



# SBP Working Paper Series

No. 53

July, 2012

## Estimation of Discount Factor $\beta$ and Coefficient of Relative Risk Aversion $\gamma$ in Selected Countries

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STATE BANK OF PAKISTAN

# SBP Working Paper Series

Editor: Riaz Riazuddin

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Editor,  
SBP Working Paper Series  
Research Department,  
State Bank of Pakistan,  
I.I. Chundrigar Road, P.O. Box No. 4456,  
Karachi 74000. Pakistan

Published by: Editor, SBP Working Paper Series, State Bank of Pakistan, I.I. Chundrigar Road, Karachi, Pakistan.

ISSN 1997-3802 (Print)  
ISSN 1997-3810 (Online)

<http://www.sbp.org.pk>

Printed at the SBPBSC (Bank) – Printing Press, Karachi, Pakistan

## Estimation of Discount Factor $\beta$ and Coefficient of Relative Risk Aversion $\gamma$ in Selected Countries

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### Abstract:

We estimate the long-run discount factor for a group of developed and developing countries through standard methodology incorporating adaptive expectations of inflation. We find that the discount factor of developing countries is relatively nearer to unity as compared to that of the developed countries. In the second part, while considering a standard Euler equation for household's intertemporal consumption, we estimate the parameter of constant relative risk aversion (CRRA) for Pakistan by using the Generalized Method of Moments (GMM) approach. The resulting parameter value of CRRA conforms to the empirical range for developing countries (as given in, Cardenas and Carpenter, 2008) The GMM estimator for the discount factor reinforces its result from the first part of the paper. Consequently we show that different combination values for both the parameters result in different (in terms of magnitude) impulse response functions, in response to tight monetary policy shocks in a simple New Keynesian macroeconomic model.

### Acknowledgment:

This research would not have been possible without the help and advice of Director Research M. Ali Choudhary. We are thankful to Hamza Ali Malik, Sajawal Khan, and Farooq Pasha for their comments and suggestions. Any errors or omissions are the responsibility of the authors. Finally, views expressed here are those of the authors and not necessarily of the State Bank of Pakistan.

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# 1 Introduction

It is well understood in economics that when there is a need for intertemporal optimization, it raises the likelihood for discounting of future returns into present day terms, *i.e.*, when economic decisions affecting both the present and the future are being made. All economic agents hence face such discounting issues and tend to have their specific discount factors, which may differ within homogeneous as well as across heterogeneous agent space. Households have to intertemporally distribute consumption in order to maximize utility, firms have to manage production over time such that their profits get maximized, policy makers on the other hand are concerned with inter-generational issues, arising due to different fiscal policy stances, and are concerned with maximization of overall social welfare, etc. In short, all economic agents are exposed to the discount factor while dealing intertemporally.

When economic agents make intertemporal decisions they do attach some degree of risk to the future realization of returns. When this degree of risk aversion tends to remain fixed intertemporally then its value is depicted by the intertemporally fixed coefficient of "constant relative risk aversion" (CRRA). Knowing the degree of risk aversion helps in determining the exact extent of the impact of certain policy shocks on real economic variables.

In this paper we make a point that the discount factor of emerging economies is different from that of the developed economies. We also estimate the parameter of constant relative risk aversion (CRRA)  $\gamma$ , through an Euler equation, which shows how households give downward risk, *i.e.*, changes in stable present consumption, a greater weight than upside risk, *i.e.*, likelihood of a windfall raise in future consumption. The purpose of the estimation of these two parameters is purely to highlight their importance, let alone bringing refinement in their representation and estimation, in being different from those of the developed countries following standard calculation and estimation techniques. Since both of these parameters require rather simple and easily available data series (despite of the data frequency issues) we motivate researchers to calibrate these parameters using country specific data rather than relying on the empirically available values for developed countries.

The paper follows with the discussion, theoretical parameterization and calculation of the discount factor in the next section. Then the third section is divided into various subsections outlining the literature, construction and estimation of Euler equation for consumption. Statistical testing procedures and results for the coefficient of CRRA follows. In the end, we also show that different values of the discount factor and corresponding CRRA coefficient provide impulse responses with different magnitudes from a simple calibrated macroeconomic model.

