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Estimation of Output Gap for Pakistan

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Abstract

A comprehensive exercise is conducted for the estimation of potential output and output gap for Pakistan while considering shortcomings of existing relevant literature. A number of approaches, combining both state-space and structural estimation have been employed for this purpose. These include Bayesian inference, multivariate filter method, vector auto regression (with identification restrictions), state-space model and univariate filtering for estimation of output gap and potential output. The study finds fall in potential output growth of Pakistan during FY09 – FY13, has increased the economy's vulnerability by making it more susceptible to demand shocks. Forecast of output gap on quarterly and annual frequencies for FY17 is also presented portraying upbeat aggregate demand going forward.

Keywords: State–Space Models, Potential Output, Output Gap.

JEL Classification: E10, C31, E52.

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Non-technical Summary

This paper presents business cycle information, especially output gap and potential level of output for Pakistan. Comparison of actual output with potential level has valuable policy information. Availability of robust estimate on output gap is directly relevant to monetary policy formulation as monetary policy deals with price stability and management of variability in macro economy in short-run to steer the economy towards a sustainable long-run path.

Output gap is vital component linking the real economy with trends in inflation and its expected path in the near term. Such information is useful for macroeconomic managers especially in the formulation of monetary policy. Output gap is also linked closely with movements along the business cycles: period of relatively high resource utilization is arguably characterized by tendencies of inflationary pressures and vise versa. Therefore, virtue of output gap as a tool for effective policy formulation is that intensity of resource use in an economy is linked closely with inflationary pressures. Especially under the circumstance that policies implementation bears its desirable impacts with a lag.

In this article we estimate and analyze trend and cyclical components of real GDP of Pakistan. We employ statistical time-series disaggregation methods based on macroeconomic indicators to disaggregate annual real GDP of Pakistan on quarterly frequency. Thereafter, utilizing several econometric methods we estimate the output gap and potential level of output on quarterly and annual data frequencies.

Estimates of output gap using methodologies utilized in this paper are less susceptible to revision in data and increase in the sample size, which is a highly desirable property from the perspective of use in economic policy analysis. Quarterly estimates of potential GDP of Pakistan over time show that growth in the productive capacity has slowed down over the last three decades. Marked decline in potential output growth increases the vulnerability of Pakistan economy and makes it more susceptible to the demand shocks, resulting in somewhat corresponding volatility in inflation.

Going forward, aggregate demand in Pakistan is expected to regain momentum given recent improvement in potential output.

1. Introduction

Potential output and output gap are two concepts that are important to macroeconomic analysis because they are related to different time horizon of economic dynamics. Potential output measures long-term movements associated with the economic growth while output gap captures short-term fluctuations (Hall and Taylor, 1991). Accordingly, institutions responsible for enforcing stabilization policy monitor output gap and its relative evolution, along with movements in business cycle and expectations regarding inflation. Kuttner (1994) original model relates the gap to inflation incorporating a Phillips curve equation in the model. Although somewhat atypical since it involves regression on unobserved variable, this model has entertained definite popularity. Kichian (1999) and Gerlach and Smets (1999) apply this methodology, with some variations, to G7 countries highlighting the analysis with respect to utility in policy making process. In addition, the European Commission uses similar specification for estimating structural unemployment, see Planas et al. (2008). OECD, also, considers a closely related version for estimating the Non Accelerating Inflation Rate of Unemployment (NAIRU), see Planas et al. (2008).

Information regarding trend output or sustainable level of output is rather difficult to pinpoint. This is due to two distinct reasons; a) trend level of output is a latent or unobservable variable (or concept) and b) characterization of trend of total national output is subjective based on how the long term sustainable level of national income is defined. Relevant literature categorizes two distinct approaches to unravel this lemma. First, purely statistical or (atheoretic) filter methods can be employed to iron-out the cyclical and irregular pattern in the actual observed output to obtain the trend level of output. Furthermore, inclusion of macroeconomic relationships in defining a model based framework for extracting the trend and cyclical components can also be used to obtain estimates of output gap and potential output.

Estimates of potential output and output gap obtained from sound approaches incorporating economic theory and robust methods have a desirable property of being less sensitive to revision of estimates as more data becomes available, see Michal (2013). It is noteworthy however that some quadratic minimization routines, exponential smoothening methods and spectrum filtering methods (collectively these can be called atheoretic, filter methods) have restrictive limitations for applicability. These methods are based on statistical smoothening procedures; often arbitrarily imposing a frequency and amplitude band in the frequency domain, at the same time ignoring the economic theory input about the time series ; while prone to limitations, these filter methods are cost and time effective. Borio et al. (2013), Hall and Taylor (1991) and Conway and Hunt (1993) contain extensive commentary on use and abuse of statistical methods for output gap estimation. [1]

Typically, literature identifies potential output with respect to optimal utilization[2] of factors and dynamics of price inflation, see Okun (1962). However, it can be argued that, in a typical economy, actual level of output as well as its long run trajectory is also influenced by movement in other macroeconomic variables, like unemployment, financial sector health, capacity utilization in the economy, to name just a few.

Indeed, favorable as well as adverse, transient and sustained movements in many macroeconomic variables characterize the dynamics of the economy, the interaction of which leads to booms and busts of the business cycles. This is important information which can be modeled in a macroeconomic (or

[1] Examples of filter methods include Hodrick and Prescott (1997), Baxter and King (1999), Christiano and Fitzgerald (2003) and numerous frequency domain filter methods.
[2] Optimal utilization of factor input is analogous to natural levels or steady state levels of factor input.

reduced form) settings to extract the potential output and output gap. Such 'economic model specific' interpretation of the long and short term trajectory of the economy appears in literature in varying degrees of complexity. Kuttner (1994) employed a Phillips curve relationship, along with an equation illustrating actual (observed) output composed of trend and cyclical component for extracting potential output as a latent variable. Benes et al. (2010) employ a more rigorously defined, small-scale macroeconomic model for this purpose. Succinctly, they model movements in unemployment, price inflation and capacity utilization to impact the long run trajectory of total output.

The existing literature tends to define potential output with reference to the full (economic) utilisation of factor inputs and to inflation developments, e.g. Okun (1962). The basic idea is that, all else equal, inflation tends to rise (fall) when output is above (below) potential. Inflation above long term sustainable levels, in other words, is one of the key indicators of macroeconomic unsustainability.

Several researchers have used multivariate models to estimate output gap. Benes and N'Diaye (2004) applied a similar model with calibrated parameters to construct output gap series for the Czech Republic. Kuttner (1994) estimated a simpler model relating the output gap to inflation through the Phillips curve using the classical (frequentist) methodology. The same model was replicated by Kichian (1999) for G7 countries, and popularized by Gerlach and Smets (1999). Apel and Jansson (1999) extended the model to include unemployment. The European Commission used it to estimate structural unemployment (Planas et al., 2008), and the OECD applied a closely related version for estimating the Non Accelerating Inflation Rate of Unemployment (NAIRU) (see OECD, 2000).

Estimation of trend and cycle GDP using multivariate filter method implemented at central banks of Canada and New Zealand in the 1990s, had two specific advantages: (a) multivariate filters incorporated economic theory to model building and estimation – see Conway and Hunt (1997), (b) these methods exhibited superior out of sample forecasting capability in case of expected future path of inflation. In line with the studies cited above, Benes and N'Diaye (2004) employ modified Multivariate HP filter method based on a small macroeconomic model utilizing dynamic relationships between the variables; capacity utilization, unemployment, inflation and real GDP to extract the output gap. The study concludes by highlighting robustness of estimates for various countries.

Output gap estimates are widely extracted as structural shocks from VAR models using structural restrictions as in Blanchard and Quah (1989). Structural representation of VAR is utilized by imposing long run restrictions on dynamic interaction of unemployment, inflation and total real output to estimate the potential output, see Scott (2000). Other studies utilizing similar model and identification restrictions with some improvement iterations include Ding et al. (2014) and Cotis et al. (2010).

State–Space methodology (or Unobserved Components method) is also widely implemented for estimation of output gap. Clark (1987) and Harvey and Trimbu (2008) estimated output gap using this methodology and concluded positively with respect to robustness of estimates. More recently Borio et al. (2014) conducted estimation of output gap and potential output for various countries, highlighting the relationship between output gap estimates and movements along the business cycles.

Worthwhile to note here is that state-space method, various modifications in Hodrick Prescott filter technique and multivariate filter methods share somewhat similar technical aspects as the output gap in these methods is estimated by sequentially evaluating the likelihood function using Kalman filter method [Chagny and Lemoine (2014), Benes, et al., (2010) and Borio et al. (2014)].

