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Rethinking the Monetary Policy Framework of the State Bank of Pakistan

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1. Introduction

An earlier paper by one of the authors of the present paper had examined the issue of how a flexible exchange rate regime can facilitate the pursuit of domestic stabilization objectives with open capital markets in Pakistan.[1] That paper had focused on several types of monetary policy frameworks, which are consistent with the central objective of maintaining low inflation under a flexible exchange rate regime. Each of those options focuses on a different long-term nominal anchor, which is implied in the names attached to those frameworks: monetary targeting, nominal income targeting, and inflation targeting. In contrast, the present paper does not discuss the issue of nominal anchor, and in particular, the pros and cons of monetary, nominal income, and inflation targeting in the context of the Pakistan economy. The reason being that it is not just that one does not want to repeat what has already been said, but it is probably not pushing too far to say that there has been a revolution in the practice and analysis of monetary policy over the last several years. Not only have many central banks adopted an explicit inflation targeting strategy, using interest rates as their operational target, but also they have mostly ignored monetary aggregates in the conduct of monetary policy. It appears that the new consensus on monetary economics, which is reflected in the new-Keynesian modeling approach, does not assign money a special role in controlling inflation.[2] Furthermore, in many of the new-Keynesian models, monetary aggregates do not even enter the monetary policy transmission mechanism.[3] Proponents of this approach have pointed out that the relation between money and prices (and output) is often so volatile and unstable, that

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[1] Zaidi (2006).

[2] See Clarida et al. (2000).

[3] In these models, the implications for causality running between money and prices or feedback mechanisms between the two series are not emphasized, and monetary aggregates do not play a major role in the workings of the model.

central banks should disregard monetary aggregates altogether in the analysis and implementation of monetary policy.[4]

This approach is in sharp contrast with the State Bank of Pakistan's (SBP) monetary policy framework and the conduct of monetary policy. Even though Pakistan has some degree of exchange rate flexibility, a lot of emphasis is given to monthly or quarterly targets on monetary aggregates in the discussions of monetary policy, both within the deliberations done inside SBP and in discussions with IMF missions. The analysis of monetary policy at the SBP, and at the IMF, remains closely related to the quantity theory MV = PQ, which means that money targets (M) are set so as to be consistent with a desired path for prices (P) given real output (Q) projections and under some assumption about the trajectory of velocity (V), which in turn is based on some estimated money demand function. It would not be a gross exaggeration to describe the SBP's monetary policy framework as an approach based mainly on targets for monetary aggregates. As noted above, however, this contrasts with most modern discussions of monetary policy, and with actual policy implementation in most advanced and emerging market countries. In the ensuing discussion, we focus on several systematic weaknesses in how SBP (and by extension IMF programs) design handles different aspects of uncertainty in the formulation and implementation of monetary policy.[5] At the risk of oversimplification, many of these concerns can be characterized as involving issues about the implicit model(s) underlying the design and implementation of IMF-supported adjustment programs.[6] One of the major problems is that because monetary targets are often missed, the policy discussions between the IMF and SBP staff have focused mainly on assessing whether those misses reflected policy errors or unexpected changes in the velocity of money, but this inordinate emphasis on monetary targets/velocity comes at the expense of neglecting many other more important analytical and policy issues (see below).

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[4] Woodford (2003) provides several convincing arguments to support this policy recommendation.

[5] See, for example, Independent Evaluation Office (2002, 2003, and 2004).

[6] For a comprehensive discussion of the IMF's programming approach, see Mussa and Savastano (1999), or IMF (2004b, Appendix I). There is rarely, if ever, an explicit comprehensive macroeconomic model linking policies to targets that underlies the formulation of programs, especially in low-income countries. Rather, program design draws upon a variety of empirical evidence and judgments about key underlying relationships to link a set of macroeconomic projections on core variables (real GDP growth, inflation, the external current account, and international reserves) with monetary, fiscal and exchange rate policy instruments. The process is typically an iterative one – reflecting the mutual dependence of instruments and targets. At the heart of the process is a "financial programming framework" that is a set of accounting identities that provides a consistency check on the macroeconomic projections. To "close" the model requires a number of simplifying behavioral assumptions that can vary depending on country circumstances but typically includes an assumption about the stability (or at least predictability) of money demand.

Another important line of research in recent years has been the examination of models in which fiscal factors replace the money supply as the major determinants of the price level, which raises even more fundamental questions about the heavy reliance on monetary aggregates in the SBP's monetary policy framework, not least because of the historical experience of fiscal dominance in Pakistan. There are at least two ways whereby fiscal policy might matter for the price level. First, if fiscal variables affect real money demand, the price level will also depend on fiscal factors. Second, the fiscal theory of the price level (FTPL) makes the important point that if there are multiple price levels consistent with a given nominal quantity of money, then fiscal policy will determine which of these price levels will be the equilibrium value. The government's intertemporal budget constraint implies that any government with a current outstanding debt must run, in present value terms, future surpluses. The traditional interpretation of the government budget constraint would run along the following lines: when the real value of debt and the present value of primary surpluses are not equal, then this would force the government to either undertake fiscal adjustment, or default on the debt, or resort to inflationary finance. The resort to inflationary finance is what gives us the monetarist unpleasant arithmetic. In the monetarist unpleasant arithmetic, it is assumed that the government's intertemporal budget constraint is satisfied for a given real value of government's liabilities by either the fiscal authority producing the primary fiscal surpluses or the monetary authority generating the seigniorage revenues required to ensure that the constraint is met. Therefore, if the fiscal authority is not adjusting the primary surpluses expected in the future, then an attempt by the monetary authority to reduce inflation and seigniorage today must lead to higher inflation and seigniorage in the future because the present discounted value of seigniorage cannot be altered. A Ricardian regime is defined to be one in which either taxes and/or seigniorage always adjust to ensure that the government's intertemporal budget constraint is satisfied. In contrast to the traditional monetary view or the monetarist unpleasant arithmetic, proponents of the fiscal theory of the price level (FTPL) view argue that the government budget constraint, which relates the real value of the debt stock to the discounted present value of expected primary surpluses in the future, is indeed an equilibrium condition, but this does not mean that the government has to follow policies that are "Ricardian". The key point in this view is that if primary surpluses evolve independently of the level of debt, then equilibrium price level has to jump to assure fiscal solvency, and Woodford (1995) has labeled this as the Non-Ricardian regime. In other words, the government's intertemporal budget equation represents an equilibrium condition rather than a constraint that must hold for all price levels. See Sims (1994) for a particularly lucid discussion of the FTPL. This line of reasoning raises a host of very interesting policy questions, including placing limits on the inflation targeting regime, but to do justice to this important research work would take us too far away from the main focus of this paper.

Given the predominance of the new-Keynesian framework in the policy and academic circles and its sharp message regarding the role, or more to the point would be to say the lack of any role for monetary aggregates in the monetary policy framework, there are several papers which have looked at how to bring "M" back into monetary policy. Since the message of the present paper is along the lines of the new-Keynesian framework, we begin our discussion with a thought-provoking paper that tries this bringing "M" back research work (Berg et al., 2010), which incidentally is also the view held by some economists at the SBP. These authors (henceforth BPU) attempt to extend the new-Keynesian model to provide a role for money in the conduct of monetary policy by making what they claim to be the minimum number of necessary changes to the standard model. To examine the conditions under which some adherence to monetary targets is optimal, BPU introduce a role for money in the model, but the introduction is done in such a way that they still would like to claim that money does not play a direct role in the monetary transmission mechanism. The approach taken by BPU is to extend the new-Keynesian framework by providing what is supposedly a deeper understanding of (a) how central banks in low income countries should set monetary targets and (b) provide the analytical framework for asking the question of how should central banks respond to deviations from these targets. BPU's main rationalization of why money matters is based on the informational role of money.

In the BPU model, the information structure is set up so that the authorities periodically choose a target for both the growth rate of the monetary aggregate and the short term interest rate over the next period (one quarter). BPU argue that information gaps in developing countries are pervasive, but when it comes to monetary aggregates, these are measured accurately and with little lag. However, their argument that "these aggregates are systematically related to key variables such as output and the relevant interest rate through money demand" is not very convincing. For one thing, the relation between these key variables is subject to shocks to the demand for money function, and the empirical support for their point that the information gaps are such that the accuracy or other measurement advantages inherent in money targets will improve the performance of monetary policy is not exactly overwhelming. Furthermore, even to assess whether money has advantages over other variables in terms of informational role, one would think that a microeconomic approach would be required, which would have to go beyond the step of introducing a simple type of information incompleteness in a standard new-Keynesian model. In a similar vein, if one were to be concerned about asset-price bubbles, then the analysis should be focused not on some narrow monetary aggregate but rather take into account broad financial variables and/or indicators of total wealth of the economy. It is doubtful that some of the claims made by BPU regarding the informational role of monetary aggregates apply to

the case of Pakistan. Is it really true that key variables like output and inflation are observed imperfectly and with substantial lags in Pakistan? Whereas output gaps may be hard to assess on the basis of standard indicators of economic activity such as measures of capacity utilization, but this does not seem to be an insurmountable task.

Regarding the BPU arguments relating to the informational channel imperfections in financial markets, it does not seem to be the case that "[e]ven key markets such as interbank and government bond markets are often thin and opaque" and that a "large fraction of the population does not even participate in formal financial markets." BPU go on to argue that a readily observed interest rate such as the interbank money market rate may bear only a loose connection to the (latent or shadow) interest rate relevant to private sector decisions. However, we question the validity of this point in the case of Pakistan, and we are not convinced that the new-Keynesian model can be extended very easily to provide an important role for monetary aggregates in the conduct of monetary policy. Indeed, BPU have to admit that "our results point to interesting, qualitative, conclusions but the limitations of the model, scarcity of data, and strong identifying assumptions argue against taking them too literally." To be sure, we are not trying to claim that there cannot be any role for monetary aggregates in the analysis of monetary policy but rather that it will not be through some simple extension of the new-Keynesian model, but probably in the context of a complementary model that focuses on the so-called "new thinking of monetary policy as financial stability regulation" and the need to introduce asset prices or broad financial variables, attributed in part to the question of how to address the bubbles problem in the conduct of monetary policy.