## 2 Intertemporal Discounting

Macroeconomic models are complex in nature as they try to establish links between different economic agents who are all optimizing their returns intertemporally. In doing so these agents need to discount their future returns all the times in order to make decisions in the present. No doubt, when we talk about discount factor it is as subjective as it can be. All households or producers in the same economy can have different basis for their different subjective discount factors. As a result, empirically speaking, there are numerous discount factors in an economy at any given time. But the real issue is that how to first capture this vast information on discount factors and second how to use them in an economic model which is already mathematically challenging. Same has been concluded by Goldin (2007).

The issue of discount factor subjectivity which makes macroeconomic modeling a challenge, both in its solution and tractability, has to be resolved so that such a proxy be found that can represent the discount factor for all the heterogeneous economic agents in the model. As a result we keep reverting back to the standard Fisher equation (see the model in section 2.2 below) that links interest rate and inflation to the discount factor. Interest rate has two definitions to it, one in nominal terms and the other in real terms. Simply, interest rate is the nominal rate of return on capital assets whereas when it is adjusted for inflation it is called real interest rate. Nominal interest rate is not the appropriate anchor of interest rate

as inflation has to be considered in decision making by the economic agents because it tends to erode the real value of assets. Future inflation expectations based on lagged inflation level (technically explained by the term "adaptive inflation expectations") are generally used to convert the interest rate from nominal to real.

The issue with incorporation of such objectivity in determining the discount factor is that we cannot comment on the intertemporal decision making practices of the economic agents because we are tending to impose such a discount factor on them that may not be the one they follow in real. However, still the logical foundation of this particular method of calculating the discount factor is strong because, while keeping other factors constant, interest rate and inflation do effect intertemporal decision making of all economic agents. The degree of their importance, when compared with other subjective factors of interest, may be relatively lower but they do matter most of the times. This is the reason that we find almost no discussion in macroeconomic calibrated models on discounting behaviors of the economic agents as the behavior has been imposed upon them. This imposition of the discount factor may be relatively better matched with the subjective empirical evidence if any. However, it is difficult to generalize even this much in case of developing countries let alone existence of any solid empirical investigation being available.

Going forward with the calculations of the discount factor, it should be kept in mind that the purpose of this paper is not to discuss the issues of having a single economy wide discount factor. The main purpose is to point out that researchers should use actual data to calculate the parameter instead of taking it as it is from empirical literature especially on developed countries.

## 2.1 Methodology to Calculate Discount Factor $\beta$

The steady-state discount factor can be easily derived from the optimization conditions of households. The infinite-lived households derive their life time utility based on consumption  $c_t$ , real money balances  $\frac{M_t}{P_t}$ , and leisure  $(1 - n_t)$ .

$$\mathbb{U}_0 = \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i U_i \left[ c_{t+i}, \frac{M_{t+i}}{P_{t+i}}, (1 - n_{t+i}) \right] \quad (1)$$

where  $\beta \in (0, 1)$  is subjective discount factor. For analytical simplicity, utility function is assumed to be separable and its specification is given as:

$$U_i \left[ c_{t+i}, \frac{M_{t+i}}{P_{t+i}}, (1 - n_{t+i}) \right] = \frac{(c_{t+i})^{1-\gamma}}{1-\gamma} + \frac{\zeta_{m,t+i}}{1-\eta} \left( \frac{M_{t+i}}{P_{t+i}} \right)^{1-\eta} + \frac{(1 - n_{t+i})^{1-\nu}}{1-\nu} \quad (2)$$

where  $\gamma > 1, (\gamma \neq 1)$  is the parameter of risk aversion,  $\eta \in (0, 1)$  is the inverse of interest elasticity of money demand,  $\nu \in (0, 1)$  is the inverse elasticity of labor supply and  $\zeta_{m,t}$  is the stochastic shock to money demand.

It is also assumed that  $\frac{\partial U_t}{\partial(c)} > 0$  and  $\frac{\partial^2 U_t}{\partial(c)^2} < 0$  which implies that the utility function follows increasing and diminishing return in each of its arguments.

