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Pakistan Economy DSGE Model with Informality – The Empirics of Calibration

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Abstract

This document serves as an empirics guide to the "Pakistan Economy DSGE Model with Informality" paper by Ahmed, et al. (2012) covering the empirical aspects regarding calibration of both the model and shocks related parameters. We have tried to focus as much as possible on micro level empirical data where ever possible in order to calibrate the parameters. We have made use of international literature for pinning down of certain parameters where the empirical and national literature exhausted.

Key words: *Calibration, Shock Process.*
JEL Classification: *E17.*

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Introduction

The calibration of parameters of a DSGE model is at the crux of the model's performance. The more realistic or more representative the parameters are to the behavior of an economy the more meaningful are the model outcomes, i.e., the policy implications derived from the impulse responses functions. The parameters are sub divided into two major groups, i.e., the parameters portraying the long run behavior of the economic agents involved in the theoretical model of the economy and the parameters of the shock processes which include persistence and dispersion of the shocks. Since the paper "Pakistan Economy DSGE Model with Informality" by Ahmed, et al. (2012) is the first serious attempt to theoretically model the Pakistan's economy we placed a lot of importance to the calibration of parameters so that our theoretical model can match the economy's stylized facts well. This is a serious task keeping in view the scarcity of literature and micro evidence for estimating deep parameters for Pakistan as well as for similar developing countries. This scarcity of data, both primary and secondary, would lead to calibrating most of the model parameters at levels prevalent in developed countries. We, on the contrary, have tried to make use of micro level information as much as possible by using data sets from various field surveys conducted by the State Bank of Pakistan (SBP) itself[1] and acquired from the Federal Bureau of Statistics (FBS)[2].

All parameters in our *DSGE model with informality* are calibrated for annual frequency. The reason for doing so is due to data limitations at higher than annual frequency although we fully recognize that this class of models would have been more useful for policy analysis if we would have used higher frequency data. However, this is a reasonable trade-off if we limit our model's scope to that of replicating the stylized facts of the economy of Pakistan and this is precisely what has been carried out through this extensive exercise. Overall, there are 22 parameters in total with 16 structural and 6 shock related parameters. Structural parameters can be categorized into four broad groups: (1) household related, (2) formal-informal consumption related, (3) formal-informal labour supply related, and (4) production function parameters. Whereas, the shock related parameters represent the mean and persistence parameters of technology, fiscal and monetary shocks.

Important point to keep in mind regarding the household and labor supply is that the representative agent in our model is not one person but a household which include persons consuming and deriving utility as a whole. They possess different levels of education on the basis of which it is determined that either

[1] Price and Wage Setting Surveys of Formal and Informal Manufacturing and Services Sectors of Punjab and Sind.

[2] Labor Force Surveys.

they work in the formal sector or in the informal sector. For this model, we have ignored the agriculture sector of the Pakistan economy. This has been made possible by using the Labor Force Survey's time periodic data set which allowed us to choose households with no labor supply to the agriculture sector.

Most of the parameters used in our model have been calibrated using partial estimation/computation approach. However, only very few of the parameters, for which estimation remained an issue throughout, are picked from existing DSGE models literature preferably for developing countries. Only when all these resources failed to pin down a parameter we turned our sight to the results from the developed world. The extensive mathematical nature of DSGE modeling also leads to existence of such parameters in the model for which there could be no reference in literature as well as no surveys have ever been conducted or can be conducted soon. Such parameters can be termed as "free parameters". Our DSGE model did have one free parameter. Its value has been calibrated to a point where the moment matching exercise yielded the best results given all the other parameters already being pinned down. The time period for the time series data used for estimation is from 1981 to 2010 [3] [4]. The discussion on the model and shock related parameters follow next.

1 Calibration of Structural and Shock related Parameters

1.1 β- Discount Factor

The discount factor β, which is a benchmark of forward looking behavior, is computed to be 0.99 by taking inverse of the average long term real interest rate. Discount factor is one of the most important and easiest to estimate parameter of all theoretical modeling exercises since it is this parameter that determines the nature of existence of the future of an economy. The discount factor is actually an inverse of the discount rate at which the economic agents are willing to let go their current utility for future utility. The standard formula is $\beta = 1/(1-d)$ where d is the discount rate. All economic agents, whether firms, government or households are assumed to have a similar discount factor in DSGE models. The reason for such objectivity comes from the complex nature of subjectivity attached with the discount rate. For example a household can have different discount rates for different factors in its utility function and these rates vary across households as well. All other economic agents tend to have similar issues as well. This makes it impossible for any social planner to collect so much information and then to process it for viable results in

[3] 1981-2010 in terms of financial year means 1980-81 to 2009-10.
[4] Table 4 provides the summary of the values used for all the model parameters.

aggregation. The objective of choosing the same discount rate, attached to the real interest rate of an economy, thus reduces the amount of confusion and technicality due to subjectivity. Another important point to understand is that the interpretation of the discount factor also alters from its subjective counterpart when its value stands imposed by the economic modeler. We are not in a position to say whether Pakistani economic agents are more patient because of the high discount factor of 0.99 because it does not come directly from the economic agents discounting their utility functions involving various factors. Similar argument is also true for the other economic agents in the model optimizing intertemporal choices. This uniformity and objectivity of the discount factor then makes the model solution easier as well as tractable. Our result is in line with the estimated value of β in Ahmed, et al. (forthcoming). The discount rate has been estimated using annual data from 1960 to 2010. Return on government bonds and change in CPI have been used to measure the long term interest rate and inflation respectively. To incorporate expectations, lagged inflation has been used to calculate the real interest rate.

1.2 χ- Household's Preference for Money Holding

The symbol χ reflects the household's preference for holding money which, in the utility function, reflects the level of utility it provides to the representative household. This parameter is usually estimated following the methodology outlined in Christiano, et al. (2005). They estimate the household's preference for holding money by exploiting the money specific first order condition of the household's utility function. This can be done, as it is, for any country but when it comes to developing countries data issues become a hindrance. Same is the case for Pakistan as there are data limitations regarding estimation of this parameter based on Pakistani data. First, if aggregate data is used then the private consumption time series data available is actually the residual of the economy's resource constraint. Issues with the validity of this residual based consumption time series data also affects the moments of the consumption related ratios as shown in the moment matching exercise of Ahmed, et al. (2012). Second, there is no existence of survey based consumption result at micro level which would have provided more realistic results when it comes to estimation of the deep preference of money holding parameter. FBS does conduct Household Integrated Economic Surveys (HIES) from time to time which record general consumption patterns of the households. However, the panel for this data set representing the behavior of a cluster of same households over a period of several years is a task in itself. After exhausting all our national empirical evidences to get some clue we reverted to the international literature. Our search for this parameter left us no clues from the developing world as well. In the end we used the parameter value specified by DiCecio and Nelson (2007) for UK. This value for χ is 0.25.

1.3 ϕ- Coefficient of Labor Supply

The coefficient of labor supply (or the inverse elasticity of labor supply) shows the impact of change in wages on the units of time for which the household supplies labor for production. In the utility function the coefficient of labor supply, denoted by ϕ, is fixed at 1.5 following Fagan and Messina (2009) who used this value for Finland, Germany, Portugal, USA and Belgium while analyzing downward wage rigidities and optimal inflation in these economies. This value is also consistent with the posterior mean of the Bayesian process reported by Smets and Wouters (2005). It is logical to say that the FBS's Labor Force Survey's data can be used to calculate the coefficient of labor supply. But this is not as easy as it may sound. The reason is that our model deals with a representative household rather than a representative economic agent/person. In order to compare the labor provisions of the same household over few different wage levels a panel having same households over a period of few years is required. We did try this by pooling the data for the Labor Force Surveys. This issue could be resolved if we knew the exact weights of the households of interest so that the representative panel can be created within the pooled data. But the provision of raw data by FBS lacked such weights. As a result we were restricted to use the literature's value for ϕ.

1.4 ω- Share of Formal Consumption; $\frac{P^F}{P}$- Ratio of Formal Prices to Aggregate Price; $\frac{P^I}{P}$- Ratio of Informal Prices to Aggregate Price; and $\frac{P^I}{P^F}$- Ratio of Informal Prices to Formal Prices

The parameters ω, $\frac{P^F}{P}$, $\frac{P^I}{P}$ and $\frac{P^I}{P^F}$ together govern the distribution of formal and informal consumption. They are all calculated using same data and value of one leads to the value of the other two in case of the pricing ratios. The reference for the method of finding these parameter values is Khan and Khan (2011). Since these parameters are interlinked we discuss them under a single section. Khan and Khan (2011) use two approaches to calculate the share of formal goods consumption ω. The first approach uses data from the Federal Board of Revenue from 1999-2010 to establish the list of 920 industrial and consumer goods and services on which the Government of Pakistan collects sales tax. If the Government earns tax from a commodity, it is classified as a formal good, whereas commodities with zero tax collection are classified as informal goods. Next, the list of 374 commodities that is used in calculating the consumer price index for Pakistan, and the weights assigned to each of these commodities are tallied with the sales tax data. This then determines the weight, allocated by the average household buying a representative basket of goods and services, to purchases from both the formal and informal sectors. The issue of housing as an item of expenditure poses a potential problem because there is a substantial weight (23.43 units) associated with the House Rent Index in the CPI, while the nature of the housing market in Pakistan is somewhat ambiguous resulting in dropping its index from the representative basket of goods and services, so that the sum of weights for the CPI is 76.57 units instead of a 100 units. A division of weights between the two sectors based on tax collection, then, yields a value of 44.05% for the informal sector consumption, and 55.95% for the formal sector consumption. This exercise pins down the share of formal consumption $\omega = 0.55$ in our calibration exercise. The second approach which uses consumption data from the FBS's Household Integrated Economic Survey (HIES) for 2007-2008, yielded similar result for ω.

The same CPI data with its commodity specific weights has been used to determine that the steady state share of formal prices in overall price level, i.e., the ratio of the two $\frac{P^F}{P}$, comes out to be 53.8 percent. This value remains the same for all the years for which the consumption basket with all of its commodity weights remains the same. Thus, we rounded off the long run ratio $\frac{P^F}{P} = 0.53$ and since the informal prices to aggregate price ratio is just the difference from $\frac{P^F}{P}$, $\frac{P^I}{P} = 0.47$. When we take the ratio of the informal and formal prices it comes out to be $\frac{P^I}{P^F} = 0.89$.

1.5 μ- *Elasticity of Substitution between Formal and Informal Consumption (free parameter)*

The value of μ- elasticity of substitution (arising due to change in income) between formal and informal consumption- has been calibrated to have the best moment matching results between the model outcomes and the empirical data used. The reason for doing so emerged as we pinned down μ to be a free parameter on the basis of its uniqueness since we found no empirical evidence for the value of this parameter neither from any survey nor from any calibrated model. We did try to focus the problem by trying to conduct a pilot survey but we were not able to pin down exactly the degree of substitution due to the change in income. For example, milk is available both formally and informally but the substitutability resulted on many issues other than the income itself making it impossible just to focus on the income component of substitutability. Hence, we refrained from conducting any more pilot or actual surveys that lead to misguided results in this regard. Fixing a value for such a parameter to match the empirical moments is the standard practice in literature as well. The value of μ best chosen is 0.7.

1.6 η- Share of Formal Labor Supply in Total Labor Supply

The share of formal labor supply in total labor supply $\eta = 0.29$ is computed by taking average of ratio of number of people employed in the formal sector to the total number of people employed in the non-agriculture sector during 1990-1991 to 2008-2009. The relevant labor force data is collected from various issues of the Federal Bureau of Statistics Labor Force Survey. The survey was not conducted in 2000-01, 2002-03 and 2004-05 and it was difficult due to the unavailability of coding scheme to decode the raw survey data for surveys earlier than 1990-91. The labor force employed in the agriculture sector was ignored since our DSGE has informality and according to international standards agriculture is not an informal sector of the economy. Since agriculture in Pakistan, like other developing countries, is not based on documentation it became difficult for us whether to classify it under the formal or the informal sector. If, contrary of the international definition, we added agriculture in the informal sector, on average the share of informal employment would have soared up to more than 90 percent. So, we found it credible to follow the convention and ignored the agriculture sector altogether. This does raise the issue when it comes to the use of aggregate series of GDP, consumption and investment during the moment matching exercise in order to check the validity of the DSGE model. However, since all these series have agriculture sector factored in, it is reasonable to continue with them without being involved in factoring out agriculture.

1.7 ϑ- Elasticity of Substitution between Formal and Informal Labor Supply

In order to estimate ϑ, we used the micro-level data from the annual Labor Force Surveys. We compiled labor force survey data from the last several waves available between 1997-98 and 2008-09. As mentioned earlier also, the survey was not conducted in 2000-01, 2002-03 and 2004-05.We estimated the elasticity of substitution between formal and informal sector separately for each wave. We ran the following regression as in Psacharopoulos and Hinchliffe (1972) for each year [5] :

$$\ln\left(\frac{W_t^F}{W_t^I}\right) = a + \vartheta \ln\left(\frac{L_t^F}{L_t^I}\right) \tag{1}$$

where W_t^F and W_t^I are the hourly wage rates of formal and informal sector employees in a household. L_t^F and L_t^I are the average hours worked in a week by employees in the formal and informal sector respectively. A caveat for our

[5] In (1) the subscript t only denotes that all the variables used in the regression are from the same time period.

estimation of elasticity of substitution between formal and informal labor is that we were limited by the nature of LFS being a survey based on individual agent's labor contributions. Our sample was reduced significantly for estimation by the fact that we could only use data from households that have more than one employee as well as at least one each in the formal and informal sector.

We considered an employee to be part of the formal sector if his/her response to the LFS question "what kind of enterprise?" about his/her work place was any one of Federal Government, Provincial Government, Local body Government, Public enterprise, Private limited company, Public limited company and Cooperative society. In addition, the respondents who answered the enterprise question with either Individual Ownership, Partnership or Other were considered part of the formal sector if and only if their enterprises kept written accounts (as asked in the next question of the survey "Does the enterprise keep written accounts?"). On the other hand those employees who responded to the enterprise question with either Individual Ownership, Partnership or Other and also answered the written accounts question with either "No" or "Don't know" were considered part of the informal sector. Using cross sectional data of Labour Force Surveys conducted between 1997-98 and 2008-09, we estimated this elasticity of labor substitution for each survey period using equation (1) and then took average of all estimated values to obtain final value of $\vartheta = 2$ (see table 1 for details).which lies well within the range as well.

Table 1: Elasticity of Substitution between Formal and Informal Labor Supply

Year	**Coefficient**	**Elasticity**
1997	-0.47	2.12
2000	-0.54	1.86
2002	-0.61	1.64
2004	-0.72	1.38
2006	-0.58	1.73
2007	-0.38	2.64
2008	-0.51	1.95
2009	-0.65	1.55
	Average	**1.9**
	Range	**(1.38-2.64)**

1.8 σ- Formal Wage Premium

The value of formal wage premium σ is set at 0.25 (Choudhary et al. (Forthcoming)). This value has been taken from the findings of the formal and informal sector wage setting surveys conducted by the State Bank of Pakistan. These surveys have been conducted in Punjab and Sind and cover the manufacturing and the services sectors of the economy's production side.

1.9 α- