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Abstract

In this paper, we compare the performance of different models, on two data frequencies, in terms of matching the business cycle moments of Pakistani economy. Out of the four models, two are simple real business cycle models for Pakistan introduced in Choudhary and Pasha (2013), and the other two are benchmark models [Aguiar and Gopinath (2007) and Garcia-Cicco et al. (2010)] from the literature for explaining the business cycles in emerging and developing economies. This paper calibrate these models for Pakistan and evaluate their performance in terms of matching second order moments from the actual data at both annual and quarterly frequency. We find that even though no single model is able to match all the relevant moments for all the important macroeconomic variables at both frequencies, the augmented RBC model with FDI shock (Choudhary and Pasha, 2013) performs relatively better.

Keywords: DSGE Model, Emerging Economies, FDI shock, Business Cycles
JEL Classification Codes: C61, D58, E32

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Non-technical Summary

In this paper, we compare the performance of different theoretical models in terms of their ability to explain short run fluctuations (business cycles) of Pakistan economy. The main objective of this research exercise is to identify a model capable of explaining business cycle dynamics for Pakistan - a country with specific economic features and limited information.

The four models considered in this paper are all on the lines of the Real Business Cycle (RBC) model. Indeed, the first is the RBC model introduced in the seminal work of Kydland and Prescott (1982). According to the simple RBC model, business cycle fluctuations can be explained by transitory shocks to technology/productivity. The second model is the augmented RBC model with FDI shock used in Choudhary and Pasha (2013) in their earlier attempt to find a model capable of capturing business cycles in Pakistan. The other two models are the ones introduced in Aguiar and Gopinath (2007) and Garcia-Cicco et al. (2010). Both of these are benchmark models in the literature for understanding and studying business cycles in developing/emerging economies.

We compare the performance of the two simple RBC models for Pakistan with the two benchmark models for emerging economies in the literature, one incorporating financial frictions as well (Garcia-Cicco et al. (2010)). On the basis of this study's moments matching exercise at both annual and quarterly frequency, we find that the augmented RBC model for Pakistan with FDI shock performs relatively well compared to the other models.

However, this does not mean that various nominal and real frictions or other sources of short run fluctuations are not important. The introduction of such important missing features based on sound microeconomic foundations may yet significantly improve the ability to model and understand business cycles in developing economies like Pakistan.

1. Introduction

This paper compares the performance of different models in explaining the business cycles of Pakistan economy. The literature on models explaining business cycles in advance economies is abundant but scant on developing /emerging economies - even more so for developing South Asian economy like Pakistan. The main reason for limited research on business cycles of developing economies in particular outside Latin America is that the basic economic features of developing economies are less known as compared to advance economies. As Choudhary and Pasha (2013) claim that the main stylized fact in case of Pakistan was the lack of any consistent stylized facts of the economy. Furthermore, high frequency and reliable macroeconomic data are generally not available for such economies. Therefore, there is a clear need for simple models that can explain business cycles for developing economies with limited information set like Pakistan.

In this paper, we compare the performance of four different types of RBC models in order to identify the best model which explains the short run fluctuation of Pakistan economy. The first two models have been discussed in detail in Choudhary and Pasha (2013). The first model is the simple Real Business Cycle model introduced by the seminal work of Kydland and Prescott (1982). According to the simple RBC model, business cycle fluctuation can be explained by transitory shocks to technology or productivity. The second model is the augmented RBC model with a FDI shock introduced for the first time in Choudhary and Pasha (2013).

There are two reasons for them to augment their model with FDI-specific shocks: 1) the FDI flows to Pakistan, though small, are highly volatile, and 2) the shock to FDI may be interpreted as technology shock because FDI may result in technology spillover. Thus introduction of alternative shocks may help their model to better account for macroeconomic fluctuations in Pakistan. There is vast literature showing the relevance of investment-specific technology shock for business cycles in both advance and emerging economies [Greenwood et al. (1988), Greenwood et al. (1997, 2000), Fisher (2006), Pakko (2002), Guerrieri et al. (2005), Schuch and Ireland (2008), Letendre and Luo (2007) and Araújo (2012)]. The augmented RBC model with FDI is closer to Araújo (2012) in that we introduce FDI-specific shock as “a part of total investment”.