This above specification is consistent with standard textbook New-Keynesian Model, see for example Gali(2008) and Walsh(2010). Each agent maximizes his lifetime utility (1) based on the following intertemporal budget constraint:

$$c_t + \frac{M_t}{P_t} + \frac{B_t}{P_t} = \frac{W_t}{P_t} n_t + \frac{M_{t-1}}{P_t} + (1 + i_{t-1}) \frac{B_{t-1}}{P_t} + \Pi_t \quad (3)$$

where  $P_t$  denotes general price level,  $B_t$  denotes interest bearing assets with  $i_t$  as nominal gross return on assets at time  $t$ .  $W_t n_t$  is the nominal labor income of the agent and  $\Pi_t$  is the real dividend.

Optimization process solves the following problem as:

$$\mathcal{L} = \mathbb{E}_t \sum_{i=0}^{\infty} \beta^i \left[ +\lambda_{t+i} \left\{ \begin{array}{l} \left\{ \frac{(c_{t+i})^{1-\gamma}}{1-\gamma} + \frac{\zeta_{m,t+i}}{1-\eta} \left( \frac{M_{t+i}}{P_{t+i}} \right)^{1-\eta} + \frac{(1-n_{t+i})^{1-\nu}}{1-\nu} \right\} \\ \frac{W_{t+i}}{P_{t+i}} n_{t+i} + \frac{M_{t+i-1}}{P_{t+i}} + (1 + i_{t+i-1}) \frac{B_{t+i-1}}{P_{t+i}} + \Pi_{t+i} - \\ c_{t+i} - \frac{M_{t+i}}{P_{t+i}} - \frac{B_{t+i}}{P_{t+i}} \end{array} \right\} \right]$$

where  $\lambda_t$  is the langrange multiplier associated with the budget constraint (3). The solution of the above problem yields the following first order conditions (FOCs):

$$c_t^{-\gamma} = \lambda_t \quad (4)$$

$$\zeta_{m,t} \left( \frac{M_t}{P_t} \right)^{-\eta} = \lambda_t \left( 1 - \beta \frac{\mathbb{E}_t \lambda_{t+1}}{\lambda_t} \frac{P_t}{\mathbb{E}_t P_{t+1}} \right) \quad (5)$$

$$(1 - n_t)^{-\nu} = \lambda_t \frac{W_t}{P_t} \quad (6)$$

$$\frac{\lambda_t}{P_t} = \beta(1 + i_t) \mathbb{E}_t \left( \frac{\lambda_{t+1}}{P_{t+1}} \right) \quad (7)$$

where  $\pi_t$  is the gross inflation rate which is defined as change in price level:

$$\pi_{t+1} = \frac{P_{t+1} - P_t}{P_t} \quad (8)$$

Euler equation of intertemporal consumption can be derived by solving (4) and (7) as:

$$1 = \beta(1 + i_t) \mathbb{E}_t \left( \frac{c_{t+1}^{-\gamma}}{(1 + \pi_{t+1}) c_t^{-\gamma}} \right) \quad (9)$$

Using standard non-linear Fisher equation,  $(1 + r_t) = (1 + i_t)/(1 + E_t \pi_{t+1})$  which can be expressed in log-linearized form as:  $\hat{r}_t \approx \hat{i}_t - E_t \pi_{t+1}$ , (9) can be simplify as:

$$1 = \beta(1 + r_t) \mathbb{E}_t \left( \frac{c_{t+1}^{-\gamma}}{c_t^{-\gamma}} \right) \quad (10)$$

Finally, steady state relationship can be obtain by setting  $t = t + 1 = t^*$ . It gives the steady-state discount factor as:

$$\beta = \frac{1}{(1 + r)} \quad (11)$$

where  $r$  is the long run average real interest rate.



## 2.2 Empirical Setup and Estimation Results

We start with simply calculating the real interest rates (RIRs) of a set of emerging and developed economies. The RIR is calculated, in general, by Fisher Equation  $r_r = i_t - E_t\pi_{t+1}$ . However, there is conflict regarding what inflation data to use. Should it be the inflation of the preceding time period, present time or an expected value for the future time period. Using the expected value of inflation is also supported by the common Fisher Hypothesis which delinks the real rate of return from all monetary phenomena. In a time series setting it is straight forward to find an expected value of inflation as it is simply the one period lead value of inflation as compared to the present value of the real rate of return. However, in practice we use the one period lag value for inflation as a proxy for its one period lead value, i.e., we make use of the adaptive expectations hypothesis.

