# Detection and Forecasting of Islamic Calendar Effects in Time Series Data

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*As the behavior of Islamic societies incorporates seasonal effects of both Gregorian and Islamic calendars, the conventional seasonal adjustment methods will yield distorted results. In this paper we extend the standard ARIMA modeling to incorporate Islamic calendar effects. This methodology is applied to the monthly data of currency in circulation. The presence of Islamic calendar effects is strongly pronounced in the results. These results are useful for policy makers of the central banks in tracking the path of currency in circulation. Furthermore, this methodology can be used for computing seasonal adjusted series and factors in Islamic countries.*

## 1. Introduction

Calendars exert profound influence on the cultural, social and economic behavior of the people. All countries of the world almost universally follow Gregorian calendar; each country set its working days and holidays accordingly. However, different societies rigidly follow their own unique calendars for observing days of religious significance and festivals. For example, Islamic societies follow Islamic Hijri calendar, which is strictly lunar based; Jewish societies follow Hebrew calendar, which is lunisolar, and Christian societies follow Gregorian calendar, which is strictly solar. Similarly, Hindu and Chinese societies also follow their own calendars.

Detection of calendar and seasonal effects in time series data of Christian societies is a well-developed field in statistics. Various methods developed at U.S. Bureau

of Census [Shiskin and Musgrave (1967)], Statistics Canada [Dagum (1988)] and Bank of Spain [Gomez and Maravall (1997)] are well suited for identifying seasonal patterns and Gregorian calendar effects. Some of these have provisions for detecting effects of Christian religious holiday of Easter as well, which is set according to Ecclesiastical calendar. Statistical methods have also been developed in the Bank of Israel [Pfeffermann and Fisher (1980)] to detect the effects of Jewish religious festivals like Passover. Compared to these developments, methods of detecting and adjusting for effects of Islamic festivals and days of significance are yet to be initiated. Applications of standard methods like X-11 or X-12 ARIMA may not reveal identifiable seasonality, or report distorted seasonal factors in time series data from Islamic countries.

In all Islamic societies and countries, socioeconomic and cultural behavior is determined both by Islamic as well as Gregorian calendars. Consumption rises during the month of Ramadhan, specially preceding Eid-ul-Fitr and the month of Dhul Hijja on account of annual Islamic Pilgrimage Hajj (Eid-ul-Azha). Consumption declines in Shawwal after festivities of Eid-ul-Fitr and in Muharram due to mourning for Ashura. Different Islamic countries also observe other Islamic holidays according to their traditions. Strength of effects for above four Islamic months may vary across Islamic countries due to differences in observance of festivities and mourning.

Detection of Islamic calendar effects would have been a simpler task if the data were also compiled according to Islamic calendar. However, all statistics are according to Gregorian calendar. Whilst the Islamic year is strictly lunar based, having 354 or 355 days; it recedes through the Gregorian calendar, which is solar based having 365 or 366 days. Another complication is that almost all Islamic societies follow an observational based Islamic calendar as religious authorities announce the beginning of each Islamic month after sighting the new moon. For this reason, algorithms of conversions of Islamic dates to Gregorian dates lead to error of up to two days. Published almanacs, based on these algorithms, are also not completely accurate. Accurate record for recent past can only be complied through daily newspapers, which can be very tedious for long time series.

Purpose of this paper is to show that time series data in Islamic countries, such as Pakistan, exhibit very strong effects for days and months of religious significance. To detect and measure Islamic calendar effects in Pakistan, we develop a simple ARIMA model with fractional indicator variables. Methodology is explained in section two, followed by data description and interpretation of results in section three. Forecasting strength of model is discussed in section four while final remarks are in last section.