The other two models evaluated for their ability to explain business cycles in Pakistan are relatively new but are already being considered as benchmark models for business cycles in emerging economies. One is by Aguiar and Gopinath (2007) and other is by Garcia-Cicco et al (2010). The Aguiar and Gopinath (2007) model explains the business cycles in both advanced and emerging economies with the help of both permanent and transitory shocks to productivity. Using data from Mexico and Canada and separately calibrating their model for both countries, they find that the permanent and transitory productivity shocks better account for the business cycle dynamics in Mexico and Canada respectively. However, Garcia-Cicco et al (2010) criticizes the use of short sample by Aguiar and Gopinath (2007) to identify the permanent productivity shift. With the expanded data span of 1900-2005, they find that Aguiar and Gopinath (2007) model is not capable of explaining macroeconomic fluctuations for Mexico and Argentina. They develop a different model and calibrate and simulate it for Argentinean economy. Their results show that their model performs well in case of Argentina. They conclude that, in emerging economies, permanent technology shock may not be solely responsible for generating short run fluctuations as generally assumed by the RBC literature.

One of the features of this paper is that we evaluate the performance of the four models in replicating the business cycles of Pakistan over different frequency and time horizons-mostly to remain consistent with existing benchmark models and research in the literature. Therefore, we compare the second order moments of simulated series from the Aguiar and Gopinath (2007) model and our two models for Pakistan economy with empirical moments from quarterly data from 1981 to 2003. Empirical moments for annual data from 1961 to 2005 are used to compare second order moments of two simple models discussed in Choudhary and Pasha (2013) with Garcia-Cicco et al. (2010) model for Pakistan.

The rest of this paper is organized as follows: the next section describes briefly the four models that we are using in this paper. Section 3 discusses calibration of various parameters used to simulate the different models. Section 4 briefly discusses the empirical moments for different frequencies and time span. Section 5 reports the findings on the moment matching exercise comparing the simulated moments from the four models and actual data moments. The last section concludes.

2. The Models

In this section, we briefly describe the four models calibrated and evaluated for matching Pakistan economy business cycles over the last few decades The detailed background, motivation, assumptions and all equations for each model is presented in the respective paper. We only cover the basic set up of each of these models here.

2.1. Simple RBC Model

In Choudhary and Pasha (2013), they compare the performance of two RBC models; this model is a simple version of the Real Business Cycles model introduced by Kydland and Prescott (1982) in their seminal article.

Our simple RBC model is a closed economy model with representative agent maximizing utility over an infinite horizon.

$$\max E_t \sum_{i=0}^{\infty} \beta^t \left[\ln C_t + \psi \frac{(1-N_t)^{1-\sigma}}{1-\sigma} \right]$$

Where β is the discount factor, Ct is consumption, Nt represents the number of hours spent working, -1/σ is the Frisch elasticity of leisure and ψ is coefficient on leisure.

Production is modeled using Cob-Douglas functional form for this economy.

$$Y_t = K_t^{1-\alpha}(A_t N_t)^{\alpha}$$

Where Y_t is aggregate output, Kt is the aggregate physical capital sock, Nt represents the number of hours spent working, At is technology and 1-α is the share of capital in production.

The aggregate resource constraint for our closed model economy is given below.

$$Y_t = C_t + I_t$$

The typical physical capital accumulation equation is given below.

$$K_{t+1} = (1-\delta)K_t + I_t$$

Finally, exogenous technology follows an AR process as below.

$$\ln(A_{t+1}) = \rho_A \ln A_t + \varepsilon_{A,t}$$

Where ρ_A is the persistence of the exogenous technology shock and $\varepsilon_{A,t}$ is the standard error associated with this shock.

2.2. Augmented RBC Model with FDI Shock

In addition to evaluating the benchmark RBC model for Pakistan Choudhary and Pasha (2013) also introduced an augmented RBC model with inclusion of FDI as an exogenous variable in the closed economy setup. They incorporated an exogenous FDI shock in their augmented RBC model in addition to the typical technology shock found in most RBC models. The augmented model is identical to the simple RBC model covered in the previous sub section in terms of the utility function, production technology and the aggregate resource constraint. Therefore, in this subsection we only highlight the novel features of the augmented model.

The aggregate investment in the augmented RBC model is a composite of domestic and foreign components. We introduce an exogenous FDI shock in our closed economy model, the main innovation in our model. This makes our model isomorphic to RBC model with investment- specific technology shock as studied by Greenwood et al (1988) and Greenwood et al (1997). The main difference between their model and ours is that they use the relative price of investment as the exogenous shock to aggregate investment and we use FDI. However, the propagation mechanism in both models works in a very similar manner.

Physical capital accumulation in this model follows the following process:

$$K_{t+1} = (1-\delta)K_t + I_t(I_t^*)$$

Where It* here represents the Foreign Direct Investment variable. The reason Choudhary and Pasha (2013) provided for introducing FDI in their augmented model was the empirical evidence supporting a non-trivial role of FDI with business cycle fluctuations of the Pakistan economy[1].

In order to model the two types of investments - domestic and foreign - they assume complementary relationship between them. However, they acknowledge the possibility that both domestic and foreign investments might respond to an underlying common shock process, such as an investment-specific technology shock, changes in tax or regulatory structure, business confidence, stability, political change etc., which drives investments decisions.

Finally, in order to remain consistent with their closed economy set up and for simplicity, they modeled foreign component of investment as an exogenous shock (like a typical exogenous technology shock is incorporated in these models).

$$\ln(I^*{}_{t+1}) = \rho_{I^*} \ln I^*{}_t + \varepsilon_{I^*,t}$$

Where ρ_{I^*} is the persistence of the exogenous shock and $\varepsilon_{I^*,t}$ is the standard error associated with the shock.

The technology shock, as in the simpler version of the RBC model, follows AR process as below.

$$\ln(A_{t+1}) = \rho_A \ln A_t + \varepsilon_{A,t}$$

[1] See Choudhary and Pasha 2013 for detail

Where ρ_A is the persistence of the exogenous technology shock and $\varepsilon_{A,t}$ is the standard error associated with this shock.

2.3. Aguiar and Gopinath Model

The model takes account of the facts that (i) GDP growth, consumption and net exports are much more volatile in emerging markets (ii) the movements of trade balance are strongly countercyclical in emerging markets as compared to developed markets and (iii) frequent regime switches (policy changes) occur in these markets. Resultantly, the authors suggest that the shocks to trend growth are the primary source of fluctuations in these markets as opposed to transitory fluctuations around the trend. They augment standard single good, single asset, small open economy model by separating transitory and trend component of productivity shocks.

Following Cobb-Douglas production function is assumed:

$$Y_t = e^{z_t} K_t^{1-\alpha} (\Gamma_t L_t)^{\alpha}$$

Where z_t and Γ_t represent productivity process given by

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z$$

Where $|\rho_z| < 1$ and ε_t^z is a normally distributed random shock with zero mean and standard deviation σ_z.

$$\Gamma_t = e^{g_t} \Gamma_{t-1} L_t = \prod_{s=0}^{t} e^{g_s}$$

Where parameter g_t is growth of productivity

$$g_t = (1-\rho_g)\mu_g + \rho_g g_{t-1} + \varepsilon_t^g$$

Where $|\rho_g| < 1$ and ε_t^g is normally distributed random shock with zero mean and standard deviation σ_g. The term μ_g is average productivity growth in the steady state.

The period utility is represented by following Cobb-Douglas form

$$u_t = \frac{(C_t^{\gamma}(1-L_t)^{1-\gamma})^{1-\sigma}}{(1-\sigma)}$$

The parameter $\gamma \in (0,1)$

Household faces following single period budget constraint:

$$C_t + K_{t+1} = Y_t + (1-\delta)K_t - \frac{\phi}{2}\left(\frac{K_{t+1}}{K_t} - e^{\mu_g}\right)^2 K_t - B_t + q_t B_{t+1}$$

Where qt represents the price of debt and is sensitive to outstanding debt. We can write as

$$\frac{1}{q_t} = 1 + r_t = 1 + r^* + \varphi\left(e^{\frac{B_{t+1}}{\Gamma_t} - d} - 1\right)^2$$

Where r* is the world interest rate, d is steady state debt level, and parameter ϕ>0 is interest rate elasticity with respect to debt.

