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Muhammad Rehman
Sajawal Khan
Zafar Hayat



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# A Small Open Economy DSGE Model with Workers' Remittances

Muhammad Rehman, Monetary Policy Department, State Bank of Pakistan, Karachi

Sajawal Khan and Zafar Hayat, Research Department, State Bank of Pakistan, Karachi

**Abstract**

In this paper, we develop and estimate a small open economy Dynamic Stochastic General Equilibrium (DSGE) model with an enriched specification, which enables us to include a variable of high significance for Pakistan viz. workers' remittances. The results indicate that a positive shock to workers' remittances help boost real growth via increased consumption and imported investment and helps ease-off the pressure on current account balance and thereby exchange rate. Too much dependence on workers' remittances to help meet the trade deficits may potentially leave the economy in doldrums in case sizable negative shocks occur to the flow of foreign remittances. Therefore there is a need for structural reforms to help the economy out of the historical trade deficits, and decrease dependence on the workers' remittances source to allow for a sustainable economic growth.

**Keywords**: Business cycles, Workers remittances, Open economy.

**JEL Classification**: E32, F24, F43.

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Contact for correspondence:
Muhammad Rehman
Joint Director, Monetary Policy Department
State Bank of Pakistan
I.I. Chundrigar Road
Karachi 74000, Pakistan.
Email: muhammad.rehman@sbp.org.pk

**Non-technical Summary**

In this paper, we develop and estimate a small open economy Dynamic Stochastic General Equilibrium (DSGE) model with workers' remittances for Pakistan, a small open economy with prolonged history of trade deficits. The country is highly dependent on external resources, especially workers' remittances from abroad, to finance this deficit. This dependency renders Pakistan's economy vulnerable to external shocks.

The recent global slowdown and the vulnerability of gulf countries—major destination to our workers abroad—in terms of depleting fiscal space may build pressures in those countries and result in foreign workers' layoffs. This may cause at-least twofold problems for Pakistan's rather ailing economy. First, the absorption of the potentially laid off workers abroad in the domestic work force, could be a giant task to deal with, given the already high unemployment rates in the country. Second, a fall in remittances may further worsen the cushion available to finance the trade deficit, thereby forcing the economic managers of the country to resort to international sources of debt. External debt not only tends to be expensive in bad economic times but also that Pakistan has already accumulated a quarter of GDP as external debt.

Our results indicate that a negative shock to remittances has a negative effect on consumption, investment, labor demand, demand for imported goods, aggregate output, current account balance as well as the exchange rate. These results imply that Pakistan's economy is at risk on so many macroeconomic fronts in case of negative shocks to the workers 'remittances. Understandably this poses a nontrivial question to the economic managers on how well prepared the economy is to cope with in case exogenous shock is significant. To avoid the risk associated with remittances as a source of financing trade gap, structural reforms are needed to help the economy out of the historical trade deficits.

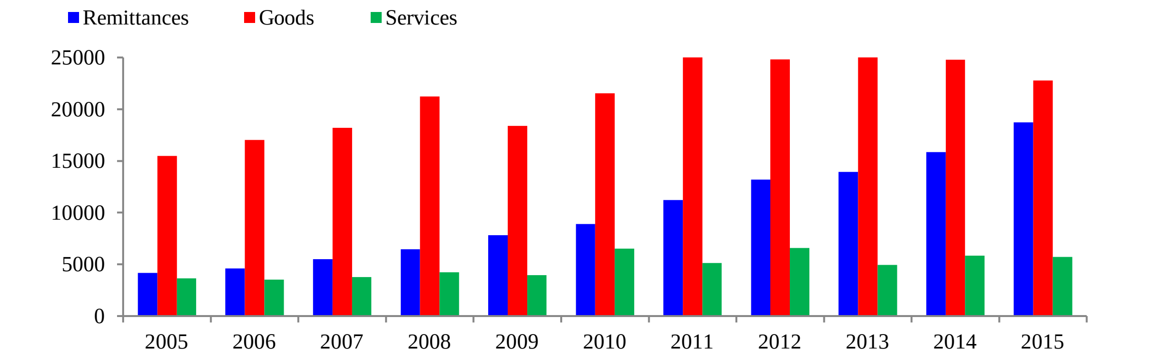
## 1. Introduction

In this paper we develop a DSGE model for Pakistan—a small open economy—with historical trade deficits. The country is highly dependent on external resources to finance its trade deficit. Although bulk of this deficit is met by workers' remittances (from Pakistanis working abroad), this dependency renders Pakistan's economy vulnerable to external shocks. Pakistan's debt levels have already been high—68% of GDP, with external debt at 24.5% of GDP —to help the economy out in case of any significant negative external shocks to workers' remittances. Therefore, there is a need for structural reforms to help bridge the trade gap to decrease dependence on remittances.[1]

The recent global slowdown and the vulnerability of gulf countries—major destination to our workers abroad—in the form of depleting fiscal space may build pressures in those countries thereby resulting in foreign workers' layoffs, hence causing at-least twofold problems for Pakistan' economy. First, the absorption of the potentially laid off workers abroad in the domestic work force would be a giant task, given the already high unemployment rates in the country. Second, a fall in remittances would further worsen the cushion available to finance the trade deficit thereby forcing the economic managers of the country to resort to international sources of debt. External debt not only tends to be expensive in bad economic times but as mentioned earlier has already been a quarter of GDP in Pakistan.

Any sizable shock to the workers' remittances is associated with huge risks of getting into traps of economic problems on many fronts with nontrivial consequences. The increasing dependence on the remittances, in case of Pakistan, can further be understood against other sources of foreign exchange such as proceeds from trade in goods and services sectors (see Figure 1).