## 2. Methodology

To detect the effect of Islamic calendar months, we use the concept of fractional indicator variable to indicate either full presence of Islamic month (say Ramadhan) in a given calendar month (say December), or partial presence of Ramadhan in November and remaining in December, with the unit value of dummy distributed in November and December to the extent of length of Ramadhan month falling in respective calendar months.  The usual dummy variables are simultaneously used to account for the effects of Gregorian calendar months. An appropriate ARIMA model is specified to the log of original variable under study. Mathematically, this is specified in the following regression equation.

$$Log(y_t) = \alpha_0 + \sum_{i=1}^{11} \beta_i D_i + \sum_j \gamma_j F_{ji} + \varepsilon_i + \sum_{p=1}^{p} \delta_p \log(y_{t-p}) + \sum_{q=1}^{q} \eta_q \varepsilon_{t-q} \quad (1)$$

$D_i = 1$, if $y$ belongs to *ith* calendar month
$D_i = 0$, otherwise

$F_{ji}$ is the fractional indicator or fractional dummy variables designed as follows:

$$F_{ji} = \frac{n_{ji}}{n_j} + \frac{n_{ji+1}}{n_j} \tag{2}$$

$n_{ji}$ is the number of days of *jth* Islamic month falling in *ith* calendar month

$n_{ji+1}$ is the number of days of *jth* Islamic month falling in $(i+1)th$ calendar month

$n_j$ is the number of days in *jth* Islamic month $(n_{ji} + n_{ji+1})$

$F_{ji} = 1$, if $y$ belongs to *jth* Islamic month falling partly in *ith* and $(i+1)st$ months

$F_{ji} = 0$, Otherwise

$\varepsilon_i$ is the usual white noise variable

$\delta_p$ are the coefficients of auto regressive process

$\eta_q$ are the coefficients of moving average process

For simplicity and conformity with socioeconomic behavior in Islamic countries, we take only four months of significance: Muharram, Ramadhan, Shawwal, and Dhul-Hijja. Deseasonalized series can be obtained by removing both calendar and Islamic month effects from our main equation. Removal of only Islamic month effects will not yield a completely deseasonalized series as the remaining series will continue to exhibit regular calendar month effects.

## 3. Data and Interpretation of Result

Specified model is tested on monthly data (compiled as on last week of the month) on stock of currency in circulation (CC) in Pakistan from July 1972 to June 2001. Underlying structure in CC was first analyzed with various correlograms. Since the selection criteria, Akaike Information Criterion (AIC) and Schwarz Information Criterion (SIC), favored difference stationary process, we differenced the series to eliminate trend growth after taking natural logarithms of stocks of CC.[1] As shown in Figure 1, differenced series exhibits a stationary process while spikes hint at the presence of seasonality. This was also confirmed by the unit root test statistic that strongly rejected the possibility of non-stationarity.

Compilation of calendar dummy variable is straightforward. However, compilation of Islamic fractional indicators required conversion of Islamic dates into calendar dates. Dershowitz and Reingold (1997) provided an algorithm implementable in computer language "LISP" to serve this purpose. Alternatives available include Jehangir's almanac of calendar conversion [Jehangir (1976)], or

**Figure 1.  First Difference of Natural Log of Currency in Circulation**



(July 1972 -  June 2001)

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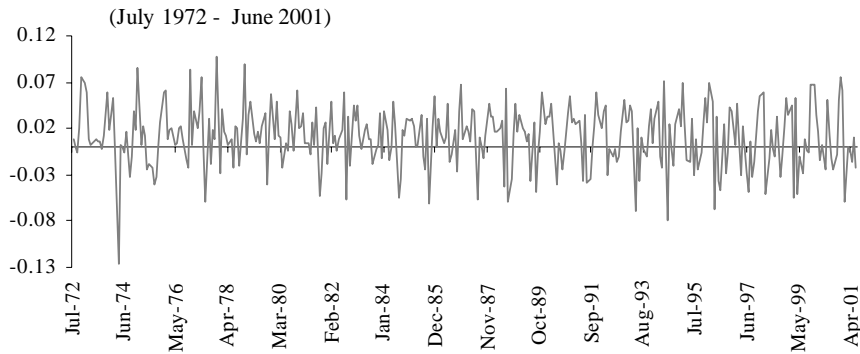
[1] Time series of log of currency in circulation displayed a unit root, as the Augmented Dickey Fuller (ADF) test statistic 1.36 was significant even at 1 percent level of Mackinnon critical value.