The representative agent's optimization problem takes the form

$$V(\hat{K},\hat{B},z,g) = \max_{\hat{C}',\hat{L}',\hat{K}',\hat{D}'} \left\{ \frac{(C_t^{\gamma}(1-L_t)^{1-\gamma})^{1-\sigma}}{(1-\sigma)} + \beta e^{g\gamma(1-\sigma)} EV(\hat{K}',\hat{B}',z',g') \right\}$$

Subject to following constraint

$$\hat{C} + e^g \hat{K}' = \hat{Y} + (1-\delta)\hat{K} - \frac{\phi}{2}\left(e^g \frac{\hat{K}'}{\hat{K}} - e^{\mu_g} \right)^2 \hat{K} - \hat{B} + e^g q \hat{D}'$$

The capital stock evolves as given below.

$$e^g \hat{K}' = (1-\delta)\hat{K} - \frac{\phi}{2}\left(e^g \frac{\hat{K}'}{\hat{K}} - e^{\mu_g} \right)^2 \hat{K} + \hat{X}$$

Given the initial levels of capital stock and debt , the equilibrium of the economy is characterized by the first-order conditions obtained from household utility maximization problem, the technology and budget constraints, along with the transversality conditions[2].

2.4. Garcia-Cicco Model

The Garcia-Cicco et al (2010) criticize the use of short samples for the characterization of observed business cycles as well as the estimation of the parameters of the theoretical model. They use long sample and show that the RBC model driven by permanent and transitory productivity shocks perform poorly in explaining the business cycles in Argentina and Mexico.

They augment the baseline RBC model by introducing preference shocks, country-premium shocks, and a realistic debt elasticity of the country premium. The latter two features are introduced to capture the international financial frictions which a small open economy is prone to. They show that their model does a better job of explaining the business cycles in Argentina. The production technology is given by the following function.

$$Y_t = a_t K_t^{\alpha} (X_t h_t)^{1-\alpha}$$

The productivity shock a_t is assumed to follow a first-order autoregressive process given below.

$$\ln a_{t+1} = \rho_a \ln a_t + \varepsilon_{t+1}^a$$

Here

$$\varepsilon_t^a \approx N(0,\sigma_a^2).$$

And the productivity shock Xt is non-stationary and its gross growth is denoted by:

$$g_t = \frac{X_t}{X_{t-1}}$$

[2] See Aguiar and Gopinath(2007) for more detail.

Where gt follows the process given as:

$$\ln(g_{t+1}/g) = \rho_g \ln(g_t/g) + \varepsilon^g_{t+1}$$

Where $\varepsilon^g_t \approx N(0, \sigma^2_g)$.

Household is subject to a no- Ponzi game constraint given as:

$$\lim_{j\to\infty} E_t(B_{t+j} / \Pi^j_{s=0}(1+r_s)) \leq 0$$

Household maximizes following utility function over an infinite time horizon

$$\max E_0 \sum_{t=0}^{\infty} \nu_t \beta^t \frac{[C - \theta\omega^{-1} X_{t-1} h^{\omega}_t]^{1-\sigma} - 1}{1-\sigma}$$

The variables ν_t represents an exogenous and stochastic preference shock following the AR (1) processes.

$$\ln \nu_{t+1} = \rho_\nu \ln \nu_t + \varepsilon^\nu_{t+1}$$

$$\varepsilon^\nu_t \approx N(0, \sigma^2_\nu).$$

Budget constraint faced by household is given below:

$$\frac{B_{t+1}}{1+r_t} = B_t - Y_t + C_t + S_t + I_t + \frac{\phi}{2}\left(\frac{K_{t+1}}{K_t} - g\right)^2 K_t$$

Here S_t represents domestic spending shock that follows AR (1) processes.

$$\ln(s_{t+1}/s) = \rho_s \ln(s_t/s) + \varepsilon^s_{t+1}$$

$$\varepsilon^s_t \approx N(0, \sigma^2_s).$$

The capital stock follows the law of motion presented below:

$$K_{t+1} = (1-\delta)K_t + I_t$$

The country premium, an increasing function of aggregate debt takes the following form:

$$r_t = r^* + \varphi(e^{\frac{b_{t+1}}{X_t} - b} - 1) + e^{\mu_t - 1} - 1$$

The household maximizes his life time utility, taking a_t, X_t and r_t as given process and initial conditions on capital and debt and subject to production function, budget constraint, no-Ponzi game constraint and capital accumulation constraint[3].