In contrast to the focus on the outcome for some monetary aggregate, we would like to stress that the formulation and implementation of monetary policy, like all macroeconomic policy-making, takes place in conditions of uncertainty about both key elements of the external environment and underlying behavioral relationships. Moreover, monetary policy decisions are arguably more forward-looking than most other instruments in the policymakers' toolkit, as documented in the extensive literature focusing on the long and variable lags in the effects of monetary policy. Therefore, good forecasts are of the utmost importance to central bankers, and this paper looks at, among other things, how effectively the current monetary policy framework of the State Bank of Pakistan (SBP) has dealt with the uncertainties in policy formulation and in communicating what are admittedly subjective forecast distributions to the public. Above and beyond this consideration, we argue that the current monetary policy framework of SBP is backward-looking rather than forward-looking and pays too much attention to past

developments in monetary aggregates in lieu of focusing on the future path of a range of important variables in the economy. As mentioned earlier, the current monetary policy framework of SBP has important similarities with the IMF approach to program design, albeit there are also important differences in the two approaches, which we will indeed discuss in this paper. The major aim of the paper is to draw some lessons on these issues from the recent experience in Pakistan and propose a few recommendations for policy reform, as well as make some specific suggestions for strengthening the research work being done at the SBP.

Turning to some criticisms of IMF programs and monetary policy implementation in Pakistan, one that is heard quite often is that relatively little consideration is given to supply-side influences and impact. There can be no debate that the formulation of macroeconomic and structural policies should aim to foster the longer-term objective of raising the growth potential of the economy, and to be sure both the IMF and SBP staff include at least some of the supply-side variables in their policy frameworks. However, there remains the question of whether sufficient consideration is given to how the supply-side might respond in the process of policy formulation and implementation for the short-term objective of inflation control. Although the analytical tools for understanding the determinants of growth are limited, the absence of any significant supply-side analysis can cause major potential problems for program design. For example, monetary policy is intrinsically a forward-looking exercise and, therefore, cannot avoid judgments about how the economy will perform in the future. In particular, there is the major issue of the volatility of the concessional and nonconcessional foreign capital inflows. Judgments about absorptive capacity in Pakistan and the appropriate monetary policy stance in an environment of high volatility of capital inflows requires an analysis of supply-side constraints, and assessments of appropriate exchange rate responses – e.g., to aid inflows – that can differ depending on how such inflows will affect capacity in traded and non-traded sectors of the economy.

Another common criticism of reports written by the IMF staff on adjustment programs and the monitoring of those programs, which also applies to some extent to the reports and forecasts made public by the SBP, is that they often do not spell out key behavioral assumptions underlying program design/forecasts. This makes it difficult to test, ex ante, whether the IMF program design and the SBP's monetary policy framework is robust to different parameter assumptions (i.e., would different assumptions – within the range of what has been observed in countries with similar economic structures – yield a substantially different linkage between policy instruments and targets). Put it differently, the SBP's and/or the IMF's "intellectual capital" should be put on the table, in order to have it tested in

discussions with other observers – including the domestic and international research community. IMF and SBP staff rightly emphasize that their approaches to monetary policy implementation allow policy adjustment in a dynamic manner in light of outcomes, i.e., reviews are carried out on a frequent and regular basis, which means that there are ample opportunities to re-assess initial assumptions, focus on reasons for why behavioral relationships may be deviating from past econometric estimation results, and why there should be some tweaking of the model calibration.[7] But there is a danger here in that if the rationale of the implicit model underlying model of the monetary policy framework is not clear, it is difficult to do any ex-post assessment in a satisfactory manner.

2. Why the move toward inflation targeting?

We argue that there are good reasons for a flexible exchange rate-cum-inflation targeting regime in Pakistan. One reason is that this regime can function as a nominal anchor and also serve as a kind of automatic stabilizer, absorbing the fluctuations in the terms-of-trade by means of exchange rate movements (see more below). Indeed, most of the recent research on exchange rate regimes for emerging market economies has suggested that, in the context of rapid international capital mobility, sustaining a fixed exchange rate in the face of terms-of-trade or international financial market shocks is very difficult. That said, the decision to adopt an inflation targeting framework should be based on an explicit comparison of the pros and cons of inflation targeting and alternative monetary frameworks. Even recognizing that the inflation targeting (IT) framework can be quite flexible; it is unlikely that it would be suitable for some countries because of their structural characteristics and limited institutional capacities. For this reason, we do not wish to oversell our argument (e.g., regarding the empirical evidence on inflation targeting and the assertion by some economists that the experience of emerging market ITers suggests that inflation targeting leads to improved macroeconomic performance and reduced vulnerability to crisis relative to other monetary policy frameworks, we would suggest the need for caution in interpreting the econometric results based on very limited data). Also, just to avoid what sometimes ends up becoming a debate on terminology, we might even be willing to go so far as to say that we are using the term IT as a short-hand for our proposed monetary policy framework for Pakistan in the medium-term. The reason for our adoption of this terminology is that we feel strongly that whatever monetary policy framework one would want to recommend, it will have to be forward-looking, based on some sort of forecast targeting of key variables, avoid

[7] In practice, some short-term contingencies are also accounted for by pre-specified automatic adjustors to program performance criteria.

fiscal dominance, give very little emphasis to monetary aggregates, build up the institutional capacity of the SBP, and adopt several other characteristics that are often taken to be the ingredients of the IT regime. What we have in mind is IT or at least "IT lite" but it may well be that some observers might prefer to call our approach the "risk management approach to monetary policy" or use some other more long-winded phrase. However, if for no other reason than simplicity (to call a spade a spade and not call it a long handle instrument used for digging) and also because our approach shares so many features of what is sometimes called "IT lite", we shall use the term "IT" as a shorthand for our proposal, and will go on to argue that SBP is now in a position to set a time frame for the adoption of an IT regime, and the change in regime would certainly mean "stop monetary targeting" or paying so much attention to monetary aggregates.

IT requires a good understanding of the monetary transmission mechanism, which is the connection between changes in the monetary stance and their effect on the operating target, and ultimately, inflation. The stronger the transmission links, and the better they are understood, the more effective will be changes in monetary instruments aimed at attaining the inflation target.[8] The central bank should have legal autonomy, and be free from fiscal and/or political pressure that would create conflicts with the inflation objective. Specifically, the central bank should have the legal autonomy and mandate to pursue the inflation target; sufficient discretion to set its monetary instruments in accordance with the inflation objective; and legal provisions that shield the central bank from pressures to monetize the fiscal deficits. It goes without saying that inflation forecasting and modeling capabilities, and the data required for their implementation should be available at the central bank. Furthermore, for IT to be the preferred monetary policy framework, the central bank's inflation forecast should outperform a money rule as an intermediate target for monetary policy.

3. The IMF's traditional net international reserves (NIR) and net domestic assets (NDA) framework

To give a specific example of the undue emphasis given to monetary aggregates in IMF supported adjustment programs in Pakistan – and indeed this would apply to programs in many other countries – one of us had worked for SBP in the mid-1990s and had been involved in negotiating those programs. The main point to

[8] We do recognize that inflation forecasting will be challenging, because Pakistan has a high degree of vulnerability to exogenous shocks; there are ongoing structural changes in the economy, not least in financial innovations and increased integration with international financial markets; the links from monetary policy instruments to inflation are not always stable; and there are data issues insofar as some requisite data are not available or there are questions with regards coverage and timeliness.

note is that if the IMF programs had succeeded in bringing about some semblance of fiscal consolidation and had checked the build-up of Foreign Currency Deposits (FCDs) – or if there were to be continued build-up of FCDs, then there should at least have been a corresponding build-up of reserves – a lot of the problems could have been tackled and the Pakistan economy would not have been so vulnerable during the 1990s. The lack of fiscal consolidation and FCDs were the two most glaring errors, and they can be explained to a large extent by the lack of emphasis on institutional reforms, lack of ownership by the Pakistan government and inconsistent monitoring by the IMF staff, and geopolitical considerations and seal of approval function of the IMF, etc. But even now, after so many years away from the SBP, one is still puzzled why the IMF missions used to spend so much time on issues which were related to the Letter of Intents (LOIs) and other IMF program-monitoring issues to the neglect of core issues that would have enhanced macroeconomic stability. One concrete example to support the contention that the focus on monetary targets had diverted attention from the core issues in overall macroeconomic adjustment would have to be the lack of discussions on FCDs. For example, one would have thought that in their negotiations with SBP management and staff, the IMF staff would have focused on the exit strategy from the reliance on FCDs and the fiscal dominance problem as it related to the conduct of monetary policy, but those important issues were rarely touched even after some prodding by SBP staff, and the bulk of the discussion would be on the quarterly ceilings for net domestic assets (NDA), reserve money expansion, floors on net foreign assets (NFA), etc. However, the main problems were the unsustainable current account and fiscal deficits, and the major reason for the reliance on short-term foreign capital was to finance the current account and fiscal deficits.[9] Accordingly, there was a need for a sustained strengthening of fiscal policy, as well as measures which would have made the domestic currency denominated assets relatively more attractive to hold. The current account deficits should have been reduced sharply through measures to foster external competitiveness and export growth, as well as prudent fiscal policy to raise public sector savings, which ought to have centered around revenue enhancing measures. At the same time, the maturity structure of the FCD deposits needed to be lengthened through various policy incentives, including better rate of return on longer maturity

[9] Despite significant external financing and domestic nonbank borrowing, the Government had relied heavily on the banking system to meet its financing needs. In addition to budgetary support, the public sector had pre-empted a significant part of total domestic credit in the form of directed credit, including lending through the specialized banks, the SBP credit to the Non-bank Financial Intermediaries (NBFIs), and various government sponsored schemes. As a result, the share of bank credit that had gone to the private sector proper in the 1990s was less than one-third of total credit, which cannot be said to be commensurate with the leading role that was expected from that sector in raising domestic investment.

deposits. Our reading of the history of that period is that there was too much emphasis on monetary aggregates in the IMF-supported adjustment programs in Pakistan.[10,11]

The point that the traditional NIR/NDA framework presents significant conceptual and practical problems in the context of inflation targeting bears emphasis. In the NIR/NFA framework, monetary conditionality consists of limits on monetary aggregates. Specifically, a floor is set for the level of net international reserves (NIR) and a ceiling established on the net domestic assets (NDA) or on base money. However, the very reason for moving to inflation targeting is the overwhelming empirical evidence that the linkage between monetary aggregates and inflation has been very weak, especially when inflation rates are low. To be sure, one can see an argument for an NIR floor, which would come from an external sustainability framework and not the traditional monetary approach to the balance of payments, but the NDA should not be given a prominent role in an inflation targeting regime. One could try to stretch the argument for an NDA ceiling by pointing out that when the NIR floor has been set to allow for a significant margin for foreign exchange intervention, the additional NDA ceiling would serve as a secondary safeguard. In particular, if the country experiences a sudden capital outflow due to an external shock, the authorities may react by drawing down NIR toward this floor to defend the exchange rate and to counteract the inflationary pressure stemming from the pass through. However, this contradicts the point that in an inflation targeting regime, the value of the domestic currency is allowed to adjust in response to supply and demand conditions in the foreign exchange market. When the exchange rate is flexible and a country's export prices decline, the domestic currency will depreciate in the foreign exchange market, which will help to increase exports and economic activity, thereby partly offsetting the initial impact on output of the negative terms of trade shock.[12] In this regard, the policy response that defends the exchange rate adds to

[10] Mirakhor and Zaidi (2006) provide a detailed discussion of the FCD problem, focusing in particular on the international liquidity shortages.

[11] Even the IEO report on prolonged use of Fund resources (IEO, 2002) notes that the Fund's surveillance documents "while factually accurate, never gave much prominence to the issue" and that this clear macroeconomic vulnerability" was never a pivotal issue of either policy or program discussions with the authorities until late 1996". In fact, the IEO report appears to hint at some sort of staff complicity: "at the authorities request, FCDs owned by residents were reported in the balance of payments "above the line" as part of private transfers, and even FCDs held by non-residents were not included in the stock of external debt." Furthermore, FCDs held by residents, even though they represented a liquid claim on the central bank's foreign exchange holdings and generated a large open position for SBP, were not netted out from the foreign exchange reserve holdings for the purpose of program monitoring of net international reserves.

[12] If the external shock is related to the asset markets (e.g., a shift away from domestic assets because of credibility or contagion effects), then that might well call for a monetary tightening but that would

the negative terms of trade shock that caused the initial contraction in output. Since exchange rate flexibility allows the economy to adjust to exogenous terms-of-trade shocks with lower costs in terms of output fluctuations, this would be the right response in an inflation targeting regime.[13] However, this runs counter to the IMF staff's traditional analysis that the ceiling on NDA would effectively limit sterilized intervention and instead prompt a monetary policy response, thereby heading off additional pressures on the balance of payments and on inflation. This last sentence is telling because the IMF staff would have the NDA ceiling be the cause of the monetary tightening to control inflation, but the whole argument for the direct inflation targeting regime is the recognition of the weaknesses in the intermediate variables, such as monetary aggregates, and therefore the monetary policy instrument setting is changed in response to forecasts of inflation that deviate from the target. Isn't there a risk that the NDA ceiling could undermine the basis of the inflation targeting framework because instead of having a transparent policy in which the monetary instrument is adjusted in response to the changes in the overriding objective of inflation control, the NDA ceiling that has no strong links to inflation would be the cause of changes in the instrument setting? As mentioned earlier, if it were desirable in the context of an open economy to also specify other objectives, such as exchange rate or international reserve holdings in the loss function for the authorities, then it would have to be done explicitly in the context of the inflation targeting framework, and not from some unreliable intermediate variable such as the NDA.

4. Inflation targeting, exchange rate bands, and a reviews-based approach in an IT regime

We argue that SBP could move toward direct inflation targeting, which has been adopted in many emerging market countries, but as with any other monetary framework, there are a number of issues that should be addressed, including meeting certain preconditions to effect a successful transition to inflation targeting. However, these preconditions are not a sine qua non in that the success of a new monetary regime depends more on the authorities' commitment to drive institutional change after the adoption of an inflation targeting framework than in

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be within the framework of the inflation targeting regime and not because of some exogenously imposed NDA ceiling.

[13] In this regard, as the authorities contemplate the transition to inflation targeting regime, this would entail that foreign exchange market interventions would be confined to smoothing the effects of temporary shocks, and the exchange rate objectives would be subordinated to the inflation target. This could lead to large swings in the real exchange rate, which highlights the point that the structural reform efforts would need to ensure that there would be sufficient flexibility in the labor and product markets to dampen any adverse effects on external competitiveness.

ensuring that all of the "preconditions" are met at the outset. Regarding the risks from having a period of transition toward more flexible exchange rates where inflation targeting is combined with some form of exchange rate bands, it should be stressed that commitment to nominal exchange rate stability could constrain monetary policy and the transparency of the inflation targeting framework, and possibly induce an element of procyclical monetary policy responses to exogenous shocks. In flexible or lite inflation targeting, SBP would aim for an inflation target, broadly defined, but it would also give weight to other objectives in the loss function. The exchange rate would be taken into account in the inflation targeting framework not only to the extent that the inflation forecast – which is the intermediate target of monetary policy in this regime – is affected by the exchange rate, but also insofar as SBP may need to adjust the monetary policy instruments to limit the impact of exchange rate changes on other objectives, e.g., external competitiveness. This is not to say that SBP should aim for an inflation target and at times switch to other objectives, but rather that a large enough target range would be specified for the inflation target, which should leave some room for maneuver, or equivalently, the SBP's loss function would give weight to other objectives.

The IMF's 2000 policy on conditionality under inflation targeting had emphasized the reviews-based approach to alleviate some of the potential tensions between inflation targeting and the traditional monetary conditionality. However, empirical evidence has shown that the reviews-based approach, or at least the way it was originally envisaged, might not be the right framework. The question remains that under this approach, monetary policy would be subject to periodic reviews focusing on recent inflation outturns, but is this not what an inflation targeting central bank should be doing in the first place? Thus, central banks under an inflation targeting regime present periodic reports on the inflation outlook, which is exactly what the reviews-based approach requires, and IMF conditionality should not be required since the IMF staff can simply rely on the periodic reports published by the SBP, if these reports are meeting the right standards.

Another difficulty with the IMF's reviews-based approach is that it calls for the IMF and the authorities to broadly agree ex ante on timely monetary responses to possible deviations from the targeted inflation path. However, it would be difficult to implement this approach in an inflation targeting country because this requires an agreement on the monetary policy reaction function. It would be an understatement to say that this is a daunting task because, in fact, defining such a reaction function in the absence of information on the magnitude and nature of the deviation has proven to be impracticable in countries that have adopted IT. This is exactly what one would have expected because the forecasts represent the

collective judgment of Monetary Policy Committee members of the central bank, and it would be very time consuming to secure agreement on quantitative projections for a wide range of variables. In particular, on the question of agreeing to an illustrative path for interest rates consistent with the deviations from the inflation target, the announcement of an illustrative path could be misinterpreted as the central bank's commitment to follow the path. There is the risk that this would not be viewed as a path conditional on the information available at a given point in time, and, as such, if this information were to be made widely available-which should be the case because of the importance attached to transparency in the inflation targeting framework-market participants may end up having destabilizing expectations whenever there were significant divergences between the interest rate forecast and outturn. These divergences are almost guaranteed because the number of contingencies to which monetary policy has to respond to is very large, and some of them are unforeseeable.

Since monetary policy is conducted in a complex setting and full information about forecasts is not a sufficient condition for guaranteeing stabilization of market expectations, a transparent policy framework and effective communication by the central bank are also necessary to improve the ability of financial markets to predict monetary policy actions. SBP should focus on demonstrating its ability to carry out a monetary policy based on a few basic principles, which would be easy for the public to understand, and thereby predict the bank's behavior under different scenarios. This would mean an inflation targeting framework, but some specific proposals, such as ex ante agreement with the IMF staff on timely monetary responses to possible deviations from the targeted inflation path for further fine tuning the monetary framework, do not appear to be required in such a monetary regime. Instead, program reviews should be used to ensure monetary policy remains on track, and as noted earlier, there is no need for a specific mechanism such as an NDA ceiling to limit sterilization.

The recommended approach would be to avoid overburdening the inflation targeting framework with new conditions, and instead to focus on making use of the framework in a way that it fits in well with IMF's need for program monitoring. One approach would be to take seriously the point that none of today's ITers – either individually or on average – had strong “preconditions” suggests that the absence of these “preconditions” is not by itself an impediment to the adoption and success of IT. In other words, the feasibility and success of IT appears to depend more on the authorities' commitment and ability to plan and drive institutional change after the introduction of IT than to ensure that all of the “preconditions” are met at the outset. These considerations suggest that SBP should focus on developing the technical expertise and institutional capacity for

forecast-based monetary policy, which looks at a range of variables for policy analysis, instead of continuing with the framework based on traditional monetary aggregates.

The monitoring of the way forward could be done in terms of structural benchmarks to be set by SBP management, which can be divided into four broad categories: institutional independence, technical infrastructure, economic structure, and financial system health. First and foremost amongst the benchmarks would be institutional independence, that is, SBP should have legal autonomy, and be free from fiscal and/or political pressure that would create conflicts with the inflation objective. The second set of benchmarks would relate to the need for a well-developed technical infrastructure, which would include a proper set of tools for implementing monetary policy in support of the inflation target. The third set of benchmarks would relate to the economic structure, which requires that prices should be fully deregulated, the economy should not be overly sensitive to commodity prices and exchange rates, and dollarization should be minimal. The final set of benchmarks would be that the financial system should be healthy, which minimizes potential conflicts with financial stabilization objectives and permits effective monetary policy transmission.