Studies also conclude that variation in estimates of output gap exist within methodologies. Bayesian methods as well as more elaborate Dynamic Stochastic General Equilibrium (DSGE) models are being employed for estimation of the trend and cyclical components of real output, see Berger and Kempa (2011), Juillard, et al. (2010) and Leist and Neusser (2010).

Quarterly estimates of output gap for Pakistan have not been estimated as yet. This article is therefore the first attempt in this direction. Previously Adnan and Safdar Ullah (2010) presented output gap estimates for Pakistan on annual frequency using statistical methods. In addition, Sadia et al (2009) and Shah (2009) also analyzed and estimated output gap for Pakistan on annual data.

2. Methodology for Estimation of Output Gap

Gauging the trend in aggregate demand at high data frequency is significant to effective steering of monetary and fiscal policy decisions. In contrast to quarterly estimates of output gap, annual estimates contain insufficient policy relevant information. It is relevant to note in this context that many developing economies lack official estimates of National Income Accounts at quarterly frequency. National Income Accounts (NIA) data for Pakistan is available on annual frequency only, thereby hampering high frequency data analysis specifically on indicators of business cycles, aggregate demand etc.

Significant contribution of this study is that we shed light on quarterly output gap estimates for Pakistan based on real GDP data disaggregated from official annual estimates. We use latest temporal disaggregation methodologies available in literature employing a novel disaggregation routine based on principal component analysis (PCA).

We implement estimation techniques which can be broadly classified in five categories

- Multi Variate Filter Method
- State-Space Method
- Structural VAR Method
- Bayesian Estimation of Output Gap
- Various Uni-variate Filter Methods

2.1. Multivariate Filter Approach

We apply the multivariate (MV) approach to estimate the output gap of Pakistan by implementing a small macroeconomic model presented in Benes et al. (2010). This model gauges potential output to be such level of output that does not entail inflationary tendencies. Estimation of a long run trend in total output is carried out by including empirical estimates of economic relationship between total output, core and headline price inflation and NAIRU within an expectations augmented Phillips curve relationship. The model consists of three major equations:

- Output gap equation is modeled from the perspective of central bank policy implementation, assuming that interest rate reaction function keeps core inflation aligned with the level of NAIRU in the long run. The impact of this exercise translates in the real economy by aligning the unemployment with its steady state.[3]
- Expectations augmented Phillips curve is implemented by including the contemporaneous impact of output gap as well as the so called speed limit effect of economic shocks.[4]

[3] $y_t = \rho_1 y_{t-1} - \frac{\rho_2}{100}(\pi 4_{t-1} - \pi 4_{t-1}^{LTE}) + \varepsilon_t^y$

y_t is the output gap, $\pi 4_t$ is the quarterly core CPI inflation, $\pi 4_t^{LTE}$ is the trend of quarterly core CPI inflation.

[4] $\pi 4_t = \pi 4_{t-1} + \beta y_t + \Omega(y_t - y_{t-1}) + \varepsilon_t^{\pi 4}$

- Equilibrium unemployment equation is modeled as containing transitory and permanent shock components, while the long term evolution of unemployment is modeled to revert to steady state. Also, included is the lag impact of output gap which incorporates partial hysteresis i.e. the impact of aggregate demand fluctuations on employment. 5 For details on the model see Appendix A.

Regularized Maximum Likelihood method is used to estimate this model. This method has two major benefits; (a) Incorporating of prior information in the mode, (b) Restricting parameters from wandering in nonsensical regions by specifying the prior distributions. Implementation of priors, calibration of selected parameters and results on posteriors can be seen in Table 1 and Table 2.

Table 1: Multivariate Filter Method: Results of Regularized Maximum Likelihood

Parameters	Prior		Posterior	
	Mode	Dispersion	Mode	Dispersion
α	0.900	0.158	0.896	0.024
β	0.400	0.316	0.410	0.046
Ω	0.500	0.316	0.529	0.047
ρ_1	0.800	0.158	0.779	0.023
κ_1	0.100	0.632	0.100	0.091
ϕ_1	0.800	0.158	0.811	0.023
ϕ_2	0.300	0.158	0.133	0.017
τ	0.100	0.158	0.123	0.023
δ	0.500	0.158	0.500	0.024
κ_2	1.500	1.581	1.500	0.237
ω	3.000	1.581	3.020	0.237
ρ_2	5.000	3.162	4.999	0.478
λ	2.000	3.162	1.983	0.469

Table 2: Calibrated Parameters

Parameter	Value
θ	0.605
$G_{SS}^{\bar{Y}}$	4.500
$G_{SS}^{\bar{U}}$	5.450
$\sigma_{SS}^{\bar{Y}}$	5.911

Data and relevant studies indicate that unemployment is also impacted by the lagged impact of change in total output; this information is also incorporated in the model. Estimates of output gap, on both, annual and quarterly frequency, are plotted along with the CPI Inflation in Figures 1 and 2 (see Appendix C). There appears to exist three distinctive periods worth highlighting. During the period Q2-FY98 to Q2-FY03, low headline inflation along with a negative output gap posits the existence of contraction in aggregate demand in Pakistan, while Q2-FY04 to Q4-FY08 depicts the existence of excess demand tendencies in Pakistan. Finally the negative and persistent output gap since Q2-FY09

$\pi4_{t-1}$ is the inflation expectations, βy_t, shows the contemporaneous impact of output gap on inflation, while the parameter $\mathbf{\Omega}$ measures the average impact of economic shocks to core inflation.

[5] $u_t = \phi_1 u_{t-1} + \phi_2 y_t + \varepsilon_t^u$, u_t is the unemployment gap.

to date along with high headline inflation during FY07 – FY11 signals lower than could be the productive capacity of the economy. [6]

Table 3 shows Mean Absolute Deviation (MAD) of estimates from HP filter and MV filter methods. Results show that MV filter estimates of output gap are robust at t-12, t-8 as well as t-4 data frequencies, where the MAD estimates of output gap obtained using HP filter are higher than those from MV filter method.

Table 3: Mean Absolute Revisions in Output Gap Estimates

quarter	t - 12	t - 8	t - 4	(Nowcast)
Output Gap (MV Filter)	0.079	0.068	0.234	0.623
Output Gap (Hodrick Prescott Filter)	0.165	0.287	0.665	1.165

Note: This table compares robustness of estimates to revisions in data. Output gap for Pakistan obtained from the multivariate method is compared with the cyclical time series component extracted using Hodrick Prescott filter on the real GDP.

We also test for the robustness of estimates on account of revision in data. Comparison of estimates of output gap from MV filter approach is compared against those from de-trended real GDP of Pakistan using Hodrick Prescott filter. Results indicate that output gap estimates are robust with respect to revisions in data (see Figures A1 and A2 in Appendix A). Estimates of output gap are obtained by de-trending the quarterly GDP of Pakistan (using the Hodrick-Prescott filter), recursively, adding one data point and obtaining the cycle estimates at each step. A similar approach is implemented for MV filter method.

2.2. State Space Model

The state-space model is flexible to formulate and modify. It easily handles missing data or non-stationary dynamics, and is a natural representation for linearized recursive dynamic economic models.

We implement a bi-variable, state-space approach to identify and decompose GDP into a trend component and cyclical component. Assume that the state-space model takes the following form

$$Y_t = HZ_t + AX_t + M\xi_t \qquad (1)$$

$$Z_t = \Lambda_0 + \Lambda_1 Z_{t-1} + N\xi_t \qquad (2)$$

The measurement variable real GDP (Y_t) is modeled in the dynamic system as explained by, vector of unobserved variables Z_t, vector of exogenous variable X_t (CPI Inflation) and errors ξ_t . Where, $\xi_t \overset{iid}{\rightarrow} N(0, \Sigma_\xi)$ and the variance-covariance matrix is defined by the scalar $\Sigma_\xi = I$. Also, the state equation represents the dynamics of the unobserved variables, Z_t. Where output gap is one element of Z_t. As is typical, structural shocks are modeled to be uncorrelated with measurement errors, by imposing the restriction, $N\Sigma_\xi M' = 0$.

The representation above of the state-space model is implemented vastly in literature for extracting output gap and potential GDP (Kuttner, 1994, Laubach and Williams, 2003, Michal, 2013). As in standard practice the Maximum Likelihood estimation of the state-space model is conducted using

[6] Fiscal Year (FY) in Pakistan spans between 1st July to 30th June.