**Figure 1: FX Proceeds from Goods and Services Exports and Remittances**




Source: Haver; (Units: Mil. USD)

With this brief background, we focus on examining the role of workers' remittances, an area, which to the best of our knowledge has not been addressed under a DSGE setting for Pakistan. It is nevertheless acknowledged that many DSGE models with distinct features have been developed for Pakistan recently, which, however are focused on other areas rather than the one we intend to examine. For example, Ahmad et al. (2012) developed a closed economy DSGE model with informality both in labor and product markets. Haider et al. (2013), on the other hand developed an open economy DSGE model based on New-Keynesian micro-foundations by incorporating informal labor and production sectors. Ahmed et al. (2014) while using Bayesian methodology estimated the structural shock parameters of Ahmed et al. (2012). Ali and Ahmad (2014) used DSGE models to explore the relative performance of inflation and price level targeting regimes under alternative monetary policy instruments. Also, Ahmad et al. (2016) went on to theoretically evaluate the role of

[1]Source: Economic Survey of Pakistan 2015–16.

money and monetary policy in propagating business cycle fluctuations for Pakistan. Finally, Choudhary et al. (2016) analyzed the pressures an expansionary fiscal policy exerts on the economy through credit markets in Pakistan.

Although in case of Pakistan, literature exploring the remittance channel through DSGE models is absent; internationally some studies have examined the role of this channel in a DSGE setting. There are two types of DSGE models in the literature that add the remittance channel. First type considers remittances as an exogenous shock (see for example Durdu and Sayan, 2008; Stepanyan and Tevosyan, 2008; and Punzi, 2009). In the second type of DSGE models, the remittances are allowed to be determined endogenously (see for example, Chami et al. 2006; Mandelman and Zlate, 2008; Acosta et al. 2009 and Rosario, 2010).

In our model one of the unique features compared to the other DSGE models developed for Pakistan is that household members are assumed to work either domestically or in a foreign country sending remittances to their families residing in their parent country. Our model is sticky-price and sticky-domestic wage small open economy model, which allows labor migration and remittances received by domestic households, and accounts for current accounts and debt dynamics.

The main findings of our research indicate that an increase in foreign remittances helps boost domestic consumption, and imported investment goods, reduces trade gap, and relaxes pressures on exchange rate and thereby enhances real growth. It has a positive impact on domestic inflation, which might have resulted from the increased level of disposable income and thereby consumption. The rest of the paper is structured as follows. In Section 2 we present the theoretical model. Section 3 describes the calibration of model parameters. Section 4 explains the impulse response functions obtained from the model while Section 5 concludes the paper.

## 2. The Model

Our main vehicle is a new Keynesian type small open economy DSGE model with three domestic agents viz. households, firms, and a government. The latter constitute both fiscal and monetary authorities. The households' preferences are a function of consumption and leisure whereas agents consume the goods produced not only by the domestic economy but imported goods as the final consumption good. The households supply a proportion of their labor to foreign firms whilst the remaining differentiated labor is provided to labor packers. These labor packers in turn supply labor to domestic intermediate firms under a perfectly competitive environment. Households accumulate capital both through domestic investment and foreign investments, and rent it out to intermediate firms. They also hold domestic and foreign bonds, the choice for which depends upon uncovered interest rate parity. The model has three tiers in production i.e. intermediate goods, final goods and exports. For the production of intermediate goods, capital and labor are used, which in turn are used to produce homogenous final goods. Part of the homogenous final goods is consumed domestically while the remaining is acquired by the exporters to be sold out to the rest of the world.

Broadly, the model contains three categories of firms; domestic, exporting and importing firms. Domestic firms are of two types i.e. intermediate goods producing firms and, final goods producing firms. For the production of intermediate goods, capital and labor are used, which in turn are used to produce homogenous final goods. Part of the homogenous final goods is consumed domestically while the remaining is acquired by the exporters who in turn package the final good for export purposes. Importing firms buy two types of homogenous good: consumption goods and investment goods in the world market. They package them and sell them to domestic households.

The fiscal authority accounts for the impact of the shocks originating from government expenditures. It issues domestic bonds and imposes taxes to generate funds for the purchase of consumption goods. Furthermore, we assume that the fiscal authority may take external debt to balance the external payments. The monetary authority on the other hand is assumed to set the nominal interest rate, which—for the sake of convenience—is accounted for through a simple Taylor rule.

### 2.1. Households

There is a continuum of households of measure one. We posit that the representative households' preferences may be expressed by the following function:

$$E_{j,0}\sum_{t=0}^{\infty}\beta^t\left(\log(C_t - hC_{t-1}) - \varphi_L\frac{(L_t)^{1+\sigma_L}}{1+\sigma_L}\right) \qquad (1)$$

Where the parameter $\beta \in (0,1)$ is a discount factor and the parameter $\sigma_L$ is the elasticity of work effort with respect to real wage. The variable $C_t$ is a composite consumption index of both domestically produced and imported goods denoted by $C_t^d$ and $C_t^m$, respectively. The $C_t$ can be expressed as:

$$C_t = \left[(1-\gamma_c)^{\frac{1}{\varepsilon_c}}(C_t^d)^{\frac{\varepsilon_c-1}{\varepsilon_c}} + \gamma_c^{\frac{1}{\varepsilon_c}}(C_t^m)^{\frac{\varepsilon_c-1}{\varepsilon_c}}\right]^{\frac{\varepsilon_c}{\varepsilon_c-1}} \qquad (2)$$

Where, the parameter $\varepsilon_c > 1$, is the elasticity of substitution between domestic and imported consumption goods, whilst the parameter $\gamma_c$ determines the share of import price index in overall price levels in the country as given below:

$$P_t = \left[(1-\gamma_c)(P_t^d)^{1-\varepsilon_c} + \gamma_c(P_t^m)^{1-\varepsilon_c}\right]^{\frac{1}{1-\varepsilon_c}} \qquad (3)$$

Given the composite demand function in Equation 2, households maximize their consumption subject to their budget constraint as:

$$C_t^d = (1-\gamma_c)\left(\frac{P_t^d}{P_t}\right)^{-\varepsilon_c} C_t \qquad (5)$$

$$C_t^m = \gamma_c\left(\frac{P_t^m}{P_t}\right)^{-\varepsilon_c} C_t \qquad (6)$$