Such a shift in the formulation and implementation of monetary policy would be analogous to the shift that has occurred in banking supervision in recent years. Banking supervision used to be backward looking, focusing on the banks' balance sheets, but this approach does not fare well in a world of financial innovations where it is easy to make huge bets and become an insolvent institution quickly if an adequate risk management system is not in place. Accordingly, bank supervisors now focus on the risk management systems and the stress testing exercises. SBP needs to take similar steps when it comes to monetary policy, and in the transition toward the inflation targeting regime, because that regime has the inflation forecast as its intermediate target, which indicates that the monetary policy framework has to be modified to become forward-looking, and downplay the emphasis on the historical data on monetary aggregates.

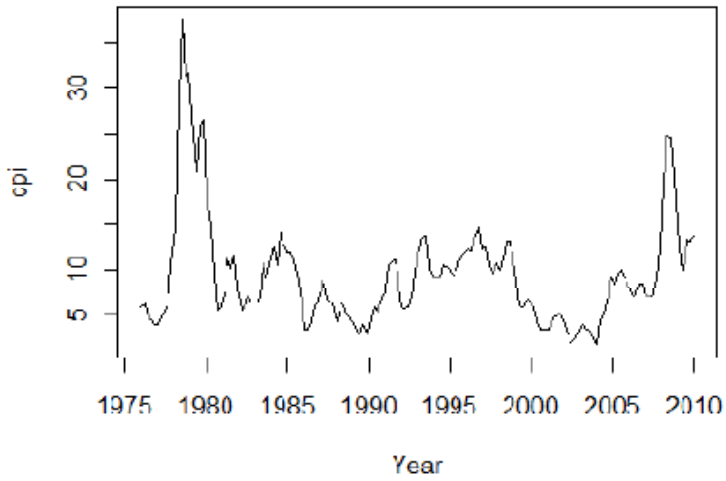
5. Two simple examples of a Bayesian approach for analyzing monetary policy in Pakistan

The model

This section undertakes a Bayesian approach for further developing the monetary policy framework in Pakistan. The procedure uses a dynamic linear model for two important series of the Pakistan economy, namely, the inflation rate and the interest rate. We first undertake a simple univariate stochastic process as model

for the CPI inflation rate of Pakistan and apply the Kalman filter to calculate the model's likelihood function.

Figure 1. CPI Pakistan, 1976-2010



After the likelihood has been obtained, an importance sampling algorithm can be applied to simulate the posterior density of the estimated parameters. Later, we use a more complicated process to model the interest rate process, i.e., a stochastic volatility model that takes into account the time-varying variances, which requires Markov Chain Monte Carlo (MCMC) methods in order to simulate the posterior density.

Implementation and empirics of the Bayesian approach

Univariate model of inflation process in Pakistan

We apply the theory described in the appendix to two different empirical estimation procedures which could serve as building blocks for a Bayesian Dynamic Stochastic General Equilibrium Model, using data on Pakistan's inflation and interest rate, which arguably are the two most important series in the inflation forecasting work. We first consider fitting inflation data, using Consumer Price Index (CPI) for Pakistan for the sample period (1976-2010). This appears to be a time varying process (see Figure 1), so it would be better to consider a stochastic volatility model rather than assuming homoskedasticity over the entire range of data. The stochastic volatility model requires the methods that are more involved than the importance sampling algorithm. We thus defer our attention to those methods until we consider the inflation process in Pakistan in coming paragraphs.

In the first example, the model we consider is a simple univariate model of the inflation process, for which we use Kalman filter to evaluate the model's likelihood function. By applying the importance sampling algorithm, we are able to generate samples from the posterior distribution of the variances. Let π_t denote the inflation rate, then the model in state-space form can be written as follows,

$$\begin{aligned} \pi_t &= \tau_t \mid \eta_t \\ \tau_t &= \tau_{t-1} + \epsilon_t \\ \begin{bmatrix} \eta_t \\ \epsilon_t \end{bmatrix} &\sim \mathcal{N}\left(0, \begin{bmatrix} \sigma_\eta^2 & 0 \\ 0 & \sigma_\epsilon^2 \end{bmatrix}\right) \end{aligned} \tag{1}$$

where η_t and ε_t are independent of τ_{t-1} and are each serially independent across time.

First we find the maxima of the posterior density of the two σ^2 in the state-space model, and we use the model values to compute a Kalman filtered series by smoothing the values of τ_t. See appendix for a brief introduction to the Kalman filter and smoother. To initiate our Kalman filter, we need an initial distribution for the state. Towards this end, we start with the a prior distribution *N* (0.05, 0.004). We calculate the filtered and smoothed values of the inflation process and the plotted results are provided in Figure 2. The graph of Figure 2 has some interesting properties. The smoothed estimates and filtered estimates coincide closely, except for the periods with the highest volatility. The trend in both values seems to be related. The smoothed estimates deviate the most in the period of the highest volatility.

Figure 2. Pakistan CPI Process

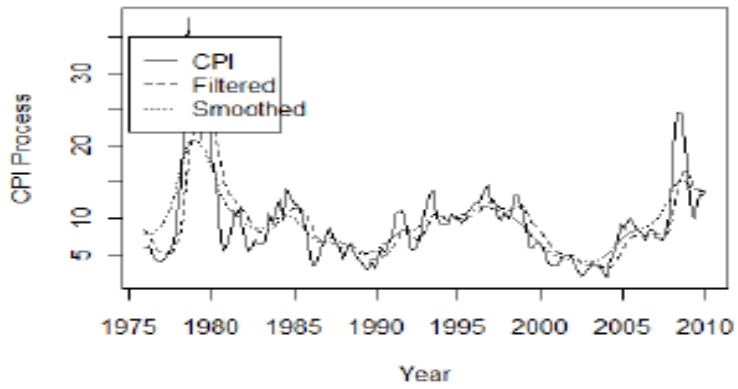
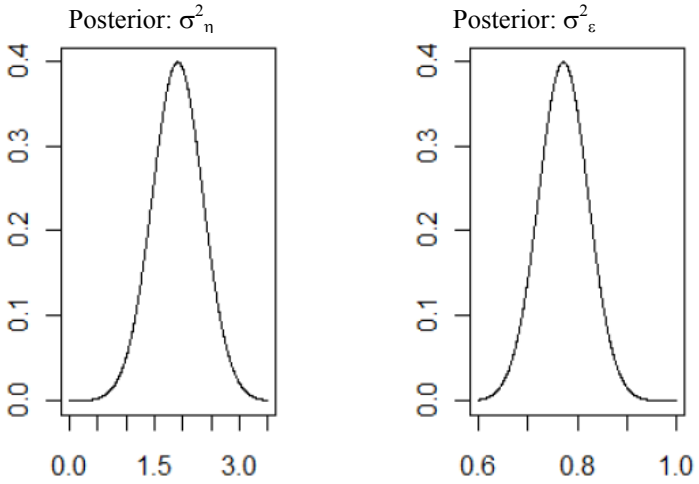


Figure 3. Posterior densities of σ^2_η and σ^2_ε



The two estimates begin to coincide more as time goes on, indicating a convergence of the Kalman filter iterations and a lower level of variance/noise. Changes in the values of the initial prior for variance give slightly different results, and in particular, keeping it too low makes it difficult for the procedure to converge, which would seem to indicate that our value for the prior was too low. Having obtained the likelihood of the parameters, the importance sampling method is a convenient Monte Carlo method for evaluating the posterior density of the estimated parameters, and the model parameters. For the current example, we assume conjugate priors, which are priors that are closed under sampling. In particular, we assume inverse gamma distributions for both σ^2_ε and σ^2_η, both distributed *i.i.d*. according to the form:

$$\sigma^2_{\epsilon,\eta}|y \sim \mathcal{IG}\left(\frac{n}{2}, \frac{(n-2)\,\hat{\sigma}^2}{2}\right) \tag{2}$$

where y denotes the CPI data, and $\hat{\sigma}^2$ is an estimate of the variance obtained directly from the observed data. Since we have conjugate priors, our posterior is also given by an inverse gamma distribution, which makes the importance sampling routine easy to implement. The densities obtained from the importance sampling algorithm are outlined in Figure 3.

The analysis above is based on headline inflation, but perhaps a word is warranted on the question of core versus headline inflation, and which is the preferred measure for actual inflation outcomes and indicators of future inflation in deciding upon the monetary policy stance. If reviews of the monetary policy stance were to be triggered by deviations of forecasted inflation from the target, it would be more

consistent with the inflation targeting framework to focus on the measure that is under the control of SBP. Some economists appear to exaggerate the problems that might arise from basing the consultation bands on forward-looking indicators of inflation, such as the question of which forecast of inflation should be used for triggering changes in the monetary policy stance. It is recognized that there is more than one measure of inflation that is relevant for the consumers, and indeed in the SBP's monetary policy framework, there may well be more than one model involved, but this does not mean that there can be no agreement on a forecast of such a central variable as the inflation rate. Furthermore, by focusing on the further institutional development, such as a well-developed technical infrastructure for the forecasting exercise and the presentation of alternative scenarios, market participants would become more comfortable with the authorities' inflation forecast. Relatedly, if the monetary policy has succeeded in establishing a credible track record, market participants will understand the deviations from the targets for responses to large unforeseeable shocks.

Stochastic volatility model for interest rates

The class of models discussed in the appendix are attractive because of their tractability, and furthermore, they can be easily modified to accommodate regime changes. One example of regime changes would be for the model to have the flexibility to allow for shifts in residual variances. In other words, we would like to have a method of modeling stochastic or time-varying volatility in the series, which is particularly important in the case of the interest rates, and we discuss this model in the next paragraph.