[3] See for more detail Garcia-Cicco et al (2010)

3. Parameters Calibration

We calibrate and simulate models on both quarterly and annual frequencies to keep consistency with exiting benchmark literature in the context of business cycles in emerging economies. In particular Garcia-Cicco et al (2010) calibrate and simulate their models at an annual frequency, while Aguiar and Gopinath (2007) use quarterly frequency in their work. In this paper, we compare the two models introduced in Choudhary and Pasha (2013) at annual frequency with the model introduced in the Garcia-Cicco et al. (2010); and at a quarterly frequency with Aguiar and Gopinath (2007) model. Therefore, we need to assign calibrated parameters for both quarterly and annual frequency.

The values of calibrated/estimated parameters are presented in Table 1. The values of parameters α, β, δ, σ, ω, γ, ϕ, and θ have been taken from the existing literature on Pakistan as well as similar studies on other emerging economies[4]. We set the values of μ_g, output growth as 1.04 on annual while 1.01 on quarterly bases in line with the data. Similarly the value of parameter d is set to 0.60 which is the average debt to GDP ratio for Pakistan over the selected period. The value of φ is also estimated from annual data on Pakistan[5]. We estimate the value of the parameters ρ_g, ρ_z, σ_g, and σ_z using Solow residual obtained from estimated production function for Pakistan. We utilize the approach given in Aguiar and Gopinath (2007) to estimate parameters directly. For the parameters related to preferences shock ρ_v & σ_v, domestic saving shock ρ_s & σ_s, and country premium shock ρ_μ & σ_μ we use the values from Garcia-Cicco et al (2010) as no information is available on these parameters for Pakistan.

Table 1: Calibrated/Estimated values of parameters

Parameter	Value		Parameter	Value	
	Quarterly	Annual		Quarterly	Annual
α	0.50	0.50	σ_μ	0.057	0.46
β	0.98	0.95	ρ_g	0.94	0.94
δ	0.025	0.10	ρ_z	0.83	0.83
σ	1.50	1.50	σ_g	0.01423	0.01423
φ	0.002	0.004	σ_z	0.005	0.005
ω	----	1.60	ρ^v	-----	0.85
γ	1	0.15	ρ_s	-----	0.21
ϕ	4	3	σ^v	-----	0.54
μ_g	1.01	1.04	σ_s	-----	0.019
ρ_a	0.82	0.90	θ	-----	2.25
ρ_μ	0.91	0.93	d	0.60	0.60
σ_a	0.015	0.02			

4. Empirical Moments

In this section, we briefly discuss the empirical moments of Pakistani economy for selected period at both annual and quarterly frequency. The time period, macroeconomic variables, de-trending approach and the reporting style has been kept as close to the two benchmark papers (Garcia-Cicco et al. (2010) and Aguiar and Gopinath (2007)) as possible. The idea is that we give those models their best chance in terms of their ability to explain business cycles for an emerging economy such as Pakistan.

[4] See for example Choudhary and Pasha 2013.
[5] The quarterly value of this parameter is taken smaller assuming that interest rate is less elastic with respect to external debt at higher frequency.

4.1. Annual Empirical Moments (1961-2005)

The annual empirical moments reported in Garcia-Cicco et al (2010), for Mexico and Argentina are for the years 1900 to 2005. One of the more novel aspects of their work is the use of this extended time period for computing empirical business cycle moments for emerging economies. They are the first authors to report such extended macroeconomic data going back more than a century for emerging economies and have reshaped the direction of the literature on business cycles in emerging economies moving forward. In order to follow their approach and utilize longest possible period for computing annual second order moments from the Pakistani data we use the data for the period 1961-2005. The reason for starting from 1961 is quite simply the fact that it is the first year for which data is available for private consumption and gross fixed capital formation (investment) for Pakistan.