Kalman smoother method. Kalman filter is a recursive process based on two distinct stages of feedback to the system i.e. prediction and correction. Suitable priors based on information about the steady states on the parameters Λ_0, Λ_1, H and A, and restriction on the variance covariance matrices Σ_ξ, M and N are implemented for prediction. Also, for the correction, posteriors of the estimates and the variance covariance matrix are utilized. Kalman filter utilizes prior information to generate posteriors, and this learning procedure is repeated iteratively until all the sample data is analyzed.

Specific functional form as implemented here for estimation and analysis is presented below. Potential output is modeled to follow a random walk with drift, where dynamics of the process governing drift follows mean reversal with long term steady state $\bar{\mu}$ and with a transversality condition, $\beta \in (0,1)$.

$$Y_t = Y_t^1 + Y_t^2 \tag{3}$$

$$Y_t^1 = \mu_{t-1} + Y_{t-1}^1 + \xi_t^1 \tag{4}$$

$$\mu_t = (1-\beta)\bar{\mu} + \beta\mu_{t-1} \tag{5}$$

$$Y_t^2 = \tau_1 Y_{t-1}^2 + \tau_1 Y_{t-2}^2 + \xi_t^2, \text{ where, } \xi_t^2 \overset{\text{iid}}{\rightarrow} \mathrm{N}(0, \sigma_t^2) \tag{6}$$

$$\pi_t = \lambda_1 \pi_{t-1} + \lambda_2 \pi_t^* + \lambda_3 Y_{t-1}^2 + \xi_t^3 \text{ where, } \xi_t^3 \overset{iid}{\rightarrow} N(0, \sigma_t^2) \tag{7}$$

Where Y_t^1 and Y_t^2 represent trend and cycle component of output, respectively. π_t is CPI inflation while π_t^* is the target CPI inflation, assumed to be the steady state level of inflation. Trend output is modeled as autoregressive process with a deterministic component, while cycle is assumed as AR(2) process. Inflation is modeled in a typical Phillips curve setting (see equation 7).

2.3. Structure VAR Model

Employing similar identification long-run restrictions in 3 variable setting, similar to Bjørland et al. (2006), we attempt to identify 3-distinct disturbances which can be interpreted as aggregate supply shock, real demand shock and nominal demand shock respectively. Details are as under.

We augment the bivariate model of Blanchard and Quah (1989), to also include CPI inflation; the ordering of the variables is thus, unemployment, change in log of real GDP and CPI inflation, respectively. This allows us to identify three different shocks: two demand shocks (nominal and real demand) and one supply shock. We assume that neither of the demand shocks can have a long run effect on unemployment. However, to distinguish between the two demand shocks, we assume that only the nominal demand shock is restricted from affecting output in the long run. This allows us to investigate the possibility that one of the demand shocks (real demand) can have a more persistent effect on output than the other, although without changing the unemployment rate permanently as a result. These assumptions may allow for the interpretation of the real demand shock as a preference shock and the nominal demand shock as a monetary policy shock; see Rabanal and Gali (2004) for further discussion.

Consider three observed series u_t, $\Delta\mathrm{gdp_t}$ and $\pi_t = \Delta CPI_t$. Where u_t is unemployment, Δgdp_t is log of real GDP and π_t is CPI inflation. Let us define $y_t = [u_t, \Delta gdp_t, \pi_t]'$, so that y_t is $I(0)$. Assuming that y_t is modeled as in the following structural representation,

$$\boldsymbol{B}\boldsymbol{y_t} = \gamma + \boldsymbol{\Gamma}\boldsymbol{y_{t-1}} + \boldsymbol{\varepsilon_t} \tag{8}$$

the SMA representation is

$y_t = \mu + \Theta(L)\varepsilon_t$.

Since y_t is assumed to be covariance stationary, we know that $lim_{s\to\infty}\,\theta_{ij}^{(s)} = 0$, meaning that no structural shock has a long-run impact on the level of y. The long-run cumulative impact of the structural shocks is captured by the long-run impact matrix; $\Theta(L)$

$$\text{Also } \Theta(L) = \begin{bmatrix} \theta_{11}L & \theta_{12}L & \theta_{13}L \\ \theta_{21}L & \theta_{22}L & \theta_{23}L \\ \theta_{31}L & \theta_{32}L & \theta_{33}L \end{bmatrix} \tag{9}$$

And the following reduced form representation,

$$y_t = a + A_1 y_{t-1} + \xi_t \tag{10}$$

$$y_t = \mu + \Psi(L)\xi_t \tag{11}$$

Where ξ_t, the error terms are contemporaneously correlated with covariance matrix Ω, while the structural shocks are modeled to be contemporaneously uncorrelated.

And $E[\varepsilon_t \varepsilon_t{}'] = D$, also D is diagonal. $E[u_t u_t{}'] = \Omega$, where $\Omega = B^{-1}DB^{-1'}$

$\Psi(L) = (I_2 - A_1 L)^{-1}$ and $\Theta(L) = \Psi(L)B^{-1}$

The structural shock $\varepsilon_{\mathrm{u}}^{\mathrm{AS}}$ is interpreted as aggregate supply shock, $\varepsilon_{\mathrm{y}}^{\mathrm{RD}}$is interpreted as the real demand shock and lastly $\varepsilon_{\pi}^{\mathrm{ND}}$ is interpreted as nominal demand shock.

Identification of the parameters of the SVAR is achieved through restrictions on the parameters of the structural moving average representation. Assuming ε_u^{AS} has no long-run cumulative impact on Δgdp_t, then $\theta_{12}L = \sum_{s=0}^{\infty}\theta_{12}^{s} = 0$, similar identification assumptions are implemented for ε_y^{RD} and ε_π^{ND}. This implements a lower triangular long-run identification restriction to extract the desired shocks, and thereby obtain the trend and cyclical component of real GDP.

The form of $\Theta(L)$ is as stated below;

$$\Theta(L) = \begin{bmatrix} \theta_{11}L & 0 & 0 \\ \theta_{21}L & \theta_{22}L & 0 \\ \theta_{31}L & \theta_{32}L & \theta_{33}L \end{bmatrix} \tag{12}$$

Estimates of output gap are plotted in Figures C1 and C2, Appendix C.

2.4. Bayesian Estimation of Output Gap

Recent literature on estimation of output gap emphasizes methods which are less sensitive to data revisions and end-point uncertainty. Bayesian inference, as pointed out by Planas et al. (2008) and Antonio et al. (2010), score highly on these merits. Estimation of output gap of Pakistan using Bayesian inference is conducted in line with Planas et al. (2008). Robustness of estimates is achieved by incorporating intelligent prior inputs with actual data thereby limiting parameters from acquiring nonsensical values. Inference can then be made on this joint posterior distribution. We proceed by specifying a stochastic trend setup for total output, the specification similar to State-Space model in Kuttner (1994) using CPI inflation and real GDP as observed variables. Real GDP is modeled as comprising two unobserved components i.e. trend ($\mathrm{p_t}$) and cycle ($\mathrm{c_t}$). We specify trend as AR(1) process with drift, while cyclical component is modeled as AR(2) process which is stated in a sinusoidal form. Economic inter-linkage between inflation and total output is modeled as expectations augmented Phillips curve. Thereafter the model can be jointly estimated for output (modeled as stochastic trend specification) and the Phillips curve, using Maximum Likelihood (ML) method.

For Bayesian inference we first undertake the above exercise and then utilize these ML estimates as prior hyper parameters of coefficients, their variance and error variances. Inference is thereafter carried out in line with Planas et al. (2008). See Appendix B for concise model specification. Also, for estimates of output gap see Figures C1 and C2 in Appendix C. Detailed information about the priors can be found in the first columns of Table 4 and Appendix B.

Table 4: Bayesian Estimates of Output Gap

Model Specification

GDP: $y_t = p_t + c_t\,, \Delta p_t = \mu_{t-1} + a_{\mu t}\,, c_t - 2Acos\{2\pi/\tau\}c_{t-1} + A^2 c_{t-2} = a_{\mu t}$

Inflation: $\phi_\pi(L)\Delta\pi_t = \mu_{t-1} + \beta_0 c_t + \lambda\Delta y_{t-1} + a_{\pi t}$

Results and Diagnostic

Variable	Mode	Sd.	NSE	RNE	CD
τ	32.09	2.51	0.017960	1.954	0.7692
A	0. 7599	0.1005	0.001321	0.5785	0.79
μ_p	1.07	0.1054	0.000986	1.141	0.2443
β	0.8554	0.2481	0.003299	0.5655	0.1407
ϕ_π	1.253	0.0838	0.000792	1.118	0.565

Note: *Sd.* stands for Standard deviation, *NSE* is the numerical standard error of the mean, *RNE* is relative numerical efficiency and *CD* is *p* value of Geweke convergence statistic.

Basistha (2007) and Planas et al. (2008) provides reference to previous estimates as a rough indication for the prior means, however we implement the prior variances fairly wide. The output gap includes one lagged dependent variable. The prior distribution of the autoregressive (AR) parameters is implemented so that 90% interval covers the range $[0.10, 0.90]$. Thereby imposing a versatile but not very persistent output gap. For potential output growth we use a tight prior (as in Planas et al, 2008). The variance parameters for the shocks to the output gap and potential output have been set according to previous estimates (i.e. utilizing estimates from the state-space method stated above). However, we leave a considerable amount of uncertainty around the prior means. Turning to inflation, the model includes one lag of the dependent variable, the output gap. The prior distribution of inflation persistence, as measured by β_π, is very flat allowing for a unit-root process of the inflation rate.
Table 4 also illustrates the mode of the posterior distribution of all parameters. Similar to other studies we find the output gap to be a relatively persistent process. Prior and posterior distribution of selected parameter as modeled in equations for trend and cyclical output are similar as can be seen in Figure 1 in Appendix B, which shows the prior together with the posterior distribution. The persistence of inflation is lower than found in the literature; see Planas et al. (2008), for further information on posteriors (Table 4).

As, Bayesian inference conducted in this model is based on theoretical model input, estimates of output gap from this method are significantly similar with the results from economic model based estimates from MV filter method, with considerable less variance, as expected.

2.5. Various Uni-variate Filter Methods

Following literature convention and also for comparison purpose, here we state consolidated estimates of output gap for Pakistan on both annual and quarterly frequency using some uni-variate filter methods.[7] In light of the principal problem of end-of-sample estimate uncertainty and the fact that estimates from uni-variate filters are prone to significant revisions due to inclusion of latest data, policy input from such methods is prone to criticism. For results see Figure C1 and C2 of Appendix C.

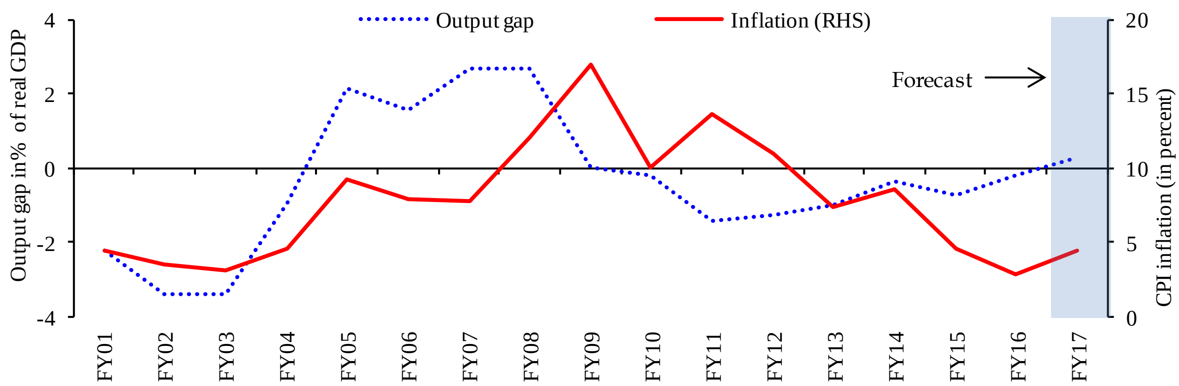
3. Data, Assumptions and Conventions

Data on CPI, core inflation (NFNE), unemployment and real gross domestic output (factor cost) is sourced from Pakistan Bureau of Statistics (PBS). Data used in our analysis spans the period between FY78–FY16. Annual data on gross domestic product is at base year FY06. Consumer price indices used in this study are at base year FY08. Unemployment is as percent (of total labor force).

For quarterly data on prices we use the quarter average of monthly consumer price indices. CPI inflation one period in the lead is used as measure of adaptive expectation of inflation. Annual unemployment statistics are interpolated for quarterly frequency using cubic-mean interpolation method. We utilize methods based on Enrique (2005) for temporal disaggregation of real GDP of Pakistan to quarterly frequency. [8] These quarterly real GDP estimates depict reasonable robustness and ease in calculation/estimation as compared to estimates in Hanif et al. (2010). Our estimates of quarterly real GDP exhibit persistence (they are not susceptible to drastic quarter-on-quarter change). In order to access the validity of disaggregation method, interpolation of quarterly real GDP of USA was conducted for the period 1960 -2007, using CPI inflation and industrial production as indicator variables. The comparison of interpolated quarterly GDP series with the official disseminated data for real quarterly GDP is very promising. The mean absolute deviation of actual compared to interpolated quarterly GDP data for USA, is calculated to be very low, thereby providing credence to the interpolation method.

For FY17, we assume annual CPI inflation to be at 4.5%[9], unemployment rate at 6% and real GDP growth at 5.2%.

Figure 1: Annual Output Gap Estimates (Average of 5 Methods) and CPI Inflation for Pakistan



[7] These methods include; Uni-variate Kalman filter, Hodrick and Prescott (1997) filter, Christiano and Fitzgerald (2003) filter, Baxter and King (1999) filter, Wavelet Filter (Motohiro, Y., 2008), Quadratic time de-trending and Beveridge and Nelson (1981) decomposition.

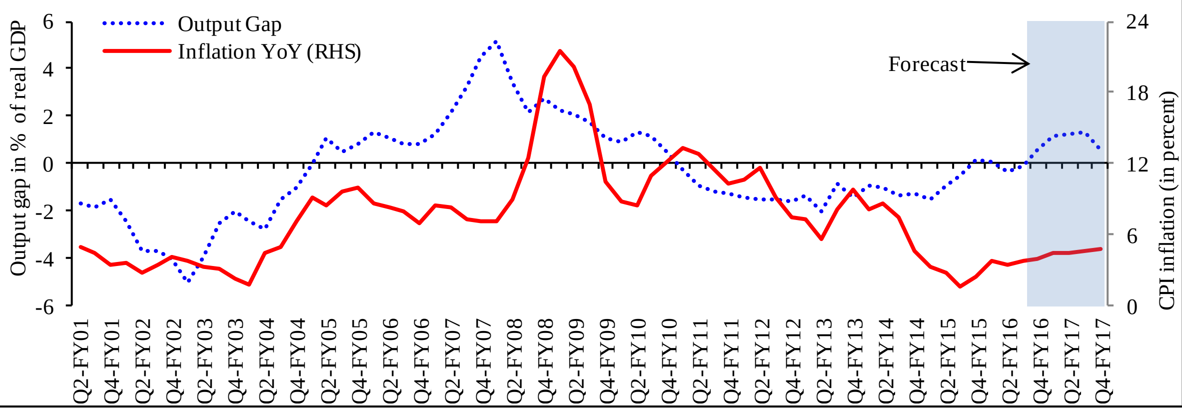
[8] *Robust Quarterization of GDP and Determination of Business Cycles for IGC Partner Countries*, by Abdullah Tahir, Jameel Ahmed & Waqas Ahmed – *Unpublished Manuscript*. This research is supported by the International Growth Center (IGC), London School of Economics and Political Science, UK.

[9] FY17 forecast is obtained by a uni-variate ARIMA model for CPI Inflation for Pakistan.

4. Results

Robustness of estimates of output gap is evident from the mean absolute revisions depicted in Table 3. The estimates of output gap using methodologies utilized in this paper are less susceptible to revision in data and increase in the sample size, see Figures A1 and A2. This is a desirable property of estimates of output gap and potential GDP from the perspective of use in economic policy, as, the most recent estimate of trend and cyclical components of real GDP is particularly interesting for policy makers. Robustness of estimates of output gap (to data revision), as elaborated, is generally not achieved using time series filtering techniques for cycle extraction. Cycle extraction using Hodrick – Prescott filter, for instance, is susceptible to significant revisions (see Figure A2 in Appendix A), primarily because the last data point of the series has exaggerated impact on the trend of the series, commonly termed as the end-point bias.[10] Ideally the cycle extracted using HP filter is robust in signal extraction over a very long data sample; this condition, unfortunately, is frequently overlooked in analysis of output gap considering that HP filter is widely used in cycle extraction from real GDP (see Bruchez (2003) for details).