Our stochastic volatility model will treat residual variance as a continuously distributed random variable with some sort of autoregressive serial dependence. We take p (N (26, 11.13) $y_t|\theta$ (.), Y_t) from the general model discussed in the Appendix to be determined by the equation:

$$r_t = \alpha_0 + \alpha_1 r_{t-1} + \epsilon_t \sigma (S_t) \qquad (3)$$

where r is the interest rate yield obtained from quarterly reports, and the ε_i's are white noise error terms. The θ (.) function is thus characterized by m + 2 parameters, the m values come from σ_j , for j = 1,…,m corresponding to the m states, plus α_0 and α_1 which do not vary with the state.

In order to avoid redundancy, we impose the requirement that $\alpha_j < \alpha_{j+1}$ for all j = 1,…,m. This monotonicity requirement does not change the implications of the model's behavior, because the states differ only in their values of σ_j, and by keeping the same permutation of subscripts in the sequence $(\sigma_j)^m{}_{j=1}$ and the transition elements in H, we do not change the implications for the behavior of the

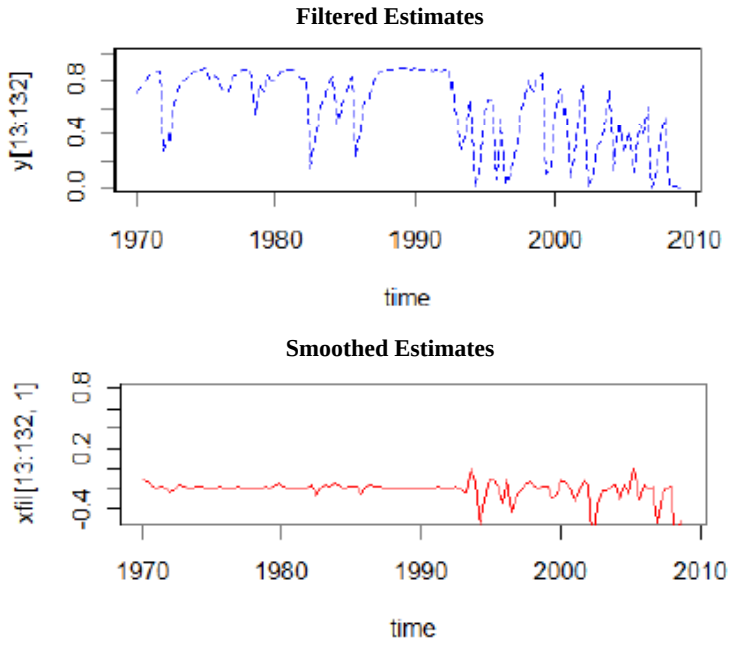
observables *y*'s. Sampling from the conditional posterior of these *m*+2 parameters conditional on *H* and $\{S_t\}$ sequence is best done in two steps. First, by conditioning on the $\{\sigma_j\}$ values, the likelihood obtained is that of a normal linear regression with known, but time varying variances. The posterior is thus normal, centered on the least-squares estimates of α_0 and α_1 parameters, which in turn are based on the appropriately weighted data.

With α's held fixed, the likelihood as a function of the σ'*s* is proportional to

$$\prod_{t=1}^{m} \frac{1}{\sigma^{n(j)}} \exp\{-s_j^2/2\sigma_j^2\} \qquad (4)$$

where *n* (*j*) is the number of occurrences of state *j* in the $\{S_t\}$ sequence and s^2_j is the number of squared residuals, which is calculated using the fixed values of α for the time periods in which $S_t = j$. The expression in Equation (4) is proportional to the product of *m* inverse χ^2 *pdf*'*s*, for which the j^{th} *pdf* has $s^2_j/2$ as its inverse-scale parameter and *n* (*j*) – 1 as its shape parameter, and therefore it is easy to sample from this conditional distribution.

Figure 4. Stochastic Volatility Model Pakistan



The likelihood function for the observable variables has been computed recursively using the Kalman filter, which is then combined with the prior distributions to form the posterior densities of the parameters. We use Monte Carlo Markov Chain (MCMC) methods which approximate the generation of random variables from the posterior distribution, after finding the parameters that maximize the posterior density using some optimization routine. Using Gibbs sampling and initial starting values for each parameter, we are able to generate draws from the posterior of the given model for the interest rate data.

Implementing this procedure on interest rate data for Pakistan from 1971-2008, we present the results in Figure 4. The filtered estimates deviate from the smoothed estimates throughout the sample period, due to the large volatility of the interest rate data, and indeed the likelihood function would only converge after heavy tinkering and innumerable attempts.

From a Kalman filter model, we are able to evaluate the likelihood of the model, and an optimization routine yields the MLE estimates of the model's parameters as: $(\hat{\alpha}_0, \hat{\alpha}_1, \hat{\sigma}_1, \hat{\sigma}_2)$ = [.11, 1.22, 43.12, 33.11]. Furthermore, applying a Gibbs sampling algorithm with 2,500 iterations on the model we are able to generate a set of 2500 Monte Carlo draws of the joint posterior *pdf* of all the parameters of the model. Although convergence isn't guaranteed from this model, as even for 2,500 draws there doesn't seem to be uniform convergence amongst the parameters. The values are given $(\hat{\alpha}_0, \hat{\alpha}_1, \hat{\sigma}_1, \hat{\sigma}_2)$ = (0.2896, 0.9662, 0.7112, 2.2785). It is clear that the Monte Carlo estimates for the variances are much lower than the estimates obtained from the previous Kalman filter and optimization routine, indicating the posterior uncertainty is lower in the Gibbs sampling method in comparison to the MLE method.

Even though this model is more general than the univariate random walk plus noise model discussed earlier insofar as it allows for heteroskedasticity, the estimates in Figure 4 indicate that these results are tentative and that there is a need for further work. In particular, it may be noted that the smoothed estimates and the filtered estimates remain divergent throughout the procedure. This indicates that there is a lot of noise in the system, which makes it difficult to achieve convergence, not least because the data is seemingly exhibiting different regimes, with the first 60 data points (1971-1975) being almost constant at 5.76, and after that period, there is a jump to 7 percent and an increasing trend with some fluctuations in the late 1970s and 1980s. A large spike occurs in the 1990s. The results indicate there is a downward tendency regarding the smoothed estimates compared to the filtered estimates. The increases in interest rates of late

1970's and 1980's gradually begin to give the smoothed estimation procedure a better chance of detecting the properties of the time series.[14]

Forecasting is a natural product of a Bayesian approach to statistical inference because of its emphasis on the explicit formulation of a model, and the requirement that the researcher condition on what is known to make statements about what is unknown. Geweke and Whiteman (2006) have rightly noted that the term "Bayesian forecasting" is a redundancy, because "forecasting is at the core of the Bayesian approach and the parameters of a model are no more known" than future values of the data; the Bayesian approach treats the two types of unknowns in a symmetric fashion. Thus, the future values of an economic time series simply constitute another function of interest for the Bayesian analysis, and this is indeed what was done in the estimation of the models in this paper. In their paper, Geweke and Whiteman provide a lucid presentation of the principles of Bayesian forecasting, and describe recent advances in computational capabilities: the Bayesian approach faced substantial technical obstacles because conditioning on known data and structures in a realistic model often lead to intractable integration problems, which could not be handled until about twenty years ago. However, advances in numerical integration "have steadily broadened the class of forecasting problems that can be addressed routinely in a careful yet practical fashion. This development has simultaneously enlarged the scope of models that can be brought to bear on forecasting problems..."[15]

In the Bayesian approach, the likelihood function is combined with a prior function before maximization to obtain the posterior distribution, where the prior is based on additional knowledge about parameters that is unrelated to the data.[16] As shown in the two examples provided in this paper, it is necessary to approximate the equilibrium dynamics of the model quickly and accurately. In the case of BDSGE models, this is particularly important because these models typically involve solving for the equilibrium dynamics thousands of times for different combinations of parameter values. The Kalman filter has provided a way to evaluate previously intractable likelihood functions in the Bayesian approach, and we have presented simple examples of the Kalman filter and how it can be used for estimating and forecasting interest rates and inflation in Pakistan. Also,

[14] We should also mention that the problem of some missing values also distorted the results, and we used the very simple median approach to find "our guesses" for the missing values. Needless to say, we could and will extend our model by using another Kalman filter to first estimate the missing values, which would clearly be the superior procedure for this application. The data is missing from the range June 2000 – November 2001.
[15] Geweke and Whiteman (2006), page 6.
[16] Smets and Wouters (2003).

the development of simulation techniques such as MCMC have allowed the likelihood function to be explored efficiently, which has driven the explosion of BDSGE modeling and the use of full-information maximum likelihood methods, and this paper has provided an example of the MCMC simulation. It should be stressed that conditioning on what is known requires the appropriate use of prior knowledge of structures that one is studying and having reasonable parameterizations for the model, and the correct use of priors is an important feature in the Bayesian approach, which our two examples have highlighted.

6. IT and DSGE models

We recognize that SBP has come a long way in the development of the monetary policy framework, but nonetheless we suggest that it should redouble its efforts on the technical challenges implicit in the inflation targeting framework. It is necessary to develop alternative methods to forecast inflation.[17] The inflation targeting system would demand a preemptive stance from SBP when setting the instruments of monetary policy due to the existence of lags in the transmission mechanisms, as was mentioned earlier. In this sense, it is essential to deal with econometric forecast techniques in order to properly assess the repercussions of eventual shocks that may lead to deviations of the future path of inflation from its previously fixed target. Space constraints do not allow us to review the use of estimated DSGE models in the formulation of monetary policy, and we instead make some brief suggestions for the research and training being done, especially in the research department of the bank. SBP staff should intensify the work to (a) estimate the parameters of a DSGE model by Bayesian methods, (b) estimate unobserved variables like the output gap; and (c) compute forecasts and forecast uncertainty. This will require special effort to train the staff to enhance their understanding of the MCMC algorithm, because analysis of the posterior distribution of parameters is central to Bayesian inference and the MCMC algorithm is the standard tool for that purpose, as discussed in the previous section and the appendix.[18]

[17] Baxter (1985) provides an interesting analysis of the dynamics of expectations during stabilization policy, applying Bayesian learning to inflation, and one could try to apply the Bayesian approach to the various episodes of attempted disinflation in Pakistan, which were often implemented under IMF-supported adjustment programs.