**Table 1. Regression Results**

| Months | Parameter | Coefficients | T-Statistics | Relative Effect |
|--------|-----------|--------------|--------------|-----------------|
| June | $a_0$ | -0.0133 | -3.35 | -1.3260 |
| July | $\beta_1$ | 0.0264 | 4.82 | 2.6742 |
| August | $\beta_2$ | 0.0009 | 0.17 | 0.0909 |
| September | $\beta_3$ | 0.0062 | 1.03 | 0.6248 |
| October | $\beta_4$ | 0.0426 | 7.78 | 4.3502 |
| November | $\beta_5$ | 0.0709 | 12.93 | 7.3496 |
| December | $\beta_6$ | 0.0372 | 6.79 | 3.7924 |
| January | $\beta_7$ | 0.0499 | 9.13 | 5.1216 |
| February | $\beta_8$ | 0.0309 | 5.64 | 3.1420 |
| March | $\beta_9$ | 0.0162 | 2.68 | 1.6334 |
| April | $\beta_{10}$ | 0.0149 | 2.73 | 1.5056 |
| May | $\beta_{11}$ | 0.0274 | 5.01 | 2.7809 |
| Muharram | $\gamma_m$ | -0.0420 | -8.27 | -4.1130 |
| Ramadhan | $\gamma_r$ | 0.0404 | 7.94 | 4.1253 |
| Shawwal | $\gamma_s$ | -0.0452 | -9.00 | -4.4192 |
| Dhul-Hijja | $\gamma_h$ | 0.0189 | 3.82 | 1.9097 |
|  | MA(9) | -0.2333 | -4.26 |  |

consulting old newspapers for compilation of Islamic dates. For convenience, we have used Al-Rumaih's "Hijri-Gregorian Converter" [Al-Rumaih (1997)]. As mentioned earlier, these algorithms can err by maximum of two days. However, the resulting distortion in monthly data is unlikely to be serious. This may not be true if one is working with weekly or daily time series.

Parsimonious selection of ARIMA with our specified model resulted in the inclusion of moving average of ninth order. The estimated results of Equation (1) along with *t*-statistics are reported in Table 1. Higher significance levels of differential coefficients, except for August and September suggest that seasonality concerns remained valid for most of the months. However, the significance of monthly differential coefficients for each month should not matter much, as it simply states that the variation in CC from the base is significant. In contrast, the significance level of Islamic calendar effect matters a lot. Being the main hypothesis of this paper, results suggest that CC displays systematic variation in selected Islamic months. It is clear from the table that four selected Islamic months significantly affect CC in addition to normal seasonality explained by the differential coefficients of calendar month specific dummy variables. This result is

reinforced, as the log likelihood ratio test strongly rejects the joint hypothesis of zero restrictions on the coefficients of Islamic months.[2]

In our analysis, the base month is June (the value of intercept) and all others represent difference from the base. Specifically, the coefficient of $D_1$, 0.0264, states that holding all other effects constant, the average value of log of ratio of CC will be higher by the amount 0.0264 during the month of July. Given these differential coefficients, one can easily arrive at average value for each month by adding month specific differential coefficient to the base. This interpretation can be made more reader-friendly by specifying left hand side of the equation as follows:

$$D[Log(CC)] = Log(CC_t) - Log(CC_{t-1}) = Log(\frac{CC_t}{CC_{t-1}}) \cong \frac{\Delta CC}{CC} \qquad (3)$$

In this case, left hand side of the equation is simply the natural log of the ratio of CC. Original series is thus transformed into a chain of log ratios of currency stocks in consecutive months. Following Halvorsen and Palmquist (1980), we transform the differential coefficients to show differential effects in terms of percent change. The relative effect for each month is calculated with the help of exponential transformation and further multiplied by 100 to show percent changes as reported in last column of Table 1. For example, the relative effect for June: $g = (e^\alpha - 1)$, or in percent term: $g * 100 = (e^\alpha - 1) * 100$.

The relative effects represent percent changes in average monthly ratio of stock of CC. For example, the average ratio of stock of CC in July will be 2.67 percent higher than the average in the base (June) month. Similarly, this will be higher by 4.13 percent in Ramadhan than the average ratio of stock of CC in June (base).

