Put differently, there is no trade-off between stabilizing the real exchange rate and domestic inflation and output gap: welfare improves as the central bank places some weight on stabilizing the real exchange rate and pursues domestic inflation targeting. (2) - As the central bank tries to stabilize the real exchange rate 'too much', that is, approaches fixed exchange rate case, loss increases. (3) - In case of a taste shock, this result is quite robust and holds regardless of the welfare criterion used: whether it includes real exchange rate movements, or focuses only on output gap and inflation movements. (4) - Also, the result remains unchanged whether the central bank adopts flexible inflation targeting or strict inflation targeting. (5) - Strict domestic inflation targeting outperforms flexible domestic inflation target regardless of the exchange rate regime. This result is sensitive, however, to the welfare criterion used. (6) - With CPI inflation targeting, there is some evidence in favour of 'dirty floating', however, the result is not that robust when alternative welfare criterion is used. (7) - Flexible CPI inflation targeting dominates strict CPI inflation targeting, and is not sensitive to the welfare criterion used.

The bottom line is that the dynamic relationship between net foreign asset position and the real exchange rate plays a crucial role in obtaining the above mentioned results.

These results, while suggestive, are subject to some limitations. For instance, introducing imports as production inputs, ala McCallum and Nelson (2000), with rigidities in the import prices could alter the conclusions as to the appropriate exchange rate regime or price index to target. Similarly, introducing labour market rigidities could alter the results as well. After all, as pointed out by Erceg, Henderson and Levin (2000), the simultaneous presence of both forms of

nominal rigidity introduces an additional tradeoff that renders 'goods' price inflation targeting policies suboptimal. Therefore, it may be interesting to analyze how that tradeoff would affect the ranking across monetary-exchange rate policy regimes evaluated in the present paper. These results would also need to be qualified, if one considers differences in price-setting across various markets, as in the case of less than complete exchange rate pass-through of nominal exchange rate changes to prices of imported (or exported) goods. Moreover, since the various currency crises episodes in the 1990s much of the discussion on exchange rate policy in emerging market economies is concerned with the interaction of exchange rate with balance sheets, borrowing constraints, dollarization of liabilities, and creditworthiness of firms. Incorporating such consideration in a model with imperfections in the financial markets, such as the one developed in this paper, should certainly be the focus of future research.

Finally, the paper deals with calibrated results. Conclusions about policy dominance and welfare consequences depend on a specific parameterization, and they should not be taken as general propositions. The paper experimented sufficiently with alternative parameterization to be confident that the results presented here are robust to relatively minor changes in assumptions. More work is clearly warranted, however, before making general policy recommendations.

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