Once the real rate of return (RIR) has been calculated from Fisher Equation the discount factor then can be calculated by the formula:  $\beta = \frac{1}{(1+r)}$  as derived in the previous section. Real interest rate (RIR) can be calculated for the short run as well as for the long run. In the short run the anchor for nominal interest rate is the 3 months government treasury bills whereas the anchor for the long run is the long-term government bond yield with maturities of 3 years and above.

For a group of developed and developing countries we calculate the discount factor based on the Fisher's Hypothesis and using the long-run dimension. The data used is from 1960 to 2010.

We provide the results both for annual and quarterly frequencies because many developing countries lack quarterly data hence using the annual frequency is the only available option. However, quarterly results provide easier comparison especially with USA for which the quarterly discount factor is very commonly known. Also quarterization of the annual parameter value is straightforward in this case, i.e., by dividing the real interest rate by four.

Table 1 below shows that the discount factor is negatively correlated with the real interest rate, i.e., higher the RIR lower is the discount factor  $\beta$  and vice versa. The values for  $\beta$

calculated on annual frequency show better comparison with each other because they are quite different for different countries whereas the quarterly values are relatively closer to each other. South Korea has the lowest discount factor whereas Pakistan, India and Nepal have discount factors almost near to one. It may be evident from this exercise that economic agents are more Impatient in countries with very high discount factors as compared to the countries with smaller discount factors. But this result cannot be generalized as such since the objectivity of the purpose of having a single discount factor for all the agents operating in an economy stops their respective subjectivities to creep in (as discussed above).

Table 1: Long-run RIR and Discount Factor  $\beta$

Countries	USA	Pakistan	South Africa	Thailand	Korea
Long-run RIR	3.2780	1.5642	3.5178	4.9380	6.7261
Beta (Annual)	0.9683	0.9882	0.9660	0.9529	0.9370
Beta (Qtr.)	0.9919	0.9968	0.9913	0.9878	0.9835
Countries	Philippines	Nepal	Malaysia	Venezuela	Jamaica
Long-run RIR	5.6024	1.6311	2.3572	-1.1144	0.1792
Beta (Annual)	0.9469	0.9840	0.9770	1.0113	0.9982
Beta (Qtr.)	0.9862	0.9959	0.9941	1.0028	0.9996

### 3 Coefficient of Constant Relative Risk Aversion (CRRA)

#### $\gamma$ for Pakistan

Since Hall (1978) much of the literature on modeling inter-temporal consumption has used Euler equation, i.e., a first order condition that shows a relation between a variable that has different values in different periods or different states, derived from the inter-temporal optimization problem faced by a general consumer. This allows one to estimate structural preference parameters and test the specification of the model without having to specify fully the stochastic environment in which the consumer operates. Usually the closed form solutions for equilibrium time paths of the variables of interest are obtained after imposing

strong assumptions on the stochastic properties of the variables of interest. Once these assumptions fail we get inconsistent estimates of the preference parameters. However, the Generalized Method of Moments (GMM) estimation procedure needs no distributional assumption. GMM estimates are consistent even in the presence of heteroscedasticity and autocorrelation in the error terms. The necessary condition for the GMM method to estimate the structural parameters is that it must hold the moment conditions (or orthogonality conditions).

The performance of non-linear GMM estimators creates a number of problems when we apply it on small sample which create measurement errors (Alan and Browning 2006, Attanasio and Low 2004). One solution of this problem is the use of log-linearized version of Euler equation which has a couple of issues of its own i.e. the log-linearized equation does not give a specification linear in parameters and second the constant of a log-linearized Euler equation includes conditional higher moments of consumption growth and interest rate, which loses identification of discount factor (Alan and Browning, 2006). Attanasio and Low (2004) showed that we can use log-linearized Euler equation even when the income process is heteroscedastic only for long panel data (about 40 periods) but availability of long panel data is not easy. Alan and Browning (2006) developed two alternative GMM estimators that deal explicitly with measurement error. They showed through Monte Carlo simulations that their estimators are better than conventional alternatives even for short panel data. Holt and Laury (2002) found that the average coefficient for developing countries is approximately 0.4 when defining utility over gain and not wealth. Cardenas and Carpenter (2007) showed that the coefficient of relative risk aversion, by ignoring wealth, from risk experiments in developing countries lies between 0.05 (Ethiopia) and 2.57 (Paraguay).