Specifically, the average ratio of stock of CC will significantly change if the select Islamic month also falls in that month. For example, if the whole month of Ramadhan falls in July the average ratio of stock of CC will be higher by 4.13 percent in addition to 2.67 percent than the mean value in June. In totality, this will be higher by 6.8 (2.67+4.13) percent over the base month (June). Average relative changes in the stocks of CC for select Islamic months are reported in

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[2] The computed value of maximum likelihood test statistic 127.2 is far above the critical value of Chi-Square at 1 percent, i.e.13.3.

**Table 2. Effect of select Islamic months on the average ratio of CC**
(in percent)

| Months | Parameter | Simple | Muharrum | Ramadhan | Shawwal | Dhul-Hijja |
|---|---|---|---|---|---|---|
| June | $a_0$ | -1.33 | -5.44 | 2.80 | -5.75 | 0.58 |
| July | $\beta_1$ | 2.67 | -1.44 | 6.80 | -1.74 | 4.58 |
| August | $\beta_2$ | 0.09 | -4.02 | 4.22 | -4.33 | 2.00 |
| September | $\beta_3$ | 0.62 | -3.49 | 4.75 | -3.79 | 2.53 |
| October | $\beta_4$ | 4.35 | 0.24 | 8.48 | -0.07 | 6.26 |
| November | $\beta_5$ | 7.35 | 3.24 | 11.47 | 2.93 | 9.26 |
| December | $\beta_6$ | 3.79 | -0.32 | 7.92 | -0.63 | 5.70 |
| January | $\beta_7$ | 5.12 | 1.01 | 9.25 | 0.70 | 7.03 |
| February | $\beta_8$ | 3.14 | -0.97 | 7.27 | -1.28 | 5.05 |
| March | $\beta_9$ | 1.63 | -2.48 | 5.76 | -2.79 | 3.54 |
| April | $\beta_{10}$ | 1.51 | -2.61 | 5.63 | -2.91 | 3.42 |
| May | $\beta_{11}$ | 1.51 | -2.61 | 5.63 | -2.91 | 3.42 |

Table 2. These relative changes for illustration are worked out on the assumption that Islamic months will completely overlap with Gregorian calendar months. This usually will not be the case and the relative effect is distributed in two consecutive calendar months in accordance with the value of fractional indicator variable. For illustration, Table 3 presents these effects for the year 2001-2002 with

**Table 3. Monthly Relative Effect for 2001-2002**

| Months | Parameter | Simple | Adjustment | Total |
|---|---|---|---|---|
| June | $a_0$ | -1.326 | 0.000 | -1.326 |
| July | $\beta_1$ | 2.674 | 0.000 | 2.674 |
| August | $\beta_2$ | 0.091 | 0.000 | 0.091 |
| September | $\beta_3$ | 0.625 | 0.000 | 0.625 |
| October | $\beta_4$ | 4.350 | 0.000 | 4.350 |
| November | $\beta_5$ | 7.350 | 1.992 | 9.341 |
| December | $\beta_6$ | 3.792 | -0.223 | 3.569 |
| January | $\beta_7$ | 5.122 | -2.062 | 3.059 |
| February | $\beta_8$ | 3.142 | 0.988 | 4.130 |
| March | $\beta_9$ | 1.633 | -1.158 | 0.475 |
| April | $\beta_{10}$ | 1.506 | -2.127 | -0.622 |
| May | $\beta_{11}$ | 2.781 | 0.000 | 2.781 |

Note: Islamic Months and Gregorian Calendar

Ramadhan: 17th November to 16th December 2001.