Since the households provide capital services to intermediate firms, they can therefore increase their capital stock by investing $I_t$ in additional physical capital. The law of motion for the household's physical capital stock $K_t$ is given by:

$$K_t = (1-\delta)K_{t-1} + I_t \qquad (7)$$

As with consumption, total investment is assumed to be given by a constant elasticity of substitution (CES), which is an aggregate of the domestic $I_t^d$ and foreign investment $I_t^m$ goods such that:

$$I_t = \left[(1-\gamma_i)^{\frac{1}{\varepsilon_i}}(I_t^d)^{\frac{\varepsilon_i-1}{\varepsilon_i}} + \gamma_i^{\frac{1}{\varepsilon_i}}(I_t^m)^{\frac{\varepsilon_i-1}{\varepsilon_i}}\right]^{\frac{\varepsilon_i}{\varepsilon_i-1}} \qquad (8)$$

Where $\gamma_i$ is the share of imports in investment, and $\varepsilon_i$ is the elasticity of substitution between domestic and imported investment goods. Assuming that prices of investment goods and consumption goods are the same, optimal allocation of investment between domestic and imported investment goods is given by:

$$I_t^d = (1-\gamma_i)\left(\frac{P_t^d}{P_t}\right)^{-\varepsilon_i} I_t \qquad (9)$$

$$I_t^m = \gamma_i\left(\frac{P_t^m}{P_t}\right)^{-\varepsilon_i} I_t \qquad (10)$$

**2.1.1. Labor supply and wage decisions for domestic and migrated labor sectors**

The households supply a proportion of their labor to foreign firms depending upon the demand. Since wages are assumed to be already higher in host countries, and households make a small portion of the total wage earners abroad, they therefore do not have bargaining power to set wages abroad. The remaining differentiated labor $L_t^d$ is provided to labor packers who in turn supply them to domestic intermediate firms, presumably under a perfectly competitive environment—especially after packing/equipping the labor as per the firms' demand. Given the aforementioned, the aggregate labor supply can be expressed as follows:

$$L_t^d = \left(\int_0^1 (L_{l,t}^d)^{\frac{\theta_L-1}{\theta_L}} dl\right)^{\frac{\theta_L}{\theta_L-1}}, L_t^{mig} = \left(\int_0^1 \left(L_{l,t}^{mig}\right)^{\frac{\theta_L^*-1}{\theta_L^*}} dl\right)^{\frac{\theta_L^*}{\theta_L^*-1}}$$

Where $\theta_L$ and $\theta_L^*$ are elasticities of substitution between different labor types in the domestic labor supply $L_t^d$ and foreign labor supply $L_t^{mig}$ sectors. The conditional demand for domestic and migrated labor from households is given as:

$$L_{l,t}^d = \left(\frac{W_{l,t}^d}{W_t^d}\right)^{-\theta_L} L_t^d \qquad L_{l,t}^{mig} = \left(\frac{W_{l,t}^{mig}}{W_t^{mig}}\right)^{-\theta_L^*} L_t^{mig}$$

and given that the total labor demand from households is:

$$L_t = L_t^d + L_t^{mig} \qquad (11),$$

the wage cost for the domestic goods producers would be:

$$W_t^d = \left(\int_0^1 [W_{l,t}^d]^{1-\theta_L} dl\right)^{\frac{1}{1-\theta_L}}.$$

The households i set their domestic wages following a Calvo setting. In each period, households face $(1-\xi_{w^d})$ probability to re-optimize wages. All other households that provide labor for domestic firms can only partially index their wages by past inflation.

$$W_{t+k}^d(i) = X_{t+w}^w W_t^d(i)$$

where, $X_{t+w}^w = \pi_t.\pi_{t+1} \ldots\ldots \pi_{t+k}$, if $k > 0$, otherwise $X_{t+k}^w = 1$,

therefore, the relevant part of the Lagrangian for the household i is:

$$\Gamma^{d}(i) = E_t \sum_{k=0}^{\infty} \left(\beta \xi_{w^d}\right)^k \left[X_{t+w}^{w} \widehat{W}_t^{d} - MRS_{t+k}^{d}(i)\right] L_{l,t+k}^{d}.$$

The households will optimize with respect to $\widehat{W}_t^d$ . The maximized function as a result is as follows:

$$\widehat{W}_t^d = \frac{\theta_L}{(\theta_L - 1)} \frac{E_t \sum_{k=0}^{\infty} \left(\beta \xi_{w^d}\right)^k L_{l,t+k}^{d} MRS_{l,t+k}^{d}}{E_t \sum_{k=0}^{\infty} \left(\beta \xi_{w^d}\right)^k L_{l,t+k}^{d} \frac{P_{t+k-1}}{P_{t-1}}},$$

the aggregate wage expression therefore can be expressed as:

$$W_t^d = \left[\left(1 - \xi_{w^d}\right) \widehat{W_t^d}^{\,1-\theta_L} + \xi_{w^d} \left(\pi_{t-1}^{d} W_{t-1}^{d}\right)^{1-\theta_L}\right]^{1-\theta_L} \qquad (12)$$

The foreign wage $W_t^{mig}$ is set abroad and is therefore treated as exogenous for domestic households:

$$\log\left(\frac{W_t^{mig}}{W_{t-1}^{mig}}\right) = \left(1 - \rho_{W^{mig}}\right) \log\left(W^{mig}\right) + \rho_{W^{mig}} \log\left(\frac{W_{t-1}^{mig}}{W_{t-2}^{mig}}\right) + \omega_{W^{mig},t},$$

where $\rho_{W^{mig}}$ is autocorrelation coefficient and $\omega_{W^{mig},t}$ is a zero-mean, serially uncorrelated shock with standard errors $\sigma_{W^{mig}}$.

**2.1.2. Intertemporal optimizing households**

The households' total income constitutes five major components. First is the labors' disposable income $\left(1 - \tau_{inc,t}\right) W_t L_t$, where $\tau_{inc,t}$ is the income tax rate. Second is the rental income denoted by $RK, tK_{j,t}$. The third is the return on both the domestic as well as foreign bonds denoted by $B_{t-1}$, and $e_t F_{j,t-1}$ respectively. Fourth are the profits of domestic firms producing final goods [$Div_t^D$] and fifth are the profits of exporting and importing firms denoted by $Div_t^X$ and $Div_t^m$, respectively. It may be noted that $e_t$ represents the nominal exchange rate.

Here it is important to mention that we introduce remittances, which is the focus of this research via households' budget constraint. Here we assume that the household labor income is the sum of domestic labor income ${W_t}^D L_{j,t}^D$ as well as remittances, which enters the model through the term $\gamma_{t,j}^{mig} e_t {W_t}^{mig} L_{j,t}^{mig}$. It is important to mention that we assume that some of the household members work within the country and the others from the same household work abroad. We further assume that the household members that work abroad send a proportion $\gamma_{t,j}^{mig}$ of their earnings ${W_t}^{mig} L_{j,t}^{mig}$ to their families in Pakistan, while the rest is presumably consumed by them.

The households maximize Equation 1 subject to the following intertemporal budget constraint:

$$\frac{e_t F_t}{(1 + R_t^*)} + \frac{B_t}{(1 + R_t)} + \left(1 + \tau_{c,t}\right) P_t C_t + P_t I_t$$

$$= e_t F_{t-1} + B_{t-1} + \left(1 - \tau_{inc,t}\right) {W_t}^d L_{j,t}^d + \gamma_t^{mig} e_t {W_t}^{mig} L_{j,t}^{mig} + R_{K,t} K_t + {Div_t}^d + {Div_t}^X + GT_t \qquad (13)$$

GT is the government transfer.

The household's first-order conditions are as follows:

$$\frac{W_t{}^d}{P_t} = \frac{\left(1+\tau_{c,t}\right)}{\left(1-\tau_{inc,t}\right)}\ \frac{\varphi_L\left(L_t^d\right)^{\sigma_L}}{((C_t - hC_{t-1})^{-1} - \beta h E_0(C_{t+1} - hC_t)^{-1})} \quad (14)$$

$$\frac{e_t}{e_{t+1}} = \frac{(1+R_t^*)}{(1+R_t)} \quad (15)$$

$$1 = \frac{1}{(1+R_t)}\left(\pi_{t+1}(1-\delta) + \frac{R_{K,t}}{P_t}\right) \quad (16)$$

**2.2 Firms**

At firms' level, we suppose three tiers in domestic production i.e. the firms that produce the final goods, the firms that produce intermediate goods and finally the exporting firms.

**2.2.1 The final goods sector**

The final homogeneous goods $Y_t^d$ are produced presumably using a CES technology with a continuum of domestic intermediate goods $Y_{i,t}^d$ as inputs such that:

$$Y_t^d = \left(\int_0^1 \left(Y_{i,t}^d\right)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}} \quad (17)$$

Here θ is the elasticity of substitution among differentiated intermediate goods as well as input and output prices of the firms—denoted by $P_{i,t}^d$ and $P_t^d$ , respectively—that produce final goods are conditional as given. Profit maximization on their part thus leads to the following first order condition:

$$\frac{Y_{i,t}^d}{Y_t^d} = \left(\frac{P_{i,t}^d}{P_t^d}\right)^{-\theta} \quad (18)$$

By integrating (18) and using (17), we obtain the following relation between the prices of the final goods and the prices of intermediate goods:

$$P_t^d = \left(\int_0^1 \left(P_{i,t}^d\right)^{1-\theta} di\right)^{\frac{1}{1-\theta}} \quad (19)$$

Here it may be noted that some of these good are consumed and invested domestically while others are exported such that:

$$Y_t^d = I_t^d + C_t^d + G_t \quad (20)$$

Where G is the government expenditures.

**2.2.2 Intermediate goods sector**

We assume that there is a continuum of domestic intermediate firms of measure one. Production is carried out by means of two major input factors—i.e. labor and capital—expressed as:

$$Y_{i,t}^d = A_t K_{i,t}^{\alpha}\, L_{i,t}^{d\ (1-\alpha)} \quad (21)$$

The parameter $\alpha$ is the share of capital in intermediate production. $A_t$ is an aggregate technology shock that follows a stochastic process given by:

$$\log A_t = (1-\rho_A)A + \rho_A \log(A_{t-1}) + \omega_{A,t}.$$

Where $\rho_A$ is the degree of persistence pertaining to technological shock and $\omega_{A,t}$ is a normally distributed, serially uncorrelated shock with zero mean and standard deviation $\sigma_{A,t}$. The firms that produce intermediate goods maximize their profits subject to the available production technology thereby setting demand for labor and capital given as:

$$\frac{K_{i,t}}{L_{i,t}} = \frac{\alpha}{(1-\alpha)} \frac{w_t^d}{r_{K,t}} \qquad (22)$$

where,

$$w_t^d = \frac{W_t^d}{P_t}, \text{ and } r_{K,t} = \frac{R_{k,t}}{P_t}.$$

The firm's real marginal cost is given by:

$$mc_{i,t}^d = \frac{1}{A_t} \alpha^{-\alpha} (1-\alpha)^{-(1-\alpha)} {w_t^d}^{1-\alpha} r_{K,t}^{\alpha} \qquad (23)$$

Each firm i produces differentiated goods and therefore has a market power in price setting decisions. As in Calvo (1983), firms are not allowed to change their prices unless they receive a random "price-change signal". We assume that in each period, there is a probability $\xi_{p^d}$ that intermediate producers will not be able to re-optimize their price and therefore index it according to:

$$P_{i,t+k}^d = N_{t+k}^d P_{i,t}^d$$

where,

$$N_{t+k}^d = \pi_t^d . \pi_{t+1}^d \dots \dots \pi_{t+k}^d, \text{ if } k > 0, \text{ otherwise } \pi_t = 1$$

$$\pi_t^d = \frac{P_t^d}{P_{t-1}^d}.$$

The probability that a given price can be re-optimized in any particular period is constant and equal to $1-\xi_{p^d}$. Profit optimization by producers that are "allowed" to re-optimize their prices at time $t$ maximizes their profit function subject to price $\widehat{P}_{i,t}^d$. Firm i optimizes following profit function:

$$\max_{\widehat{P}_{i,t}^d} \Omega^p = E_t \sum_{s=0}^{\infty} \left(\beta \xi_{p^d}\right)^s \left[N_{t+s}^d \widetilde{P}_{i,t}^d - {MC^d}_{i,t+s}\right] Y_{i,t+s}^d$$

which results in:

$$\widehat{P}_{i,t}^d = \left(\frac{\theta}{\theta-1}\right) \left( \frac{E_t \sum_{s=0}^{\infty} \left(\beta \xi_{p^d}\right)^s {MC^d}_{i,t+s} Y_{i,t+s}^d}{E_t \sum_{s=0}^{\infty} \left(\beta \xi_{p^d}\right)^s Y_{i,t+s}^d \pi_{t+s}^d} \right) \qquad (24)$$

The aggregate price level is given by:

$$P_t^d = \left[\left(1 - \xi_{p^d}\right)\left(\widehat{P}_{i,t}^d\right)^{1-\theta} + \xi_{p^d}\left(\pi_{t-1}^d P_{t-1}^d\right)^{1-\theta}\right]^{\frac{1}{1-\theta}} \qquad (25)$$

Log-linearizing and combining equations (21) and (23) yields the following aggregate Phillips curve relationship:

$$\tilde{\pi}_t^d == \frac{1}{(1+\beta)}\tilde{\pi}_{t-1}^d + \frac{\beta}{(1+\beta)}\tilde{\pi}_{t+1}^d + \frac{\left(1-\xi_{p^d}\right)\left(1-\beta\xi_{p^d}\right)}{\xi_{p^d}(1+\beta)}\widetilde{mc}_t^d \qquad (26)$$

### 2.2.3 The exporting firms sector

Here we suppose that there is a continuum of exporting firms indexed by s ∈ [0 1]. Each exporting firm buys a homogeneous final good and sells it to the rest of the world. The demand for these goods comes from the households of foreign economy, who decide whether to consume or invest the goods they import. We assume that their total consumption and investment basket follows a CES function. Therefore total demand for the goods of the exporting firms can be written as:

$$Y_t^X = \left(\frac{P_t^x}{P_t^*}\right)^{-\theta^{mx}} Y_t^* \qquad (27)$$

Where $\theta^{mx}$ is the elasticity of substitution between domestic and foreign goods in a foreign economy. $P_t^X$ is export price index, whereas $Y_t^*$ and $P_t^*$ are the output prices and the prices in the rest of the world, respectively. We assume that all the macroeconomic variables in a foreign country are exogenous and follow an AR(1) process such that:

$$\log\left(\frac{P_t^*}{P_{t-1}^*}\right) = \left(1 - \rho_{\pi^*}\right)\log(\pi^*) + \rho_{\pi^*}\log\left(\frac{P_{t-1}^*}{P_{t-2}^*}\right) + \omega_{\pi^*,t}$$

and,

$$\log(Y_t^*) = \left(1-\rho_{Y^*}\right)\log(Y^*) + \rho_{Y^*}\log(Y_{t-1}^*) + \omega_{Y^*,t}.$$

Where $\pi^*$ is the steady-state rate of foreign inflation whereas $\rho_{\pi^*}$ and $\rho_{Y^*}$ are the autocorrelation coefficients. $\omega_{\pi^*,t}$ and $\omega_{Y^*,t}$ are zero-mean and serially uncorrelated shocks with standard errors $\sigma_{\pi^*}$ and $\sigma_{Y^*}$, respectively. Just like the case with domestic prices, we assume that export prices also follow a Calvo process. The log-linearized aggregate export price inflation thus is given by:

$$(1+\beta)\tilde{\pi}_t^x = \tilde{\pi}_{t-1}^x + \beta\tilde{\pi}_{t+1}^x + \frac{\left(1-\xi_{p^x}\right)\left(1-\beta\xi_{p^x}\right)}{\xi_{p^x}}\widetilde{mc}_t^x \qquad (28)$$

where,

$$\widetilde{mc}_t^x = \widetilde{P}_t - \tilde{e}_t - \widetilde{P}_t^x \qquad (29)$$

### 2.3 Imports

The import sector consists of foreign firms that buy a homogenous good in the world market at price $P_t^*$. We presume that there are two different types of these importing firms: one that turns the imported product into a differentiated consumption good $C_t^{m1}$, and the other that turns it into a differentiated investment good $I_t^m$. In each of these two categories, there is a continuum of importing firms that sell their differentiated consumption goods to households $C_t^m$ and the government $G_t^m$, whereas the investment goods are being sold to the households $I_t^m$.

$$Y_t^m = C_t^m + I_t^m + G_t^m \qquad (30)$$

let the private and public consumptions be expressed as:

$$C_t^{m1} = C_t^m + G_t^m.$$

The different importing firms buy the homogenous good at price $P_t^*$ in the world market. The final imported consumption and investment goods are a composite of a continuum of i and r differentiated imported consumption and investment goods, each supplied by a different firm, which we assume to follow the CES function:

$$C_t^{m1} = \left(\int_0^1 \left(C_{i,t}^{m1}\right)^{\frac{\theta^{mc}-1}{\theta^{mc}}} di\right)^{\frac{\theta^{mc}}{\theta^{mc}-1}} \qquad (31)$$

and,

$$I_t^m = \left(\int_0^1 \left(I_{r,t}^m\right)^{\frac{\theta^{mi}-1}{\theta^{mi}}} dr\right)^{\frac{\theta^{mi}}{\theta^{mi}-1}} \qquad (32)$$

The demand for the respective differentiated imported consumption and investment goods is given by:

$$C_{i,t}^{m1} = \left(\frac{P_{i,t}^m}{P_t^m}\right)^{-\theta^{mc}} C_t^m \qquad (33)$$

and,

$$I_{r,t}^m = \left(\frac{P_{r,t}^m}{P_t^m}\right)^{-\theta^{mi}} I_t^m \qquad (34)$$