While exploring the small sample properties of GMM estimators in consumption-based models Kocherlakota (1990), Mao (1990), Hansen et al. (1996) and Holman (1998) show that there is a bias in GMM estimates in small sample data with poor results for the coefficient of risk aversion. Pozzi (2003) showed by conducting a Monte Carlo study that the estimate

of the coefficient of relative risk aversion tends to have a negative bias due to small sample problem. He also showed that the other estimates in the equation are not biased. In the next sections we estimate and test the consumption based asset pricing model for Pakistan's aggregate data on consumption with annual frequency (from 1960 to 2010) due to its non availability at higher frequencies. The GMM outcome also provides the parameter value for the discount factor as an externality.

### 3.1 Estimation of Euler Equation of Consumption

The Euler equation of consumption (10) shows the expected rate of return on the assets as well as relative expected consumption stream which is negatively related to the risk aversion parameter. This is what shows if the consumers prefer to trade-off their consumption in the present with that of more of it in the future. In order to estimate preference parameters of the Euler equation, CRRA  $\gamma$  and discount factor  $\beta$ , GMM technique is used. The necessary condition for GMM method to estimate the structural parameters is that it must hold the moment.

To get the moment condition from equation (10) it is necessary to rearrange this equation as:

$$\beta(1 + r_t)\mathbb{E}_t \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma} - 1 = 0 \quad (12)$$

According to Hansen and Singleton (1982) the discrete-time models of the optimization behavior of economic agents often lead to first-order conditions of the form:

$$\mathbb{E}_t [h(x_t, b_o)] = 0 \quad (13)$$

where  $x_t$  a vector of variables is observed by agents at time  $t$  and  $b_o$  is a  $p$  dimensional parameter vector to be estimated. By comparing (12) and (13) we get:

$$h(x_t, b_o) = \beta(1 + r_t)\mathbb{E}_t \left( \frac{c_{t+1}}{c_t} \right)^{-\gamma} - 1 \quad (14)$$

where  $x_t = [r_t, c_t]$  and  $b_o = \begin{bmatrix} \gamma \\ \beta \end{bmatrix}$ . Let  $z_t$  denote a  $q$  dimensional vector of variables with finite second moments that are in agents information (e.g., consumption, interest rates, etc...) set. This is also called as a set of instrumental variables shown below:

$$f(x_t, z_t, b_o) = h(x_t, b_o) \otimes z_t \quad (15)$$

From (13) and (15) let us assume (as in Hansen and Singleton, 1982) that:

$$E[f(x_t, z_t, b_o)] = 0 \quad (16)$$

Now we will construct an objective function that depends only on the available information of the agents and unknown parameters  $b$ . Let  $g_0(b) = E[f(x_t, z_t, b_o)]$  according to Singleton (1982), if the model in (7) is true then the method of moment estimator of the function  $g$  is:

$$g_T(b) = \frac{1}{T} \sum_{t=1}^T [f(x_t, z_t, b)] \quad (17)$$

The value of  $g_T(b)$  at  $b = b_0$  should be close to zero for large values of  $T$ . In this paper, we follow Hansen and Singleton (1982) and choose  $b \in \omega$  (parameter space) to minimize the function  $J_t$  below:

$$J_T(b) = g_T'(b)W_T g_T(b) \quad (18)$$

where  $W_T$  is a symmetric, positive definite weighting matrix. The choice of weighting matrix  $W_T$  is such that which makes  $g_T$  close to zero. In order to implement this optimal procedure and to conduct asymptotically valid inference we need a consistent estimator of

$W_T^*$ . This can be estimated as:

$$W_T^* = \frac{1}{T} \sum_{t=1}^T [f(x_t, z_t, b) f(x_t, z_t, b)'] \quad (19)$$

To compute  $W_T^*$  a consistent estimator of  $b_o$  is needed. This can be obtained by initially using  $W_t = I_{r \times r}$  (identity matrix) and suboptimal choice of  $b$  in minimizing (18) and we get the values of  $b_T$ . By using this value of  $b$  in (19) we get  $W_T^*$ . Again by using the new values of  $W_T^*$ ,  $b_T$  can be obtained by minimizing equation (18). We repeat this process until the estimates converge. According to Puzzi (2003) this iterative GMM process is more efficient in small sample than a simple standard two-step procedure given by Hansen and Singleton (1982). There are two advantages of estimating non-linear Euler equation under GMM as given in Hansen and Singleton (1982):

(a) Unlike the maximum likelihood (ML) estimator, the GMM estimator does not require the specification of the joint distribution of the observed variables.

(b) The instrument vector does not need to be economically exogenous. The only requirement is that this vector be predetermined in the period when the agent forms his expectations. Both past and present values of the variables in the model can be used as instruments. Model estimator is consistent even when the instruments are not exogenous or when the disturbances are serially correlated.

### 3.2 Test for Validity of the Model

Following Hansen and Singleton (1982) we test the over identifying restrictions of the model. The test statistics is:

$$P = T \times J_T(b) \quad (20)$$

where  $T$  is the number of observation and  $J_T(b)$  is the statistic given in equation (18). Under the null hypothesis this test statistics (called  $J$  Statistics) distributed as  $\chi^2$  with

$(qm - r)$  degree of freedom. Where  $q$  is the number of equation,  $m$  is the number of instruments and  $r$  is the number of parameters estimated. The null hypothesis for the  $J$ -test is that the over identifying restrictions (moment's conditions) are accepted. The rejection of the null provides evidence for mis-specification of the econometric model estimated, as well as the underlying economic model from which the Euler equation is obtained.

### 3.3 GMM Estimation Results

We estimate Euler equation (10) using annual data for Pakistan for 1960-2010. The choice of the data frequency is important for econometric practice. Most of the previous work is on the US monthly data series but the monthly or quarterly aggregate consumption and expenditure series are not available for Pakistan. The number of observations, even for the annual data, is only marginally sufficient for estimation however using annual data is the only available option for this study. Annual consumption per capita series for Pakistan for 1960-2010 has been extracted from Economic Survey of Pakistan. The requirement for Generalized Method of Moment estimation is that all variables in the model should be stationary. Before applying GMM stationarity of consumption growth and real interest rate need to be checked. We apply Augmented Dickey- Fuller unit root test to do so. Results in Table 2 show that both the variables are stationary at level.

Table 2: Augmented Dickey- Fuller Unit root Test Results

Variables	ADF test Statistics	Probability	Remarks
Consumption growth	-8.8422	0.0001	Reject null
Interest rate	-3.1214	0.0312	Reject null

We have used two sets of instruments in the estimation procedure. Hansen and Singleton (1982) use different combinations of these variables (consumption growth and interest rate) as instrumental variables. Hall (1988) suggested that all instruments are lagged at least two periods to deal with possible time aggregation problems. We follow the both. For simplicity

we assume,  $\frac{c_{t+1}}{c_t} = c^*$ .

Instrument sets which we have used in estimation of Euler equation are given below:

Set 1: *Constant*,  $r_{t-2}$ ,  $r_{t-3}$ ,  $c_{t-2}^*$ ,  $c_{t-3}^*$

Set 2: *Constant*,  $(1 + r_{t-2})$ ,  $(1 + r_{t-3})$ ,  $c_{t-2}^*$ ,  $c_{t-3}^*$

Table 3: GMM Estimation Results with Different sets of Instruments

Model No.	$\beta$	$\gamma$	J statistics= $(T \times J_T(b))$	Degree of Freedom	P-values
1.	0.992 (0.004)	0.576 (0.045)	4.903	1(=5-2)	0.18
2.	0.986 (0.008)	0.571 (0.048)	4.921	1(=5-2)	0.22

Note: The terms in parenthesis are standard errors

The estimated results of Euler equation (10) for Pakistan's annual data are reported in Table 3. From the table, over identifying restrictions are valid for both models, as p values of  $J$ -statistic indicate at 5% significance level. For both sets we get similar results, i.e., CRRA parameter's value is 0.576 and average discount factor of two alternative specifications is 0.9885 and their standard errors indicate that both are different from zero. Also the estimate for the discount factor confirms the result from the simple Fisher equation used earlier in the paper.

According to Ogaki and Reinhart (1998), there is a negative relationship between  $\gamma$  and  $\beta$  for US data when estimating Euler equation. Thus if high estimate for  $\beta$  is found then it is intuitive that the estimate for  $\gamma$  is low. This estimated coefficient of relative risk aversion lies within the ranges of CRRA for developing country (Cardenas and Carpenter. 2008), i.e., from  $[0.05 - to - 2.57]$ .



## 4 Conclusion

We have shown the calculation of the discount factor and estimation of the CRRA coefficient in a small data set for Pakistan. By doing so we have distinguished between the values obtained by us and the values of these parameters generally available in empirical literature. However, we have stayed away from interpreting the results of our parameter values. But we show in Figure 1 and Figure 2 that when the values of these parameters change so does the magnitude of the impulse response functions of a simple New Keynesian model in response to policy shocks. This change in magnitude of impulse response functions is relatively higher in models with nominally rigid, or sticky, prices.

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# Appendix

For counterfactual simulations, we have considered the following log-linearized version of textbook New-Keynesian model:

$$\tilde{y}_t = E_t \left\{ \tilde{y}_{t+1} \right\} - \frac{1}{\gamma} (i_t - E_t(\tilde{\pi}_{t+1}) - r^n) \rightarrow (\textit{Linearized} - \textit{IS} - \textit{Curve}) \quad (21)$$

$$\tilde{\pi}_t = \beta E_t(\tilde{\pi}_{t+1}) + \kappa \tilde{y}_t \rightarrow (\textit{Linearized} - \textit{NK} - \textit{Phillips} - \textit{Curve}) \quad (22)$$

Where,  $\kappa = \frac{(1-\theta)(1-\beta\theta)\Theta}{\theta}$

$$r^n = \rho + \gamma\psi(\rho_a - 1)a_t \rightarrow (\textit{Linearized} - \textit{natural} - \textit{rate} - \textit{equation}) \quad (23)$$

$$\tilde{y}_t = a_t + (1 - \alpha)n_t \rightarrow (\textit{Linearized} - \textit{production} - \textit{function}) \quad (24)$$

$$\tilde{m}_t = \tilde{\pi}_t + \tilde{y}_t - \eta(i_t) \rightarrow (\textit{Linearized} - \textit{money} - \textit{demand} - \textit{function}) \quad (25)$$

$$i_t = \rho + \varphi_\pi \tilde{\pi}_t + \varphi_y \tilde{y}_t + v_{i,t} \rightarrow (\textit{Linearized} - \textit{Taylor} - \textit{rule}) \quad (26)$$

$$a_t = \rho_a a_{t-1} + \epsilon_{a,t} \rightarrow (\textit{Pr} - \textit{oductivity} - \textit{Shock}) \quad (27)$$

$$v_{i,t} = \rho_v v_{i,t-1} + \epsilon_{v,t} \rightarrow (\textit{Monetary} - \textit{Policy} - \textit{Shock}) \quad (28)$$

Table 4: Selected parameter values for simulations

Parameters	Description	Benchmark Values
$\alpha$	Share of Capital in production function	0.50
$\beta$	Subjective Discount Factor	0.99
$\theta$	Measure of price stickiness	0.75
$\kappa$	Slope Coefficient in NKPC	$\frac{(1-\theta)(1-\beta\theta)\Theta}{\theta}$
$\rho$	Real interest rate in the steady state	$-\text{LOG}(\beta)$
$\gamma$	Parameter of Risk Aversion	2.00
$\phi_y$	Sensitivity of the central bank with respect to the output gap	0.50
$\phi_\pi$	Sensitivity of the central bank with respect to inflation	1.50
$\eta$	Elasticity of the money demand with respect to the nominal interest rate	2.00
$\rho_a$	Persistence of the technology shock	0.99
$\rho_v$	Persistence of the monetary policy shock	0.85

Figure 1: Impact of Positive Monetary Policy Shock: When  $\theta = 0$  (i.e., Flexible Prices)

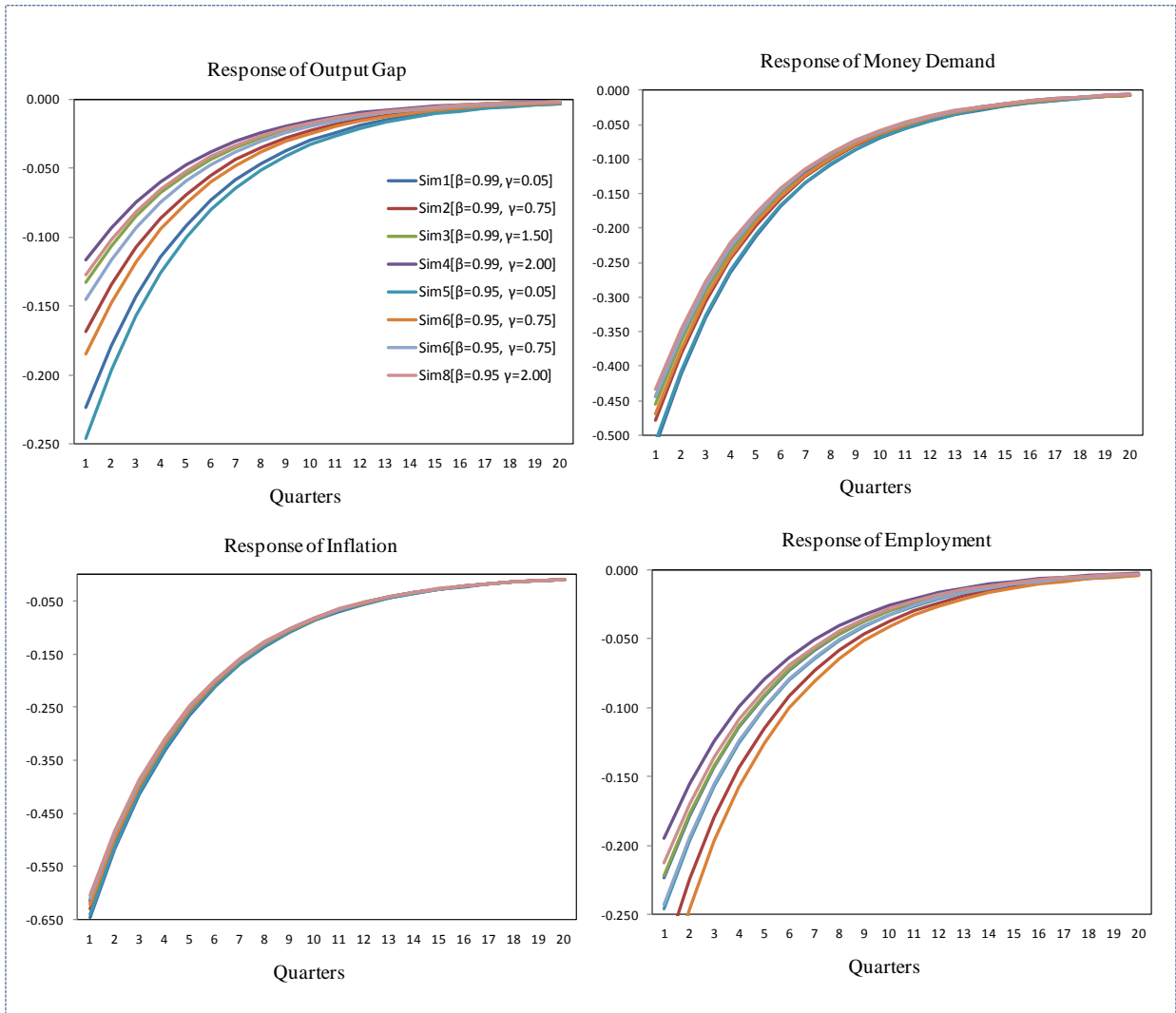


Figure 2: Impact of Positive Monetary Policy Shock: When  $\theta = 0.75$  (i.e., Sticky Prices)