[18] Sims (2007) rightly points out that the Bayesian approach is a way of thinking, rather than a basket of methods. In classical or frequentist statistics, the researcher searches for the correct model, but the Bayesian statistician takes a subjective view by admitting that all models are false and tries to find the one that will provide the best description of the data. Classical statisticians look for procedures that will deliver the correct answer a certain percentage of the time in repeated samples, whereas the Bayesian statistician makes probabilistic statements about unknown parameters by

What we have tried to do in the previous section is to take a few tentative steps regarding the application of Bayesian econometrics in the context of the Pakistan economy, focusing in particular on variables that are of great significance to the SBP, namely, the inflation and interest rates. The main aim was to show that Bayesian methods can be used even with the limited data available, and therefore to provide some encouragement to other researchers to start working on building Bayesian Dynamic Stochastic General Equilibrium (BDSGE) models for Pakistan, which would be useful for policy analysis. Those models would have to incorporate all of the key features of the canonical DSGE models, but in addition they should give special attention to the constraints and particular issues that dominate the Pakistan economy.[19]

In a companion paper, Zaidi and Zaidi (2010), we discuss the construction and use of dynamic stochastic general equilibrium (DSGE) models for the analysis of monetary policy in Pakistan. In that paper, we review the solution and estimation of DSGE models, the use of maximum likelihood and Bayesian estimation methods (BDSGE), and other approaches. Although we are tempted to review the various features that appear in modern DSGE models such as sticky wages, adjustment costs in investment, a banking sector, financial frictions, and open economy considerations, such an undertaking is clearly beyond the scope of the present paper. In the companion paper, we have argued that SBP should pursue the BDSGE approach, not least because it is a promising way of taking macroeconomic models to the data. However, we would be amiss if we did not mention here that these models have been criticized by both researchers and policymakers. For example, Chari et al. (2009) have stressed that the structural

making use of Bayes's theorem. The latter approach has several advantages, and the vast majority of DSGE models have been estimated using the Bayesian approach.

[19] We would suggest giving considerable importance to the cost channel of monetary policy in a new-Keynesian model; for example, a model with nominal and real rigidities could be extended by assuming that a fraction of firms need to borrow money to pay their wage bill. In any such model, a monetary policy tightening would increase effective unit labor costs of production, and might imply an increase in inflationary pressures from the supply side, and therefore the model could be used to examine the conditions under which there might be a positive response of inflation to a monetary contraction that would tend to offset, at least in part, the demand-side effects which are necessarily supposed to dominate the stochastic process if one is aiming at controlling inflation. Another interesting approach would be to develop a dynamic factor model with Markov switching to examine secular and business cycle fluctuations in Pakistan's unemployment rates. The model could be used to extract the common dynamics among unemployment rates disaggregated for different age groups. Such a framework would allow for the analysis of the contribution of demographic factors to secular changes in unemployment rates, as well as examining the separate contribution of changes due to asymmetric business cycle fluctuations.

shocks, backward indexation of prices, and use of the Taylor rule in describing monetary policy in the these model raises more questions than answers. In particular, the variances of the shocks are too large, backward price indexation is inconsistent with microeconomic data, and the Taylor rule cannot generate the observed behavior of the long-term nominal rate.

7. Inflation targeting and application of rules

At the outset of the discussion of inflation targeting and the application of rules, we should reiterate what Blinder (1997) has said about the "time-inconsistency problem that allegedly bedevils monetary policy. Because the Phillips curve embodies a trade-off between unemployment and unanticipated inflation, well-meaning central bankers are constantly tempted to reach for short-term employment gains by engineering inflation surprises." [20]In this section, we have very little to say about this time inconsistency problem precisely because of Blinder's perceptive comment that "this theoretical problem is a nonproblem in the real world because central bankers have found simple, practical ways to solve it." Our experience working at SBP was definitely not one in which the government had to worry about whether to adopt precommitment strategies or rules reminiscent of Ulysses and Sirens to make sure that SBP management resisted the temptation to inflate the economy with a view to exploit the trade-off in the short-run Phillips curve. Moreover, there was never any question about designing incentive-compatible compensation schemes for SBP management to help solve the Barro and Gordon (1983) reputation problem. To the contrary, our experience was similar to Blinder's comment that the "noun "central banker" practically cries out for the adjective "conservative" and it was the government which was frequently pressuring SBP to have more expansionary monetary policy than what SBP management had deemed to be optimal for the situation at hand. Our focus in this section is on a forward-looking monetary policy framework and how BDSGE models can help in the pigeonholing of particular situations into the general framework/historical context, or alternatively, the tension between the specific nature of a situation at hand and the application of general rules.

In a forward-looking perspective that is the hallmark of the IT regime, the application of rules is of paramount importance because one weighs prospects for overall inflation by assessing the likely path of underlying inflation. This path hinges in good part on growth prospects and the balance of shocks (oil prices, foreign capital inflows, and so on) likely to hit the headline inflation rate, as well as their potential reverberations. To motivate the discussion of this topic of

[20] Blinder (1997), page 13.

uncertainty and rules, let us start with a concrete example or situation. If SBP staff were to see signals of, say, a fading economic recovery, underlying inflation remaining unusually low, and a relative absence of shocks, that does not necessarily mean smooth sailing. For instance, the extent of domestic and international weakness might be unclear. One approach might be for SBP management to take a wait-and-see attitude as regards immediate action, while the alternative policy action might be that the case for a cut in the interest rate is materializing, notwithstanding headline inflation moving above the target. In this regard, we want to stress that we give great importance to the application of rules, and recognize that for a large percentage of our actions and judgments, whether in the implementation of monetary policy or in our own individual lives, we act according to rules. This is because most of the time we are moving among situations of certain standard types, and we respond in such a way to obtain certain standard results. Thus, action according to rule is a very important and the most common approach because often the best way to proceed in tackling a range of issues, and the first question which we should ask when we find ourselves in a situation of any kind, is: "What are the rules for acting in this kind of situation?" It is equally important, however, to ask the alternative question, i.e., "How is this situation different from others?" Thus, one would need to know what exceptions have been made in the administration of the particular rule, and how does this case of monetary policy/inflation targeting fit in with those previous cases.

In various periods in Pakistan's monetary history, inflationary expectations became imbedded in long-term lending rates and volatile expectations about future inflation made long-term rates highly sensitive to macroeconomic events, not least the fiscal slippages and excessive borrowing from the banking system by the government. These entrenched expectations based on fiscal/monetary interactions and associated imperfect credibility about future monetary policy actions put severe constraints in the conduct of monetary policy, not least of which was that they made long-term rates stubborn in the face of monetary policy actions. Since the management of inflationary expectations was a crucial but a very difficult part of SBP's job with fiscal dominance, a host of questions arise regarding the application of rules and taming inflation in the face of a weakening real economy. To go deeper into this important topic would take us too far away from the main thrust of this paper, but suffice it to say that whereas fiscal dominance may be par for the course in Pakistan, it makes it difficult to consistently pursue an anti-inflation policy, and there are no ready-made rules for this job. Bayesian econometrics can help to understand the dynamics of expectations and the inflationary psychology that is a recurrent constraint faced by SBP.

The point that action according to rules is not the only kind of action cannot be overemphasized, because there are occasions in which another kind of procedure, namely, human understanding and insight, is necessary. One such occasion might be when we find ourselves in a situation that does not fit in with any of our known types, and therefore there is no rule that can tell us how to act in this regard. In this particular case, the policymakers would have to look at the situations of other countries where an exception may have been made on, say, the inflation forecast targeting rule. If there have been no such cases, then we really don't have a ready-made rule for this particular type of situation. This means that we would need to rely on human insight to help tell us how to proceed in this situation in which we have actually found ourselves, and not succumb to the temptation and insist on putting it in a situation of a specific type. In other words, there may be a particular case which might well be a unique situation, and to reach the correct decision, one would need to focus on all sorts of issues. In particular, the policymakers should not rely solely on a strict/narrow interpretation of the rule, but should look at the complete picture, or in common parlance, "the totality of the things." Going back to the example just discussed, suppose international developments suggest that the worst may be over on economic growth but point to further upward pressure on headline inflation because of oil prices and/or depreciation of the exchange rate. In such a scenario, the wait-and-see/rate-is-appropriate stance might seem to be more appropriate than a cut in the interest rate, particularly if SBP wanted to demonstrate that it took its inflation objective seriously. In a similar vein, if there were to be little or no inflationary pressures, one would see no scope for even thinking about raising interest rates until a clear and firm recovery in both output and demand were to be in place. Indeed, one would see a case for further cuts if signals for a faltering recovery accumulated during the period under consideration.

It is not just the uncertainties involved with regard to the output-inflation trade-off, but moreover, there is the question of whether SBP should try to deflate the stock market bubble or the housing price bubbles that have occurred from time to time in Pakistan's history. A counter argument would be that it is impossible to identify bubbles at the right time, and, in any event, interest rates are an imprecise instrument to prick asset price bubbles. This view asserts that it is safer to let bubbles burst of their own accord and then ease monetary policy to support the economy. Given the recent financial turmoil in the advanced economies and the bursting of the US housing bubble, there may be lessons for Pakistan here now it is less uncommon to hear that the "mop-up strategy" may not be the best way to deal with bubbles. Needless to say, it is important to recognize the differences between the advanced economies and the emerging market economy like Pakistan; however, we speak from personal experience in saying that the question of asset

price bubbles has been a recurrent topic in monetary policy deliberations at SBP but with widely divergent viewpoints amongst policymakers.