Table 2: Annual Empirical Second Moments for Pakistan (1961-2005)

	Symbol	Pakistan Data (1961-2005)
	Volatility	
Growth Rate of Output	$\sigma(g^Y)$	2.22
Growth Rate of Consumption	$\sigma(g^C)$	3.08
Growth Rate of Investment	$\sigma(g^I)$	8.29
Trade Balance-to-Output Ratio	$\sigma(TBY)$	3.80
	Correlation with g^Y	
Growth Rate of Consumption	$\rho(g^Y, g^C)$	0.66
Growth Rate of Investment	$\rho(g^Y, g^I)$	0.38
Trade Balance-to-Output Ratio	$\rho(g^Y, TBY)$	-0.12
	Correlation with TBY	
Growth Rate of Consumption	$\rho(g^C,TBY)$	-0.18
Growth Rate of Investment	$\rho(g^I,TBY)$	-0.24
	Serial Correlation	
Growth Rate of Output	$\rho(g^Y)$	0.16
Growth Rate of Consumption	$\rho(g^C)$	0.10
Growth Rate of Investment	$\rho(g^I)$	0.13
Trade Balance-to-Output Ratio	$\rho(TBY)$	0.85

Table 2 displays empirical second order moments of output growth, consumption growth, investment growth and the trade balance to output ratio for Pakistan from 1961 to 2005. From the first block of the table we can see that consumption growth, investment growth and trade balance to output ratio are all more volatile than the per capita output growth. This is consistent with findings for Argentina and Mexico from the Garcia-Cicco et al (2010) paper as well as other literature on business cycle stylized facts for developing and emerging economies.

The correlation of both consumption growth and investment growth is positive with output growth. However, the correlation is stronger with consumption growth over the selected period than investment. On the other hand, there is a negative but insignificant relationship between trade balance to output ratio and output growth. Furthermore, both consumption and investment growth are negatively correlated with trade balance to output ratio.

The last block of Table 2 shows the serial correlation of the four macroeconomic series. The first order autocorrelations of output growth, consumption growth and investment growth are small but positive. In contrast, the trade balance to output ratio is significantly persistent.

4.2. Quarterly Empirical Moments (1981-2003)

The quarterly empirical moments reported in Aguiar and Gopinath (2007) for both developing and developed economies were for the period 1981-2003. In order to stay as close to their approach as possible, given our main goal for this paper, we also use the same period. Here it is important to mention that there is no official quarterly series available for output, consumption and investment for Pakistan. Therefore, we use the quarterly series of the relevant macroeconomic variables reported by Hanif et al (2013) for computing the quarterly empirical second moments for Pakistan.

Table 3: Quarterly Empirical Moments for Pakistan (1981-2003)

	Symbol	**Data (1981-2003)**
	Volatility	
Output	σ(y)	1.61
Output Growth	σ(Δy)	2.14
	Relative Volatility with Output	
Consumption	σ(c)/σ(y)	1.52
Investment	σ(I)/σ(y)	2.70
Net Exports	σ(NX)/σ(y)	1.40
	Serial Correlation	
Output	ρ(y)	0.10
Output Growth	ρ(Δy)	-0.35
	Correlation with Output	
Net Exports	ρ(y, NX)	0.36
Consumption	ρ(y, c)	0.52
Investment	ρ(y, I)	0.05

The moments reported in Table 3 have been computed after seasonally adjusting the quarterly series as well as de-trending the seasonally adjusted series by using Hodrik Prescott filter with λ =1600. The volatility block of the above table shows that output growth during 1981-2003 was more volatile than the de-trended output. Furthermore, cyclical component of consumption, investment and net exports are all more volatile than cyclical components of the output. This is consistent with the finding for emerging markets from Aguiar and Gopinath (2007); however, the magnitude is different for Pakistan as both investment and net exports are relatively less volatile compared to other emerging countries.

The middle block of Table 3 reports the autocorrelation for both de-trended output and output growth. Both serial correlations are low and in particular the autocorrelation coefficient for output growth is negative and not insignificant. This alludes to volatile nature of Pakistani economy during the selected period i.e. 1981 to 2003. This result is also quite contradictory when compared to empirical findings for emerging economies in the Aguiar and Gopinath (2007); as they found ρ(y) and ρ(Δy) to be 0.76 and 0.23 respectively.

Finally, the last block of the above table shows the correlation between de-trended net exports, consumption and investment with de-trended output. We find some puzzling but interesting results here. In particular, we find no significant relationship between de-trended investment and de-trended output. This finding is quite odd when compared to the literature for both emerging economies and Pakistan. This can be due to changing dynamics of investment during the period under study as well as the fact that actual quarterly data is not available. We also find positive co-movement between de-trended net exports and de-trended output as well as for de-trended consumption and de-trended output.

5. Results

In this section, we present the second order simulated moments from different models and compare them with their counterpart from the data. Table 4 reports the annual simulated moments obtained by the simple and augmented RBC models and the Garcia-Cicco et al. (2010) model.

The simple RBC model overestimates the volatility of GDP per capita growth while underestimates the volatility of consumption growth and investment growth as compared to their empirical counterpart. Consumption growth is less volatile relative to output growth in this model. The relative volatility of investment growth to GDP growth is smaller as compared to its counterpart from data. The respective simulated moments from the augmented RBC model are relatively closer to those shown by the data. However, it underestimates the relative volatility of consumption growth to GDP growth. Overall, augmented RBC model seems to outperform the other two models in terms of matching the volatility of output per capita growth and consumption per capita growth and does quite well for investment growth as well. The Garcia-Cicco et al (2010) model predicts lager volatility for all the three variables comparative to data. The relative volatility of both consumption growth and investment growth to GDP growth as reported by simulated moments of this model is much higher than reported empirical evidence as well as from the other two models.

In term of correlation of other variables with GDP growth, all models perform poorly. The simple RBC model shows excessive association of consumption growth and investment growth with GDP growth as compared to those shown by data. The other two models show negative correlation of these variables with GDP growth in contrast to the values implied by the data. The Garcia-Cicco et al (2010) model, however, is able to predict negative correlation between ratio of trade balance to GDP and GDP growth closer to value calculated from the data. It is also successful in generating negative correlation between ratio of trade balance to GDP and investment growth, but miss out on the negative correlation between ratio of trade balance to GDP and consumption growth.

In terms of serial correlations of GDP growth and consumption growth augmented RBC model clearly outperforms the other two models and the reported simulated moments from the model are quite close to actual data moments. Interestingly, all three annual models report negative autocorrelation for investment growth which is different from positive but very small autocorrelation for investment growth reported in the data. All in all, augmented RBC model outperforms the other two models in terms of matching the simulated volatility and serial correlation moments with actual data moments for Pakistani economy at an annual frequency for data from 1961-2005.

Table 4: Annual Moments of Data and Models

Statistic		g^Y	g^C	g^I	tby
Volatility	Data	2.22	3.08	8.29	3.80
	Simple RBC Model	3.06	1.27	7.12	------
	Augmented RBC Model	2.26	2.10	10.66	------
	Garcia-Cicco et al (2010) Model	2.87	10.42	20.42	10.75
Correlation With g^Y	Data	-----	0.66	0.38	-0.12
	Simple RBC Model	-----	0.89	0.99	------
	Augmented RBC Model	------	-0.19	0.86	------
	Garcia-Cicco et al (2010) Model	------	0.16	-0.18	-0.39
Correlation With TBY	Data	------	-0.18	-0.24	------
	Simple RBC Model	------	------	-------	------
	Augmented RBC Model	------	------	------	------
	Garcia-Cicco et al (2010) Model	------	0.42	-0.45	------
Serial Correlation	Data	0.16	0.10	0.13	0.85
	Simple RBC Model	0.00	0.39	-0.07	------
	Augmented RBC Model	0.11	0.18	-0.06	------
	Garcia-Cicco et al (2010) Model	0.70	-0.04	-0.12	0.58

Note: g^Y, g^C, and g^I denote the growth rates of output per capita, consumption per capita, and investment per capita, respectively, and tby denotes the trade balance-to-output ratio. Empirical moments are computed using Pakistan data from 1961 to 2005. Standard deviations are reported in percentage points.