Figure 2: Quarterly Output Gap Estimates (Average of 5 Methods) and CPI Inflation for Pakistan



Figures 1 and 2 depict the principal findings of this paper. In these figures we plot output gap on annual and quarterly frequency with CPI inflation, output gap forecasts for FY17 is also illustrated.[11] The output gap presented here is the mean of estimates of the five methods discussed above.[12]

Estimates of output gap from various methods generally coincide in magnitude and direction, in both annual and quarterly frequencies. In addition, output gap estimates from MV filter method, Bayesian inference and State–Space method show good overall robustness i.e. these estimates are less susceptible to drastic end of sample and temporal revisions than estimates from uni-variate methods. This is in contrast to earlier studies for Pakistan[13] that highlighted significant deviations in magnitude and direction of output gap estimates. For illustration of robustness of output gap estimates from methodologies, see Figures A1 and A2 in Appendix A.

[10] The Hodrick-Prescott filter assumes that the series y_t is composed of trend ψ_t and cyclical components ρ_t. The filter defines ψ so as to minimize the penalty function; $\sum_{t=1}^{T}(y_t-\psi_t)^2+\lambda\sum_{t=2}^{T-1}[(\psi_{t+1}-\psi_t)-(\psi_t-\psi_{t-1})]^2$. Where λ imposes a penalty on changes in slope of the trend. The second term in the penalty function excludes the first and the last data points; this leads to biased weight of the first and last data points on the trend of the time series.

[11] CPI inflation forecasts on quarterly data frequency are obtained by a ARIMA model.

[12] Standard deviation of output gap estimates from the 5 methods is obtained; weighted average is then calculated by giving more weight to the method which exhibits lower standard deviation.

[13] See discussion on results in Adnan and Safdar Ullah (2010).

Also, Furceri and Mourougane (2009) and IMF in its World Economic Outlook, October 2009 argue that persistent and prolonged impact of reduction in economic activity, like the recent global financial crisis, brings upon permanent reduction in productive capacity of an economy. In recent years economy of Pakistan has faced multiple macroeconomic shocks, attributable to adverse economic, political and law and order situation. Figures 3a and 3b below illustrate growth in potential output of Pakistan. Near stagnation in productive potential is evident as the growth in potential output has come down from 6.2% on average during FY78-FY89 to 4.3% in FY09-FY17.

Commenting on the current and expected short-run outlook; the forecast for FY16–FY17 depicts that aggregate demand in Pakistan is expected to regain momentum by H2-FY16. This is concluded on the basis of trend in potential output, real output, output gap and the projections for CPI inflation.

Prevailing low potential output growth of Pakistan, as argued above (see Figure 3), increases the vulnerability of Pakistan economy and makes it more susceptible to the demand shocks, resulting in somewhat corresponding volatility in inflation which can lead to decline in investment, causing further decline in productivity and potential output.

Figure 3a: Growth in Trend and Real GDP of Pakistan - Quarterly Data Frequency

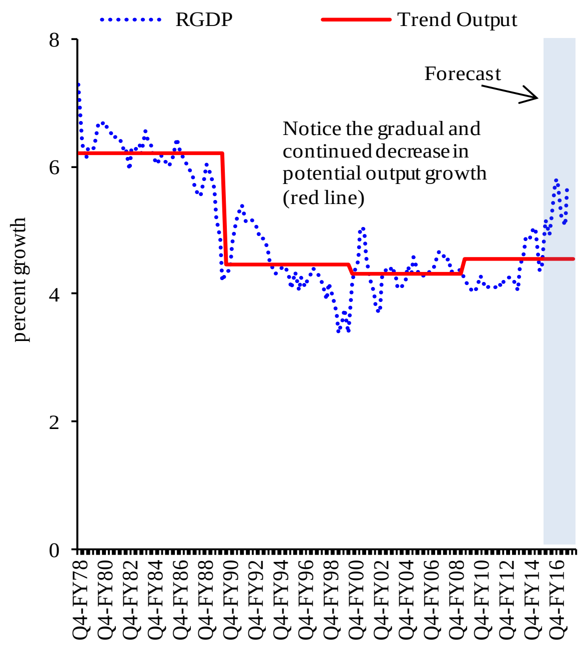
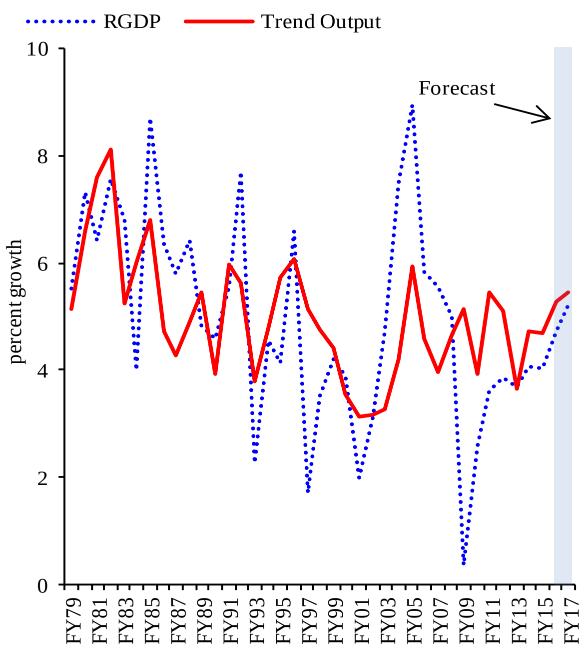


Figure 3b: Growth in Trend and Real GDP of Pakistan - Annual Data Frequency



References

Adnan, S., and Safdar Ullah, K. (2010), "Estimation Output Gap for Pakistan Economy: Structural and Statistical Approaches", State Bank of Pakistan Working Paper Series, No. 24.

Antonio, B., Michele, C., and Locarno, A. (2010), "Modeling Italian Potential Output and the Output Gap", Banca D'Italia, Working Paper Series No. 771.

Apel, M., and Jansson, P. (1999), "A Theory-Consistent System Approach for Estimating Potential Output and the NAIRU", Economics Letters, 64(3), pp.271-275.

Basistha, A. (2007), "New measures of the output gap based on the forward-looking new Keynesian Phillips curve", Journal of Monetary Economics, 54(2), pp.498-511.

Baxter, M. and Robert G. K. (1999), "Measuring business cycles: Approximate band-pass filters for economic time series", The Review of Economics and Statistics 81: 575-593.

Benes, J., and N'Diaye, P. (2004), "A Multivariate Filter for Measuring Potential Output and the NAIRU", IMF Working Paper No.04/45.

Benes, J., Clinton, K., Garcia-Saltos, R., Johnson, M., Laxton, D., and Manchev, P., (2010), "Estimating Potential Output with a Multivariate Filter", IMF Working Paper No.10/285.

Berger, T., and Kempa, B. (2011), "Bayesian Estimation of the Output Gap for a Small Open Economy: The Case of Canada", Economics Letters, Elsevier, vol. 112(1), pp.107-112.

Beveridge, S. and C. R. Nelson (1981), "A New Approach to Decomposition of Economic Time Series into Permanent and Transitory Components with Particular attention to Measurement of the 'Business Cycle' ", Journal of Monetary Economics 7 (1981) 151-174.

Bjørnland, H. C., Brubakk, L., and Jore, A. S. (2006), "Forecasting Inflation with an Uncertain Output Gap", Memorandum No. 11, Oslo: Department of Economics, University of Oslo.

Blanchard, O., and Quah, D. (1989), "The Dynamic Effects of Aggregate Supply and Demand Disturbances", American Economic Review, Vol. 79, pp.655–673.

Borio, C., Disyatat, P., and Juselius, M. (2013), "Rethinking Potential Output: Embedding Information about the Financial Cycle.", BIS Working Paper No. 404.

Borio, C., Disyatat, P., and Juselius, M. (2014), "A Parsimonious Approach to Incorporating Economic Information in Measures of Potential Output", Bank of International Settlements (BIS) Working Paper Series.

Bruchez, P. (2003), "A Modification of the HP Filter Aiming at Reducing the End-Point Bias", Swiss Federal Finance Administration Working Paper Series No. 3.

Chagny, O., and Lemoine, M. (2014), "An Estimation of the Euro Area Potential Output with a Semi-Structural Multivariate Hodrick-Prescott Filter", (HAL 00972840).