Blinder (1997, 1998) has provided insightful analyses of why central bankers should rely more heavily on models and forecasting in evaluating the impact of their interest rate setting decisions. In particular, he rightly points out that without attention to the dynamic effects of their actions, central bankers risk leaving monetary policy too loose or too tight for too long. Monetary policy is implemented in an environment of huge uncertainties and imperfect knowledge, and central bankers do not have an obviously "correct" answer for the question of when and by how much to move interest rates. It is not just that the information on the current state of the economy is subject to big revisions, but there is uncertainty about critical measures such as the size of the output gap and the stability of the parameters of the econometric models. However, the fact that SBP (and indeed all central banks) does not have a completely trustworthy model of how the economy works and forecasting is probably more of an art than a science, it does not mean that one should not invest resources in this area. To the contrary, Blinder points out that if interest rates were to be set according to the economy's current situation and without regard for the future, then they will probably be raised by too much and kept high for too long, or vice versa. Accordingly, he suggests that monetary policy must be forward-looking, and incorporate future inflation in what might be called a strategy of "dynamic programming." In other words, the interest-rate decision that the monetary policy committee takes should be thought of as the first step along a path of future interest-rate decisions. For example, whenever a central bank begins a cycle of tightening, it should have a good idea about where it is going in the medium-term and that idea will be based at least in part on its econometric models. The central bank must project a likely path of future interest-rate decisions and for key economic variables, and the central bank should continue to follow the planned path only if those economic variables turn out as expected. However, if the economy slows sooner than expected, the bank should tighten by less than what had been originally planned, or even be willing to cut interest rates.

There is also another occasion in which rules do not help us, that is, when we think that we can classify the situation in a known type, but we are not comfortable with the implementation of the readymade rule, because we know that action according to the rule involves a certain amount of misfit between the general rule and the particular situation at hand. The issue raised in this paragraph is different from the one discussed in the earlier paragraphs, because in those paragraphs the point was that the case might well be a situation for which there is no rule for the particular type, but in this paragraph the point is that even if we

were to assume for the sake of argument that there is, and if we were to attempt to act according to a rule, we are still not dealing with the situation in which we stand, but we are only dealing with a certain type of situation under which we class it. Put it differently, macroeconomic forecasts aim to provide the policymakers with the best possible set of information to support the decision-making process, but there are several different approaches to either forecast an event itself or its magnitude. What we are trying to say here is that even if this case might appear at first glance to be similar to the general rule, it is still nonetheless true that although the general rule provides us with a handle with which to grasp the particular situation, but we still have to recognize that the generality of the rule comes between us and the specific aspects of the situation we are trying to grasp. In other words, because of the special and unusual aspects of some particular case, it may well be that SBP management and staff may conclude that in the "totality of things" that one ought to consider, the difference is not just a difference in degree but a difference in kind between the generality of the rule and the specific situation at hand. Thus, insight and human understanding is required in some cases of decision making, and we cannot say that 100 percent of our decisions can be easily determined by strict application of rules.

Relatedly, some observers have argued that monetary policy is nothing but risk management, namely, looking not only at the most likely path for the economy but at all the possible paths it might follow and the associated costs. The idea is to attach extra weight to outcomes that would do severe harm. Thus, even if there was only a small risk of deflation but if the costs of such an occurrence were deemed to be very high, the central bank would push interest rates downward. It is for these reasons that SBP management should review individual cases when there are, for example, sharp deviations between headline and core inflation, between projected and actual GDP growth rates, sharp fluctuations in the interest and exchange rates, and the like. The policy analysis done with BDSGE would be of great value in reaching informed decisions, because it would help to identify the situation and what it means for policy implementation. As mentioned earlier, the use of priors in the Bayesian approach gives due regard to the principles that one should fully specify what is known and what is unknown in statistical inference, and then condition on what is known in making probabilistic statements about what is unknown. Moreover, Bayesians stress that the only objective way to measure a model's accuracy and/or compare it with others models is to generate forecasts which can be described in a probabilistic sense, that is, as part of a probability distribution. This is a uniquely Bayesian notion, and to fix ideas, consider a not uncommon situation in which the SBP Governor (or Monetary Policy Committee) weighs the implications of the models provided by the research staff against his/her own knowledge (experience gained from previous such

episodes in Pakistan or from the examples provided by other countries, hunches, insights, etc.) before making decisions. To properly carry out such an exercise, there is no option but to have the results generated by the models be weighted by some measures of their reliability, and Bayesian inference (or probability models) provides exactly this measure by the very construction of the model that is deemed to be appropriate in this approach. In other words, Bayesian inference enables the researchers to bring pre-sample information, including personal beliefs and results from earlier empirical work, into the model via priors in an explicit and systematic way.

8. Concluding remarks

Rather than summarize the major issues discussed in this paper, let us comment on the need for further intensive work on strengthening the SBP's macroeconomic framework and the institutional and technical infrastructure. One of the authors of this paper (Iqbal Zaidi) had the privilege to work on developing under the close guidance of Ex-Governor Dr. Muhammad Yaqub the macroeconomic framework that was used quite effectively for the formulation of monetary policy, guidance to the government about their borrowing limits from SBP and commercial banks, and as a basis for negotiating Fund programs.[21] The macroeconomic framework proved its worth in helping to bolster the arguments advanced by SBP management for the need to: reduce the excessive budgetary financing of the public sector from the banking system and government borrowings at below market rates; raise interest rates to provide positive rate of return on financial savings; emphasize the importance of increased domestic savings, both to promote growth and to reduce the current account deficit of the balance of payments; and to lower inflation and stabilize relative exchange rate. The framework demonstrated that the past experience of reliance on exchange rate depreciation to offset expansionary fiscal policy and low interest rates and low savings rate did

[21] We must say that to our knowledge nobody has labored harder – or more effectively – for the SBP than the maestro Ex-Governor Yaqub, and that there was a sea change in the SBP in all aspects of central banking during his tenure. The staff of the international financial institutions (e.g., IMF and World Bank) had commented very favorably in writing on the build-up of the core staff, new recruiting, training, etc. during the "Yaqub Years of the SBP." For the first time during those years, the SBP staff were carrying out financial programming exercises similar to what was previously done for Pakistan by IMF missions, a weekly reserve money management program was put in place, and a lot of technical work was done on the problem of the Foreign Currency Deposits (FCDs) and how to tackle the difficult constraint which those deposits imposed on the conduct of monetary policy. It would take an entire paper to talk about the improvements in banking supervision and prudential regulation that were effected in the Yaqub period, as well as the considerable progress made in attacking the huge nonperforming assets in the banking system, so we will not even discuss those and many other contributions made by governor Yaqub.

not contribute to strengthening of the external current account in the long run and was at best a stop-gap measure. The analytical and empirical results were quite clear-cut in showing that prudent fiscal and monetary policies were better instruments for dealing with the current account problem than exchange rate depreciation. It goes without saying, however, whereas improvements have been made to this "Governor Yaqub Macroeconomic Framework" over the succeeding years, a lot more needs to be done. For one thing, we strongly urge that this macroeconomic framework with its focus on flow of funds analysis be supplemented with a Bayesian Dynamic Stochastic General Equilibrium (BDSGE) model and other technical tools that economists have developed over the past fifteen years. Moving forward, we can acknowledge that a lot of work has been done in the flow of funds analysis part of the monetary policy framework, but also recognize that there is clearly more research to be done in the area of econometric modeling of the Pakistan economy.

The point that SBP should have an in-house facility to formulate and evaluate an independent monetary policy that is in line with the 1997 revisions in SBP Act bears emphasis. The first order of business should be to further bolster the technical infrastructure for macroeconomic analysis, which should become an integral part of the set of tools for implementing monetary policy at the SBP, and to make the strengthened macroeconomic framework operational, including having the team carry out several alternative medium-term scenarios as well as quarterly projections for the fiscal year. The close involvement of SBP management in the development of the macroeconomic framework would be of the utmost importance, because the macroeconomic framework is a tool that management will be using, and it should be designed in such a way that it is fully consistent with the priorities and requirements established by the governor. For example, the close involvement of the governor would also be required to ensure that the macroeconomic framework is fully matched with the monetary policy stance, and moreover, will help SBP management to get quantitative answers to key questions such as the following: Given the monetary overhang from the previous years and the need to bring inflation down this year, is the projected interest rate target appropriate? Are there reasons to believe that even the interest rate setting might be masking the true underlying demand pressures because some of these might take the form of leakages in the balance of payments?

At the risk of oversimplification, monetary policy is as much an art as a science, and its conduct at SBP will be the outcome of a weighted average of the governor's and SBP Board's experience and insights, on the one hand, and the quantitative projections derived from the macroeconomic framework, on the other hand (e.g., the arguments put forth in Section 7). We will venture to say that based

on our experience, the weight will and should go more to the side of management and less to the macroeconomic framework. However, like in many other countries in situations similar to Pakistan, SBP management will find that whatever investment they make into the macroeconomic framework in terms of the hours spent on it by supporting the technical staff and asking for revisions, updates, improvements, and the like, the rate of return will not just be higher but will be multiples of the rates of return in the economy!

Finally, the SBP's communications strategy in recent years has been broadly effective but not without some problems. There is merit in SBP providing a more explicit statement of its inflation objective, and being more forthcoming in letting the public know when it becomes clear to SBP management that the inflation target will not be met for whatever reason (e.g., fiscal dominance, external terms of trade shocks, unanticipated increases in indirect taxes). In spite of the misgivings expressed by some SBP staff members, we feel that this candid approach could help further anchor inflation expectations without undermining confidence in the SBP's commitment to its mandate of focusing on inflation control. The paper has attempted to show that a formal inflation target and using the inflation projections as the intermediate targets for the conduct of monetary policy might bring additional gains to the SBP's well-established credibility, while having positive implications for the SBP's other policy objectives. We conclude, by saying that perhaps consideration could be given to the establishment of a committee to examine the SBP's monetary policy framework and overall communication policy, including refining the macroeconomic framework and econometric toolkit for policy analyses, the definition of price stability, and the question of how providing more frequent Monetary Policy Reports with a greater focus on future developments could further increase the SBP's already high level of transparency.