Table 5 below presents the simulated moments generated by different models i.e. simple RBC, augmented RBC and Aguiar and Gopinath (2007) model at quarterly frequency. We use HP filtered series for all variable presented here. The standard deviation of GDP predicted by simple RBC model is closer to the value obtained from data. However, the augmented RBC model beats other models in matching the relative volatility of consumption to GDP, which is considered one of the most important distinguishing features of developing economies in the literature. The relative volatility of investment computed from simulated series of investment and output from both the Augmented RBC and Aguiar and Gopinath (2007) model are quite close to the actual data counterpart. The relative volatility of net export to GDP implied by Aguiar and Gopinath (2007) model is also generally in line with data[6].

All models depict positive correlation between consumption and GDP. However, the simple RBC model overstates this positive association, while simulated correlation between de-trended consumption and de-trended output is quite close to the actual reported correlation for both

[6] The other two models represent closed economy.

Augmented RBC and Aguiar and Gopinath (2007) models. All models predict positive correlation between investment and GDP but overestimate the coefficient by sizable difference. As mentioned in the previous section, the insignificant correlation between investment and output is quite puzzling and can be due to changing nature of the economy or data related issues. Aguiar and Gopinath (2007) model generates positive correlation between net export to GDP and GDP but the magnitude is lower than what is reported from data.

The auto correlation of GDP is also over estimated by all models but with correct sign. The Aguiar and Gopinath (2007) model fails to match the negative auto- correlation of GDP growth. In contrast, both RBC models do better job in producing negative and compatible magnitude for this moment.

Table 5: Quarterly Moments of Data and Models

Statistic		**Data**	**Simple RBC Model**	**Augmented RBC Model**	**Aguiar and Gopinath Model**
Volatility/ Relative volatility	σ(y)	1.61	1.41	2.05	2.79
	σ(Δy)	2.14	1.18	1.69	1.90
	σ(c)/σ(y)	1.52	0.21	1.14	0.74
	σ(I)/σ(y)	2.70	3.31	2.91	2.85
	σ(NX)/σ(y)	1.40	-	-	0.99
Correlation with Output	ρ(y, NX)	0.36	-	-	0.07
	ρ(y, c)	0.52	0.88	0.63	0.64
	ρ(y, I)	0.05	0.64	0.60	0.68
Serial Correlation	ρ(y)	0.10	0.65	0.66	0.79
	ρ(Δy)	-0.35	-0.11	-0.13	0.14

Note: Empirical moments are computed using Pakistan data (HP filtered Quarterly) from 1981 to 2003. Standard deviations are reported in percentage points.

In sum, it is hard to identify a single model that excels in moments matching exercise for all variables and all moments. Each model surpasses others in one aspect but lags behind in other aspects. It depends upon the objective the model is intended to achieve. In general the performance of models calibrated and simulated on quarterly basis, in replicating the stylized facts, is better than the same exercise using annual models. However, it is quite clear that overall performance of the Augmented RBC model with FDI shock introduced in Choudhary and Pasha (2013) is relatively superior to others at both annual and quarterly frequencies. However, it does not necessarily mean that this model may outperform others in case of other economies too. The performance of any model may vary from economy to economy, over different time horizons as well as frequencies. Furthermore, our research raises the need to develop a single measure of "goodness of fit" that can help in ranking different models in terms of their ability to explain business cycles in different economies for different time periods.

6. Conclusion

In this paper, we compare the performance of different models, on two different frequencies, in terms of their ability to explain business cycles in Pakistan. The broad objective here is to identify a model capable of explaining the business cycle dynamics for a country like Pakistan with specific economic features and with limited information. We compare the performance of the two quite simple existing

RBC models for Pakistan to two benchmark models for emerging economies, one incorporating financial frictions as well. On the basis of our moments matching exercise, we find that the augmented RBC model for Pakistan with exogenous FDI shock performs well vis-à-vis the other models studied in this paper. However, we do not claim that various nominal and real frictions or other sources of short run fluctuations are not important. The introduction of these missing features based on sound microeconomic foundations may yet significantly improve our ability to model and understand business cycles in developing economies such as Pakistan.

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