Where, for the sake of computational convenience, we assume that $\theta^{mc} = \theta^{mi}$. In order to allow for incomplete exchange rate pass-through to the consumption and investment import prices, we assume local currency price stickiness. Phillips curve relations for the imported consumption and investment good can be expressed as:

$$(1+\beta)\tilde{\pi}_t^m = \tilde{\pi}_{t-1}^m + \beta\tilde{\pi}_{t+1}^m + \frac{\left(1-\xi_{p^m}\right)\left(1-\beta\xi_{p^m}\right)}{\xi_{p^d}} \widetilde{mc}_t^m \quad (35)$$

where,

$$\widetilde{mc}_t^m = \tilde{P}_t^* + \tilde{e}_t - \tilde{P}_t^m.$$

### 2.4 The fiscal sector (authorities)

Fiscal authority purchases the composite consumption good $G_t$, issue bonds $B_t$, and raises taxes $\tau_c$ and $\tau_{inc}$. Like private consumption, we also assume that the government consumption $G_t$ is a CES index of domestically produced goods as well as imported goods such that:

$$G_t = \left[ \left(1-\gamma_{cg}\right)^{\frac{1}{\varepsilon_c}} \left(G_t^d\right)^{\frac{\varepsilon_c - 1}{\varepsilon_c}} + \gamma_{cg}^{\frac{1}{\varepsilon_c}} \left(G_t^m\right)^{\frac{\varepsilon_c - 1}{\varepsilon_c}} \right]^{\frac{\varepsilon_c}{\varepsilon_c - 1}} \qquad (36)$$

As was the case with private consumption, here we posit that the government purchases have the same intra-temporal elasticity of substitution i.e. $\varepsilon_c > 1$. The parameter $\gamma_{cg}$ in Equation 36 determines the share of imports in public consumption goods. The government demand for these goods can be expressed as:

$$G_t^d = \left(1-\gamma_{cg}\right)\left(\frac{P_t^d}{P_t}\right)^{-\varepsilon_c} G_t \qquad (37)$$

$$G_t^m = \gamma_{cg}\left(\frac{P_t^m}{P_t}\right)^{-\varepsilon_c} G_t \qquad (38)$$

#### 2.4.1 Government budget and deficit

The government revenues $GR_t$ are defined as follows:

$$GR_t = \tau_{c,t} P_t^c C_{j,t} + \tau_{inc,t} W_t L_{j,t} \qquad (39)$$

$$G_t = \tau_{c,t} P_t^c C_{j,t} + \tau_{inc,t} W_t L_{j,t} + B_t - (1+R_{t-1})B_{t-1} + e_t D_t^* - (1+R_{t-1}^*) e_t D_{t-1}^*.$$

Where, income from taxes is the essential revenue of the state budget. Personal income tax revenues are dependent on wages and employment level $(W_t L_{j,t})$ and the tax rate imposed, which here is represented by implicit personal income tax rate $\tau_{inc,t}$. The excise taxes are modeled by one implicit tax rate on consumption $\tau_{c,t}$, which is imposed on nominal consumption $P_t^c C_{j,t}$. By subtracting revenues from expenditures we may easily derive a primary government deficit:

$$DF_t = G_t - GR_t \qquad (40),$$

which is accumulated into debt such that:

$$\frac{B_t}{(1+R_t)} = DF_t + B_{t-1} - e_t D_t^* + (1+R_t^*) e_t D_{t-1}^* \qquad (41).$$

The variable $D_t^*$ represents external debt at time t, and the government consumption $G_t = \bar{g} * g_t$ while the tax rates follows an AR(1) process such that:

$$g_t = \left(1-\rho_{1,g}\right) g + \rho_{1,g} g_t + \omega_{g,t}$$

$$\tau_{c,t} = \left(1-\rho_{\tau_c}\right)\tau_c + \rho_{\tau_c}\tau_{c,t} + \omega_{\tau_c,t}$$

$$\tau_{inc,t} = \left(1 - \rho_{\tau_{inc}}\right)\tau_{inc} + \rho_{\tau_{inc}}\tau_{inc,t} + \omega_{\tau_{inc},t}.$$

Where $\omega_{g,t}$ , $\omega_{\tau_c,t}$ and $\omega_{\tau_{inc},t}$ are normally distributed, serially uncorrelated shocks with zero mean and standard deviation $\sigma_{g,t}$, $\sigma_{\tau_c,t}$ and $\sigma_{\tau_{inc},t}$, respectively. Since the variable $D_t^*$ represents external debt at time t, the government therefore faces the following budget constraint:

$$G_t = \tau_{c,t}P_t^c C_{j,t} + \tau_{inc,t}W_tL_{j,t} + \frac{B_t}{(1+R_t)} - B_{t-1} + e_tD_t^* - (1+R_{t-1}^*)e_tD_{t-1}^* \quad (42)$$

**2.5 Monetary sector (authorities)**

Consistent with Smets and Wouter (2003), we assume that the monetary authority sets the nominal interest rate according to a simple Taylor rule such that:

$$\frac{R_t}{R} = \varepsilon_{R,t}\left(\frac{R_{t-1}}{R}\right)^{\rho_R}\left(\left(\frac{\pi_t}{\pi}\right)^{\psi_1}\left(\frac{Y_t}{Y^p}\right)^{\psi_2}\right)^{1-\rho_R} \quad (43)$$

Where $R$ and $\pi$ are steady state real interest rate and inflation respectively, whereas $Y^p$ is potential output at time t. The parameter $\rho_R$ is the degree of interest rate smoothing in interest rate rule and $\varepsilon_{R,t}$ denotes nominal interest rate shocks.