Shawwal : 17th December to 14th Hanuary 2002

Dhul-Hijja: 14th February to 14th March 2002

Muharram: 15th March to 13th April

**Figure 2. Forecasting Strength**
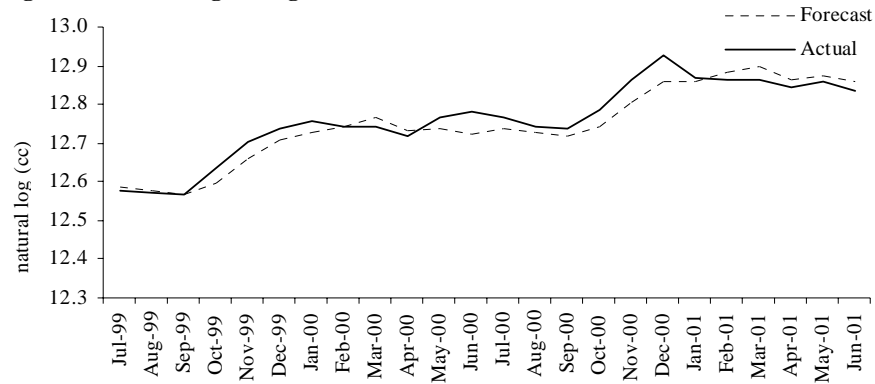


distributed coefficients. To gauge the forecasting power of the model we rerun the same specification on the data from July 1972 to June 1999, and estimate the forecast values for the last two years (July 1999 to June 2001). The actual and forecast values of CC (in natural log terms) are depicted in Figure 2, while the forecast performance statistics are reported in Table 4. Our model tracks the actual values quite impressively as evident from various forecasting performance statistics. More specifically, a very small value of Theil's Inequality Coefficient (0.0012) indicates good forecasting. Stepping back, the marginal forecast errors could be traced by looking at the last three proportion statistics. In total forecast error, bias proportion is just 17.0 percent, suggesting that the difference between means of forecast and actual values is very low. Similarly, variance proportion is also on lower side, indicating just 8.3 percent variation in actual and forecast variances. Covariance proportion is high that suggests that the major source of

**Table 4. Forecast Performance**

| | |
|---|---|
| Root Mean square Error | 0.032 |
| Mean Absolute Error | 0.026 |
| Mean Absolute Percent Error | 0.504 |
| Theil's Inequality Coefficients | 0.001 |
| Bias Proportion | 0.170 |
| Variance Proportion | 0.008 |
| Covariance Proportion | 0.822 |

**Figure 3. Forecasting for FY02**
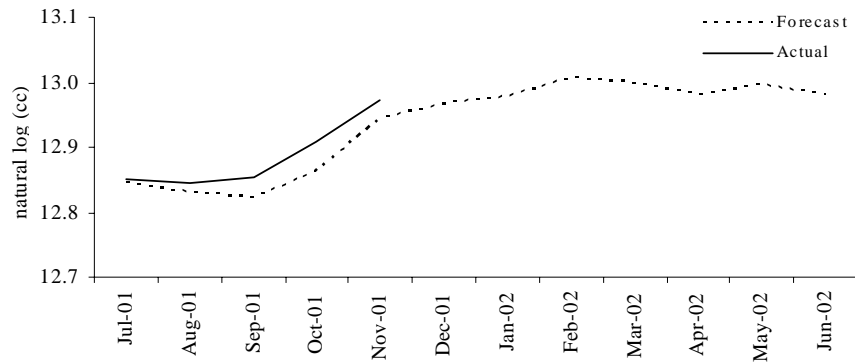


discrepancy in actual and forecast series is their imperfect co-variances. This microanalysis again highlights the strength of the model for forecasting purpose, as it efficiently tracks the actual mean and variance of the series.

We also used this model to forecast CC for the current fiscal year (July 2001 to June 2002). The actual values of CC for first quarter and forecast values for the year are shown in Figure 3.

## 4. Final Remarks

Application of this model and its results bear important implications for researchers and policy makers. At national level, central banks of Islamic countries can practically benefit by incorporating this method in improved phasing of their annual monetary targets into monthly targets thereby smoothing the implementation of their monetary polices. Systematic projection of CC is always an important part of liquidity forecasting mechanisms of central banks. Demand for liquidity can be projected with lower error in special Islamic months, as discussed earlier and subsequent injections or withdrawals through open market operations may lead to better targeting of monetary aggregates.

At the global level, international agencies can fine-tune their assistance programs, at operational level, for Islamic countries. For example, monetary programs can be designed more effectively by the IMF for Islamic countries through reduction in forecasting error of conditional variables. Countries with large segment of Muslims can also improve their projections of demand for goods and services in those regions during the Islamic months of significance.

We hope that this method will become the starting point for further research in development of methods of seasonal adjustment of time series compiled in calendar months but influenced by Islamic Hijri months. Modification of existing standard methods like X-12 ARIMA may also be possible in conjunction with implementable algorithms of conversions of Hijri dates into Gregorian dates.

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