Christiano, L. J. and Fitzgerald J. (2003), "The band pass filter", International Economic Review 44: 435-465.

Clark, P. K. (1987), "The Cyclical Component of US Economic Activity", The Quarterly Journal of Economics , 41 (3), pp.475-512.

Conway, P., and Hunt, B. (1997), "Estimating Potential Output: A Semi-Structural Approach", Discussion Paper G97/9, Reserve Bank of New Zealand.

Cotis, J. P., Elmeskov, J., and Mourougane, A. (2010), "Estimates of Potential Output: Benefits and Pitfalls from a Policy Perspective", OECD Economics Department.

Ding, D., John N., Roshan P., and Volodymyr T. (2014), "Estimating Sri Lanka's Potential Output.", IMF Working Paper No.14/40.

Enrique, Q. M. (2005), “Benchmarking Techniques in the Spanish Quarterly National Accounts”, EUROSTAT-OECD Workshop on Frontiers in Benchmarking Techniques and Their Application to Official Statistics. Luxembourg: European Commission.

Furceri, D., and Mourougane, A. (2009), “The Effect of Financial Crises on Potential Output: New Empirical Evidence from OECD Countries”, OECD, Economics Department, Working Paper No. 699.

Gerlach, S., and Smets, F. (1999), “Output Gap and Monetary Policy in the EMU.”, European Economic Review, Vol(43) , pp.801-812.

Hal, R. E., and Taylor, J. (1991). Macroeconomics. New York: Norton.

Hanif, M. N., Iqbal, J., and Malik, J. (2013), “Quarterization of National Income Accounts of Pakistan”, SBP Working Paper Series. No. 54.

Harvey, A., and Trimbu, T. (2008), “Trend Estimation and the Hodrick-Prescott Filter”, Journal of Japan Statistical Society, Vol. 38, No.1. , pp.41-49.

Hodrick, Robert J. and Edward C. Prescott, (1997), “Postwar U.S. business cycles: An empirical investigation”, Journal of Money, Credit, and Banking 29: 1-16.

Juillard, M., K.Ondra, M. Kumhof, and D. Laxton, (2010), "Measures of Potential Output from an Estimated DSGE Model of the United States." Bank of Canada Working Paper Series No. 8.

Kichian, M. (1999), “Measuring Potential Output within a State-Space Framework”, Bank of Canada Working Paper Series No.99-9.

Kuttner, K. (1994), “Estimating potential output as a latent variable”, Journal of Business and Economic Statistics Vol.12 (3), pp.361-368.

Laubach, T., and Williams, J. C. (2003), “Measuring the Natural Rate of Interest”, Review of Economics and Statistics, 85 (4), pp.1063–1070.

Leist, S., and Neusser, K. (2010), “Measuring the Natural Output Level by DSGE Models: An Empirical Investigation for Switzerland”, Swiss Society of Economics and Statistics, Vol. 146 (1), pp.275–300.

Michal, A. (2013), “What is in Your Output Gap? Unified Framework and Decomposition into Observables”, IMF Working Paper No.105.

Motohiro, Y. (2008), “Measure Business Cycle: A Wavelet Analysis of Economic Time Series”, Economics Letter (100), pp.208-212.

OECD. (2000). OECD Economic Outlook. OECD Publishing.

Okun, A. M. (1962), “Potential GNP: Its Measurement and Significance”, Reprinted as Cowles Foundation Paper 190.

Pau, R., and Jordi, G. (2004), “Technology Shocks and Aggregate Fluctuations: How Well Does the RBS Model Fit Postwar U.S. Data?”, NBER Working Paper No. 10636.

Planas, C., Rossi A. and Fiorentini G. (2008), “Bayesian Analysis of Output Gap”, Journal of Business and Economic Statistics, 26, 1, pp.18-32.

Scott, A. (2000), “Stylized Facts from Output Gap Measures”, Discussion Paper DP2000/07, Reserve Bank of New Zealand.

Shah, S. S. (2009), “Output Gap Measures for Pakistan: Methodologies and Challenges for the Monetary Policy”, Journal of Applied Research in Finance (2), pp.206-227.

Sadia S., Faiza A. and Naheed Z. K. (2009), “Output Gap and its Determinants: Evidence from Pakistan”, Journal of Economic Cooperation and Development, 30 (3), pp.75-82.

Appendix

A. Multivariate Filter by Benes et al (2010).
The small-macroeconomic model for estimation of output gap and potential output as implemented in Benes et al. (2010) is state below.

I: Three Gaps: The Output Gap y_t is the log difference between actual GDP Y_t and potential GDP $\bar{Y}_t$, $y_t = 100 * LOG(Y_t/\bar{Y}_t)$

The Unemployment gap u_t is the equilibrium unemployment rate or, NAIRU $\bar{U}_t$, minus the actual unemployment rate U_t, $u_t = \bar{U}_t - U_t$

II: Three Identifying relationships:

a. Inflation: The level y_t and the change in output gap $y_t - y_{t-1}$ influence current core inflation; πc_t, $\pi c_t = \pi c_{t-1} + \beta y_t + \Omega(y_t - y_{t-1}) + \varepsilon_t^{\pi}$

b. Dynamic Okun's Law: $u_t = \phi_1 u_{t-1} + \phi_2 y_t + \varepsilon_t^{u}$

III. Laws of motion for Equilibrium Variables

a. NAIRU: $\bar{U}_t = \bar{U}_{t-1} + G_t^{\bar{U}} - \frac{\omega}{100} y_{t-1} - \frac{\lambda}{100}(\bar{U}_{t-1} - U^{ss}) + \varepsilon_t^{\bar{U}}$

Where:

Transitory shocks	$\varepsilon_t^{\bar{U}}$
Persistent shocks	$G_t^{\bar{U}} = (1-\alpha)G_t^{\bar{U}} + \varepsilon_t^{G^{\bar{\bar{U}}}}$
Steady state level of unemployment	U^{ss}

b. Potential Output: $\bar{Y}_t = \bar{Y}_{t-1} - \theta(\bar{U}_t - \bar{U}_{t-1}) - (1-\theta)\,{}^{(\bar{U}_{t-1} - \bar{U}_{t-2})}/_{2} + G_t^{\bar{Y}} + \varepsilon_t^{\bar{Y}}$

Where;

Labor Share	θ
Captures the Impact of change in Equilibrium level of Unemployment on the growth of Potential output.	$\theta(\bar{U}_t - \bar{U}_{t-1})$
Captures the induced changes in the capital stock	$(1-\theta)\,{}^{(\bar{U}_{t-1} - \bar{U}_{t-2})}/_{2}$
Trend Growth rate of Y	$G_t^{\bar{Y}}$
Where;	$G_t^{\bar{Y}} = \tau G_{SS}^{\bar{Y}} + (1-\tau)G_{t-1}^{\bar{Y}} + \varepsilon_t^{G^{\bar{\bar{Y}}}}$
Steady state growth rate of Y	$G_{SS}^{\bar{Y}}$

c. Perceived Long Term Inflation Objectives. $\pi 4_t^{LTE} = \pi 4_t^{LTE} + \varepsilon_t^{\pi c^{LTE}}$

Where; $\pi 4_t^{LTE}$ denotes Long Term expectations for core inflation

IV. Output Gap Equation: $y_t = \rho_1 y_{t-1} - \frac{\rho_2}{100}(\pi 4_{t-1} - \pi 4_{t-1}^{LTE}) + \varepsilon_t^{y}$

Figure A1: Real Time Output Gap Estimates (red line) vs Full Sample Output Gap Estimates (blue line) using HP Filter - (The last iteration is plotted) - Notice the drastic deviation in blue and red lines since Q2FY03) interpreted such that the estimates of output gap using HP filter are non robust estimates.