**2.6 Aggregate equilibrium conditions**

Broadly, the current account equation can be expressed as:

$$CA_t = P_t^xY_t^x - \frac{P_t^mY_t^m}{e_t} + \gamma_t^{mig}W_t^{mig}L_{j,t}^{mig} \quad (44)$$

also,

$$Y_t = Y_t^d + Y_t^X \quad (45)$$

**3. Calibrations**

Since calibration requires a number of values for the parameters—22 in our model—utmost care has been taken to adopt values from comprehensive and latest possible available studies/sources. In case where the parameter values are not available from relevant research studies on Pakistan, we rely on international studies. Table 1 summarizes the parameter values for the purpose of calibration. The discount factor $\beta$ is given a standard value 0.978.[2] The weight on leisure in the utility function $\varphi_L$ is calibrated at 1.27 (see Ahmad, 2016) so that the representative household spends about one third of its total working time in the steady state.[3]

We assume unit elasticity of work effort with respect to real wage. The consumption habit persistence parameter h is calibrated from quarterly consumption series from Hanif et al. (2013) and set at 0.63. The share of imports in aggregate consumption $\gamma_c$ is calibrated using the expression

[2]The steady-state discount factor is estimated by $\beta = \frac{1}{1+R}$. Where $R$ is the long run average real interest rate. In Pakistan average of 3-month T-bills rate from July–1998 to October–2014 is 8.922. Corresponding to this value quarterly $\beta$ would be 0.978.

[3] Standard working time in Pakistan is 8 hours.

$$\frac{C_t^m}{C_t^d} = \frac{\gamma_c}{(1-\gamma_c)}\left(\frac{P_t^m}{P_t^d}\right)^{-\varepsilon_c}$$

and assuming that: $\frac{P_t^m}{P_t^d} = 1$, in steady state.

where, $C_t^d$ =Total private consumption–0.7422*Total imports, and $C_t^m$ =0.7422*Total imports.[4] The results implies that $\gamma_c = 0.22$. The share of imports in aggregate investment $\gamma_i$ is calibrated in similar fashion i.e.

$$\frac{I_t^m}{I_t^d} = \frac{\gamma_i}{(1-\gamma_i)}\left(\frac{P_t^m}{P_t^d}\right)^{-\varepsilon_i}$$

whilst assuming that: $\frac{P_t^m}{P_t^d} = 1$, in steady state. Where, $I_t^d$ =Total private investment–0.1837*Total Imports, and $I_t^m$ =0.1837*Total Imports.[5] The substitution elasticity between domestic and imported consumption/investment goods $\varepsilon_c/\varepsilon_i$ is set at 2 to balance composite consumption/investment index equation. Consistent with Ali (2014), we set the share of imports in aggregate investment $\gamma_i$ and the share of imports in real government consumption $\gamma_{cg}$ equal to 0.27 and 0.15, respectively. We set the share of import prices $\gamma_i$ equal to 0.091 based on the share of the imported items in the CPI basket. The substitution elasticity between domestic and imported investment goods $\varepsilon_i$ is set at 6 to balance composite investment index equation.

Following Ahmed et al. (2012), we set the parameter of depreciation rate $\delta$ at 0.016. Based on the study of Choudhary et al. (2014) the domestic price and wage stickiness parameters are set at 0.34 and 0.25. Consistent with Smets and Wouter (2003) we assume Calvo parameter for imported goods same as domestic Calvo parameter. Similarly, we also assume the price stickiness $\xi_{p^x}$ parameter for exports at 0.6. The price stickiness $\xi_{p^x}$ parameter for exports is assumed to be 0.6. This is domestic Calvo parameter usually used for USA. Consistent with the data collected by the Federal Board of Revenue, the steady state taxes on consumption and labor are chosen so that $\tau_c = 0.10$ and $\tau_{inc} = 0.05$ respectively. We estimate log-linearized version of Taylor rule using quarterly data of 3-Month Treasury bill rate. The estimated coefficients of regression are used in our model[6]. We fix α=0.6 to replicate the capital share in real GDP. The elasticity of the derived demand for the imported/exported good $\theta^{mc}/\theta^{mx}$ is set at 0.98/5 as in Beltran (2008).

[4] Note that 0.7422 is the ratio of consumption goods in total imports. This number is calculated by Ali (2014).
[5] Note that 0.1837 is the ratio of investment goods in total imports. This number is calculated by Ali (2014).

[6] The detail of procedure is available in Ahmad et al. (2012)

**Table 1: Parameter values for calibrations**

| Parameter definition | value | Parameter definition | value |
|---|---|---|---|
| 1. Discount factor( $\beta$) | 0.978 | 12. Calvo Parameter (Domestic) ($\xi_{p^d}$ ) | 0.25 |
| 2. Weight of leisure in the utility function ($\varphi L$) | 1.27 | 13. Calvo Parameter (Imported) ($\xi_{p^m}$ ) | 0.25 |
| 3. Elasticity of work effort with respect to real wage ($\sigma L$) | 1 | 14. Calvo Parameter (Export) ($\xi_{p^x}$ ) | 0.6 |
| 4. Consumption habit persistence (h) | 0.63 | 15. Steady state consumption tax rate ($\tau_c$ ) | 0.10 |
| 5. Subs. elast. b/w domestic/imported consumption goods ($\varepsilon_c$) | 2 | 16. Steady state income tax rate ($\tau_{in,c}$ ) | 0.05 |
| 6. Subs. elast. b/w domestic / imported investment goods ($\varepsilon_i$) | 6 | 17. Interest rate smoothing Parameter ($\rho_R$) | 0.59 |
| 7. Share of import prices in CPI index ($\gamma_c$) | 0.091 | 18. Weight assigned to inflation in Taylor rule ($\psi_1$) | 2.845 |
| 8. The share of imports in aggregate investment ($\gamma_i$) | 0.27 | 19. Weight assigned to output gap in Taylor rule ($\psi_2$) | 0.25 |
| 9. Share of imports in real government consumption($\gamma_{cg}$) | 0.15 | 20. Capital share in intermediate domestic production ($\alpha$) | 0.6 |
| 10. Capital depreciation rate($\delta$) | 0.016 | 21. Subs. elast. b/w in domestic/imported consumption goods in foreign economy ($\theta^{mx}$) | 5 |
| 11. Wage Calvo Parameter ($\xi_{w^d}$) | 0.34 | 22. The elasticity of the derived demand for the imported good ($\theta^{mc}$) | 0.98 |