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Appendix

Markov Chain Monte Carlo Methods

We consider a class of models in which we have a parametric model for the conditional distribution of the observation:

$$y_t \mid \{y_s, s < t\} \sim p\,(y_t \mid \theta_t, \{y_s, s < t\}) \tag{A1}$$

where the right hand side denotes the *pdf* of the observation. The parameter θ_t varies over time and is determined by an unobserved state variable S, hence $\theta_t = \theta(S)$. We force the unobserved state to take a finite number of values $j = 1,\ldots,m$ and follows a Markov chain, so that

$$P[S_t = i \mid S_{t-1} = j] = h_{ij} \tag{A2}$$

For this class of models, it is a straightforward procedure to evaluate the *pdf* of the data conditional on the initial observation y (1), the transition matrix H, a known form of parameter function $\theta(S_t)$, and a prior distribution for the initial state, S_1. Furthermore, filtered estimates of $\{S_t\}$ emerge as byproducts, as in the Kalman Filter. These filtered estimates can then be recursively "back–filtered" to generate smoothed estimates of $\{S_t\}$, yielding $P\,[S_t = i \mid \{y_s;\ s \le T\}]$. Furthermore, a recursive algorithm much like the smoother will generate draws from the conditional *pdf* of $\{S_t\}$ given data $\{y_t\ ,\ t = 1,\ldots,\ T\}$ and other parameters, $\theta(.)$ and transition matrix H. Thus, the *MLE* and posterior simulation via MCMC are straightforward, which can be done using a Gibbs sampling algorithm from the posterior density of the parameters. For notational convenience, let $p\ (.)$ denote the *pdf* of the random variable in its arguments, and let Y_t refer to $\{y_s \mid s \le t\}$. We would like to form $p\ (y_{t+1} \mid Y_t)$, and a product of terms in this form will give us a tractable likelihood function. Observe that

$$\begin{aligned} p(y_{t+1}, S_{t+1}, S_t \mid Y_t) &= p(S_t \mid Y_t) \cdot p(S_{t+1} \mid Y_t, S_t) \cdot p(y_{t+1} \mid Y_t, S_t, S_{t+1}) \\ &= p(S_t \mid Y_t) \cdot p(S_{t+1} \mid S_t) \cdot p(y_{t+1} \mid Y_t, \theta(S_{t+1})) \end{aligned} \tag{A3}$$

For Equation (A3), the right hand side is composed as a product of three different distribution functions. The first $p\ (.)$ is the one assumed to be given from the data, the second is read off the transition matrix H, and finally the third is the model of y given the initial parameter values. Hence, all the necessary distribution functions are available to evaluate $p\ (y_{t+1},\ S_{t+1},\ S_t \mid Y_t)$,

$$p(y_{t+1} \mid Y_t) = \sum_{i,j} p(y_{t+1}, S_{t+1} = i, S_t = j \mid Y_t) \tag{A4}$$

$$p(S_{t+1} \mid Y_{t+1}) = \frac{\sum_j p(y_{t+1}, S_{t+1}, S_t = j \mid Y_t)}{p(y_{t+1} \mid Y_t)} \tag{A5}$$

Initial Conditions

To form a complete likelihood, however, we need to know the unconditional distribution of S_1. In most instances, the observation y_1 will be informative enough about S_1 so that we could calculate the unconditional joint distribution for y_1 and S_1 implied by the dynamic model in question, but this calculation is rather cumbersome and could also be problematic. For example, if the model is non-stationary, then the unconditional distribution will not even exist. Thus, a more simple approach is to take y_1 as non-stochastic and assume initial state S_1 is drawn from the unconditional distribution of S alone. Taking the right eigenvector of H associated with its unit eigenvalue provides us with the necessary S_1 distribution, which will be a vector p satisfying $H\,\overline{p} = \overline{p}$.

Hence, $\overline{p}$ is a steady-state for the unconditional *pdf*. Existence of such an eigenvalue is guaranteed by the Perron-Frobenius Theorem (see Theorem 5.13 in Sinai and Koralov, 2007, and Section 5 in Chapter 4 of Çinlar, 2011), but uniqueness is not. In the case where uniqueness is not satisfied by the matrix, then we simply take p to be the average of all the right eigenvectors corresponding to the unit roots. (In the case that the matrix has multiple unit-eigenvalues, we say the Markov process is not ergodic. In such circumstances, different initial values can lead to different steady-state distributions. The interested reader is again referred to Sinai and Koralov (2007), in particular, Theorem 5.9).

Recursion method for obtaining Smoothed State Distributions

Using p in place of $p\,(S_1|Y_1)$, we can begin our recursion process. In order to generate smoothed distributions of states, we need to start with $p\,(S_{t+1}\,|Y_T\,)$ and to have available the filtering output from the forward recursion, i.e., we need to make available to our recursion process the probability density functions $p\,(S_t|Y_t)$ for t = 1,…,T. Our aim is to find $p\,(S_t,\,|Y_t)$ from which we can continue the recursion. Thus, we find:

$$\begin{aligned} p(S_{t+1}, S_t \mid Y_T) &= p(S_{t+1} \mid Y_T) \cdot p(S_t \mid S_{t+1} \mid Y_T) \\ &= p(S_{t+1} \mid Y_T) \cdot p(S_t \mid S_{t+1}, Y_t) \\ &= p(S_{t+1} \mid Y_T) \cdot \frac{p(S_{t+1} \mid S_t) \cdot p(S_t \mid Y_t)}{\sum_j p(S_{t+1} \mid S_t = j) \cdot p(S_t = j \mid Y_t)} \end{aligned} \tag{A6}$$

The second equality above was obtained by taking advantage of the fact that y_{t+s} for s ≥ 1 depends on S_t only through S_{t+1}, and thus when S_{t+1} is known, the values of y date after t don't provide any additional information.

The two terms in the final product in the right-hand-side of (A6) only contain terms that we are assuming to know, hence it is easy to evaluate the expression. Conveniently, taking the sum of $p\ (S_{t+1},\ S_t|Y_T\ )$ over all possible values of S_{t+1} we indeed arrive at the desired *pdf* of $p\ (S_t\ |Y_T)$.

The recursion to generate a sample path of S_t from its conditional distribution given the data can also be based on equation (A6). The method we use is backwards recursion, which is permissible since we note that

$$p\left(S_t|S_{t+1}, Y_T\right) = p\left(S_t|\left\{S_{t+s}, s \geq 1\right\}, Y_T\right) \quad \text{(A7)}$$

This equality is again used by exploiting the fact that all future states *S's* (as future are *y*'s), only depend on the current S_t via S_{t+1}. We initiate this backwards-recursion by drawing S_T from the filtered path $p\ (S_T\ |Y_T)$ and we then draw $S_{T\text{-}1}$ from

$$p(S_{T-1}\,|\,S_T, Y_T) = p(S_{T-1}\,|\,S_T, Y_{T-1}) = \frac{p(S_T\,|\,S_{T-1})\cdot p(S_{T-1}\,|\,Y_{T-1})}{\sum_j p(S_T\,|\,S_{T-1}=j)\cdot p(S_{T-1}=j\,|\,Y_{T-1})} \quad \text{(A8)}$$

and so on back to $t = 1$.

The Gibbs sampling loop

We have described in the previous two sections a method of making a draw from the distribution of $\{S_t\}$ conditional on Y_T . This algorithm can form one component of a Gibbs sampling scheme, which we will now complete with a sampling of a second component. In order to obtain the second component of a Gibbs sampling scheme, we need to sample from the posterior density of $\theta\,(.)$ and H conditional on $\{S_t\}$. The tractability of this problem will ultimately depend on the form of the density $p\ (y_t|Y_{t\text{-}1},\ \theta\,(.,\ H))$ and what restrictions are placed on the form of the transition matrix H.

As a function of the elements h_{ij} of H, the likelihood with the S's held constant is proportional to

$$\overline{p}\,(S_1)\prod_{i,j} h_{ij}^{n(i,j)} \quad \text{(A9)}$$

where $n\ (i, j)$ is the number of dates t, $2 \leq t \leq T$ at which $S_{t-1} = i$ and $S_t = j$. The steady-state probability distribution of p depends on H via $Hp = p$. The part of equation (A9) following p term has the form of m independent Dirichlet *pdf* 's. The Dirichlet distribution is a multivariate analogue of the Beta distribution, and easy to sample from. An efficient sampling scheme incorporates the Metropolis-Hastings algorithm, in which the values of H are drawn from the Dirichlet priors, with rejection criteria based on the value of p term under the new and old draw.

Kalman Filter and Smoother

The Kalman filter is simply a rule for starting with a prior $s_t \sim N(\mu_t, \Sigma_t)$ and using it, plus observation of y_{t+1}, to update to a new distribution $s_{t+1} \sim N(\mu_{t+1}, \Sigma_{t+1})$: At each date, the Kalman filter involves a forming a normal distribution for $y_{t+1}|I_t$, and so the *pdf* of the entire observed sample of y's is then

$$\prod_{i=1}^{T} p(y_t \mid I_{t-1}) \qquad \text{(A10)}$$

since we assume the information available at t consists of the time zero information I_o plus a sequence of y_s values for all $s \leq t$. The Kalman filter then gives a derivation of $p(.|I_{t+1})$ from $p(.|I_t)$ and y_t. The initial distribution $p(.|I_o)$ is determined by an initial Gaussian prior on the initial state s_0. Therefore, the log-posterior density is just the sum of the log $(p(y_t|I_t))$ terms.

With the Kalman filter results we can use a recursive method to obtain the distribution of $s_t|I_T$, where I_T is the end of the sample. At each t, the smoother uses the distribution of $s_{t+1}|I_T$ and that of $s_t|I_t$ to produce the distribution of $s_t|I_T$. This will eventually require generating a posterior distribution of $s_o|I_T$ that is different from the prior on s_0.