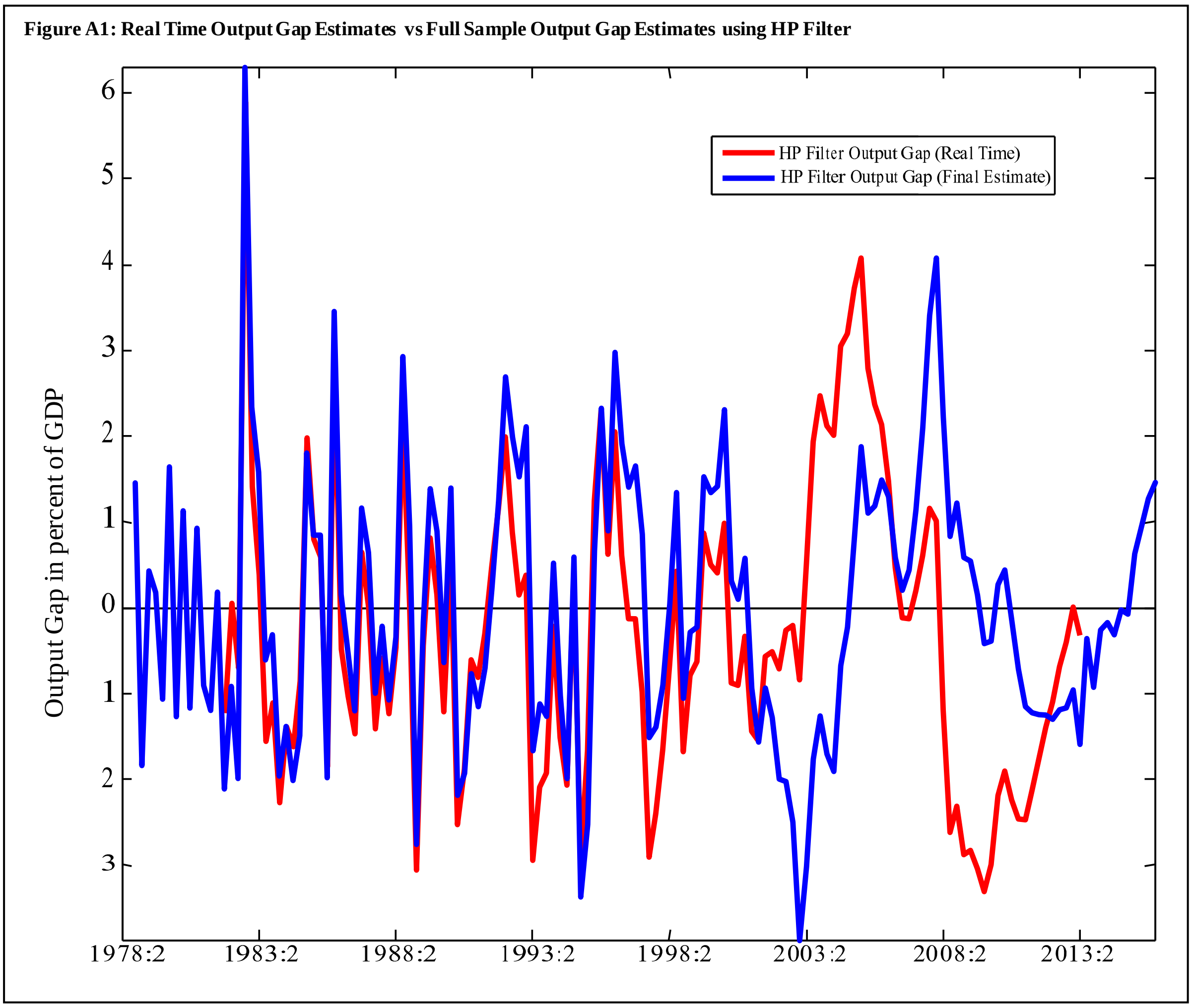
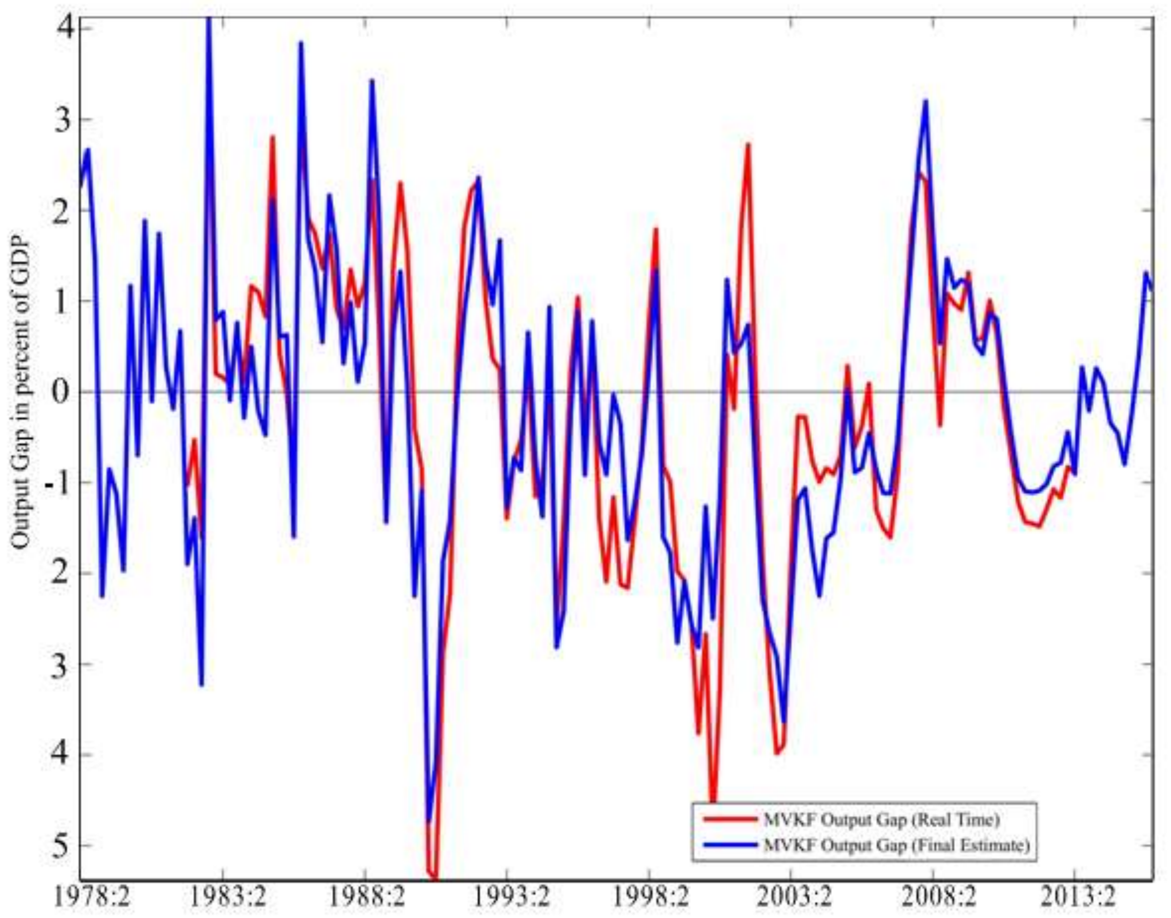


Figure A2: Robustness Real Time Output Gap Estimates from MV Filter Method - (The last iteration is plotted) - real time output gap estimates (as grey line) vs final output gap estimates from MV filter method (as blue line). Estimates are robust to change in data periods, as the grey line and blue lines are relative close together.

Figure A2: Robustness Real Time Output Gap Estimates from MV Filter Method



B. Bayesian Estimation of Output Gap

Model and description below is adopted from Planas et al. (2008). The model is specified as;

$$y_t = p_t + c_t$$

$$(1 - L)p_t = \mu_p + a_{pt}$$

$$\phi(L)c_t = a_{ct}$$

Where L is the lag operator, μ_p is a constant drift and $\phi(L) = 1 - \phi_1 L - \phi_2 L^2$ is an AR(2) polynomial. The permanent (a_{pt}) and transitory shocks (a_{ct}) are independent Gaussian white noise terms with variances V_p and V_c. In line with the model specification in Kuttner (1994) we also specify a Phillips curve of the following specification.

$$\pi_t = \mu_\pi + \beta c_{t-1} + \lambda \Delta y_{t-1} + \alpha_1 \pi_{t-1} + \alpha_2 \pi_{t-2} + a_{\pi t}$$

Where $\Delta = 1 - L$ and $a_{\pi t}$ is Gaussian white noise with variance V_π

In addition the cyclical component of the real total output is specified in the following form of sinusoidal or polar coordinates of the polynomial roots;

$$\left(1 - 2A\,cos\left(\frac{2\pi}{\tau}\right)L + A^2L^2\right)c_t = a_{ct}$$

Where A and τ represent the amplitude and periodicity of the cyclical movements.

Initiating Bayesian inference in the model setting above, we specify natural conjugate distributions for the parameters of the model above, mainly in order to reduce computational complexity (for explanations on choice of prior distributions for parameters see Planas et al., 2008). Therefore we state the following joint prior distribution;

$$p(\theta) = p(A, \tau, \delta, vech(\Sigma), \mu_p, V_p) = p(A)p(\tau)p(\delta)p(\Sigma)p(\mu_p, V_p)$$

In order to characterize the unobserved potential output and output gap we intend to condition the latent variables on observed data i.e. $p(c^T, p^T, \theta | Y^T)$, this involves inference on the Gibbs sampler draws from the following two conditional posterior distributions, as analytical solution is not possible.

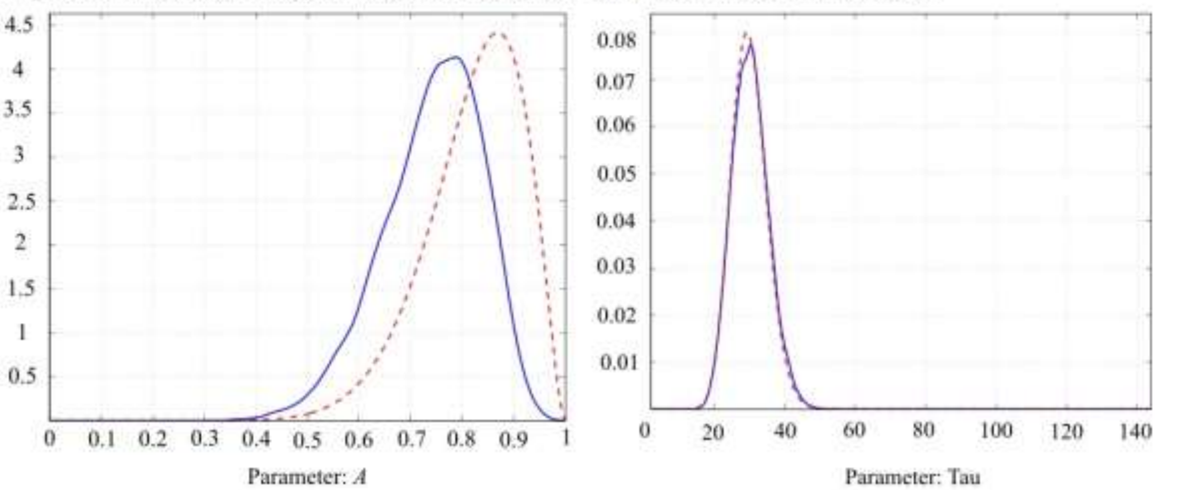
$$p(c^T, p^T | \theta, Y^T)$$

$$p(\theta | c^T, p^T, Y^T)$$

Bayesian inference for draws on parameters is conducted in the sequence below;

- Draws for state are obtained using the state sampler detailed Planas et al. (2008)[14]
- The space representation of the model i.e. $p(\theta | c^T, p^T, \pi^T) = p(A, \tau, \delta, \Sigma | c^T, p^T, \pi^T) p(\mu_p, V_p | p^T)$ is sampled independently.
- $p(\mu_p, V_p | p^T)$ is sampled as in Planas et al. (2008).
- The conditional density $p(A, \tau, \delta, \Sigma | c^T, p^T, \pi^T)$ is further distributed conditionally as $p(\Sigma | A, \tau, \delta, c^T, p^T, \pi^T)$ and $p(A, \tau, \delta | \Sigma, c^T, p^T, \pi^T)$.
- $p(\Sigma | A, \tau, \delta, c^T, p^T, \pi^T)$ is obtained using Yule-Walker method.
- $p(A, \tau, \delta | \Sigma, c^T, p^T, \pi^T)$ is obtained by distributing as the conditionals; $p(A, \tau, \delta | \Sigma, c^T, p^T, \pi^T) = p(, \delta | A, \tau, \Sigma, c^T, p^T, \pi^T) \times p(A, \tau | \delta, \Sigma, c^T, p^T, \pi^T)$
- The above steps are undertaken sequentially until convergence is achieved to the joint posterior
- $p(c^T, p^T | \theta, Y^T)$. Intuitively this can be interpreted as characterizing the latent output gap and potential output on observed data.

Figure B.1 Prior (dashed line) and Posterior Distribution (slid line) for Selected Parameters



[14] $p(c^T, p^T | \theta, Y^T) = p(c^T, p^T | Y_T) \prod_{t=2}^{T-1} p(c_t, p_t | \theta, Y^t, c_{t+1}, p_{t+1}) p(c_1, p_1 | \theta, Y_1, c_2, p_2)$

C. Estimates of Output Gap on Quarterly and Annual Data Frequencies

Figure C1: Annual Output Gap Estimates and CPI Inflation for Pakistan

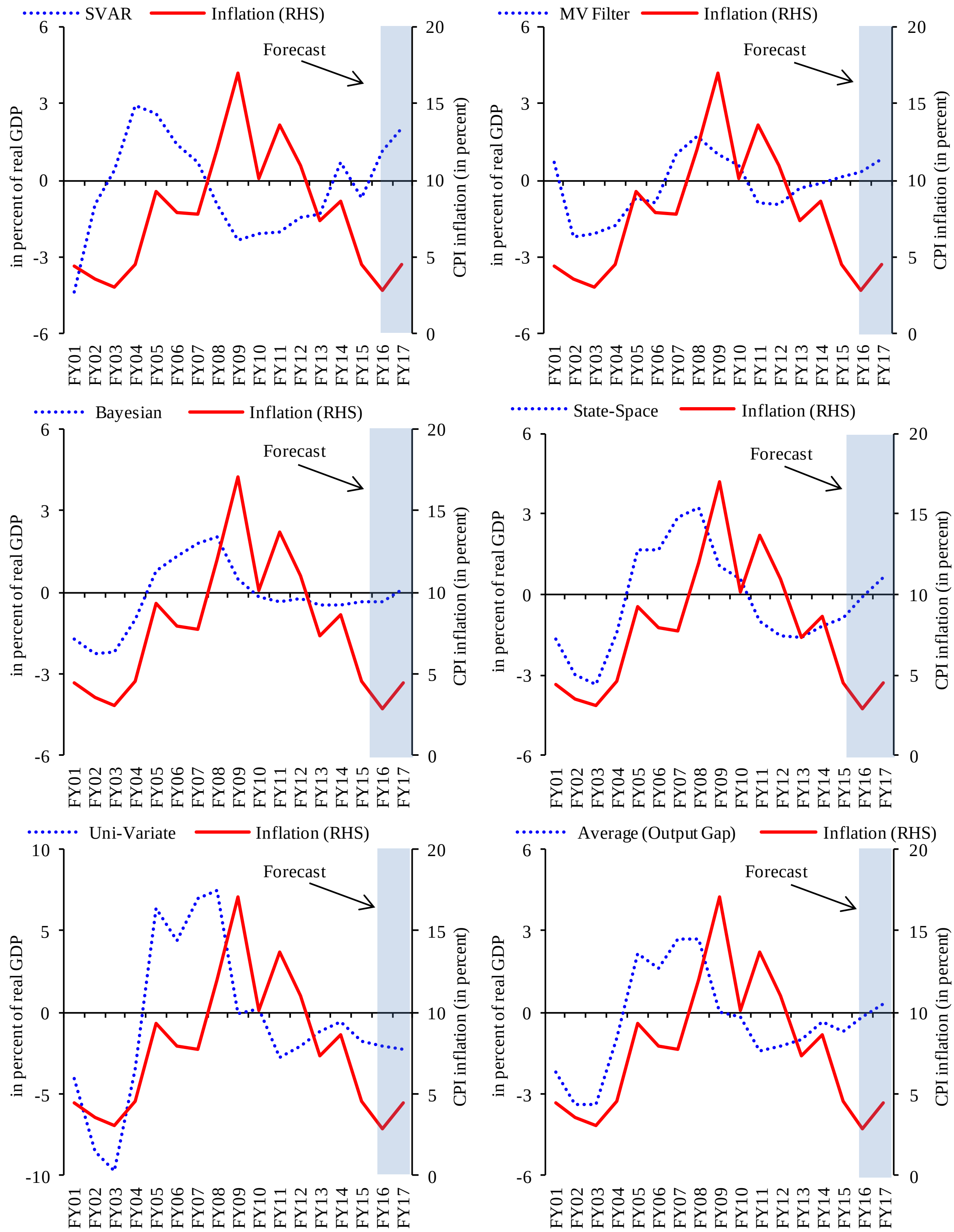


Figure C2: Quarterly Output Gap Estimates and CPI Inflation for Pakistan