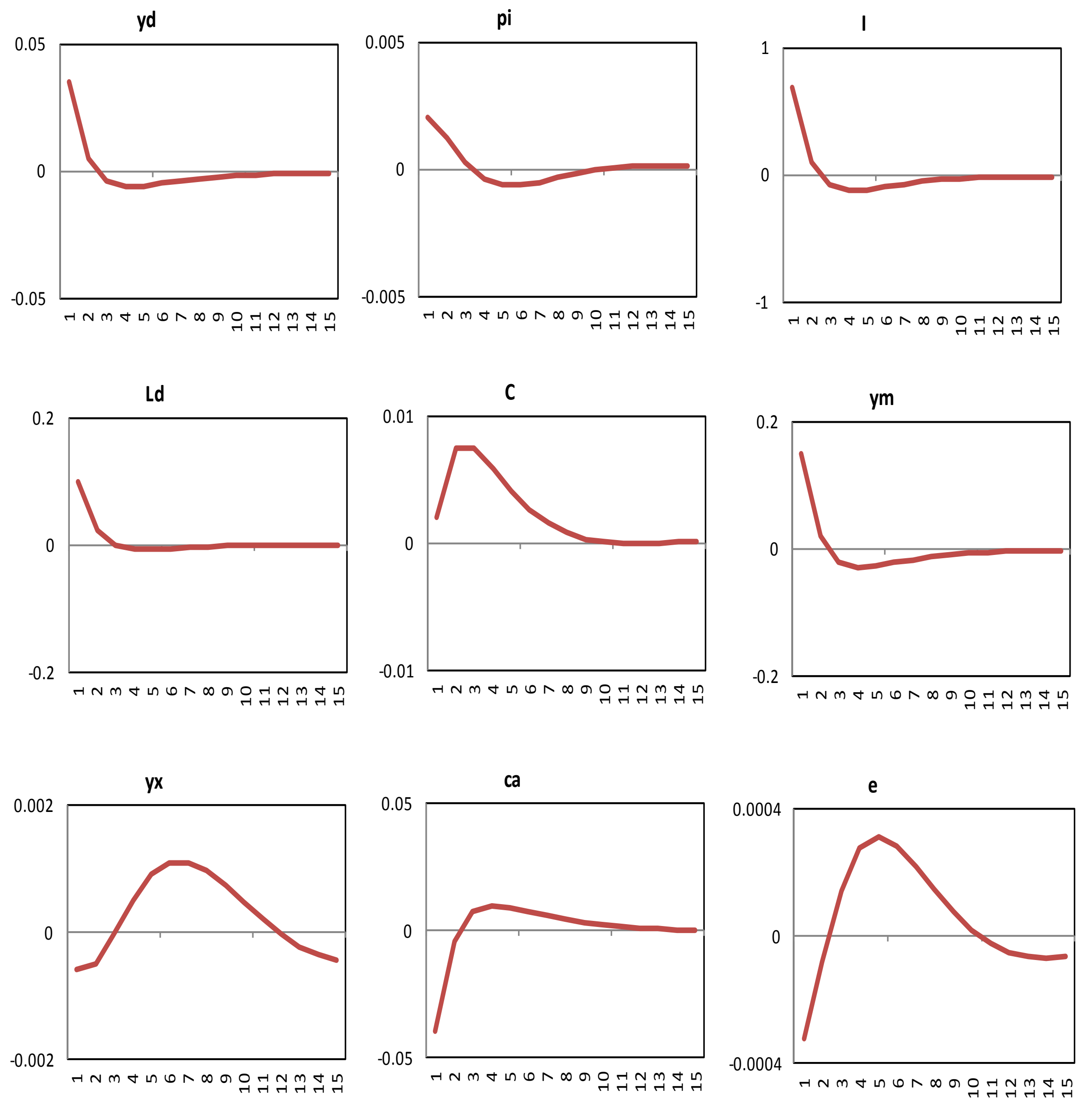
## 4. Impulse response functions

Since our main interest lies in the analysis of the responses of different macroeconomic indicators to an exogenous shock to remittances, we present the impulse responses obtained on the basis of calibrated estimations in Figure 1. As expected, a positive shock to remittances increases the demand for domestic as well as foreign consumption goods due to increased households' disposable income. Labor demand and investment spending, by the domestic firms, increases as a result of higher consumption which help boost the real output.

Initially the exchange rate appreciates due to increase in remittances but later it comes under pressure. This phenomenon in part may be explained by the notion that, although with a certain lag, the increase in imports (due to increase in aggregate consumption and aggregate demand) is more than the increase in exports thereby putting pressure on exchange rate to depreciate. This might be the case because the export sector of Pakistan has been stagnating historically due to multifaceted structural and entrepreneurial problems (SBP, 2015), which might not allow a catch up in growth in exports to the extent to more than off-set the increase in import demand, thereby causing the current account balance to deteriorate.

With an increase in workers' remittances, initially the inflation increases up to 3 quarters but then slightly falls below its steady state level for up to 7–8 quarters. The initial increase in inflation seems to be explained by an increase in aggregate consumption, investment, labor demand and output.

**Figure 1: Impulse Response Functions for a Positive Shock to Workers' Remittance**




## 5. Conclusion

In this paper, we have developed a small open economy DSGE model for Pakistan to examine the role of remittances in terms of explaining crucial macroeconomic indicators of interest. Using an enriched model specification, to allow for the analysis of the remittances channel, we found that a positive shock to remittances helps boost growth in consumption, investment, labor demand, demand for imported goods and aggregate output, whereas it improves both the current account balance as well as exchange rate, temporarily. The positive effect nevertheless is more pronounced and prolonged in case of aggregate consumption than any other variable followed by inflation, which is a downside of workers' remittances. In terms of the negative effects, there is an initial adverse effect on the aggregate exports, which might be due to exchange rate appreciation.

These results indicate that, on balance, the overall positive macroeconomic effects of remittances are nontrivial. On the flipside these results imply that Pakistan's economy is at risk on so many macroeconomic fronts in case negative shocks occur to the workers' remittances. Understandably, this poses a nontrivial question to the economic managers vis-à-vis how well the economy is prepared to cope with such exogenous shock. Given external debt already one quarter of GDP, too much dependence on workers' remittances calls for structural reforms to fix the trade deficit.

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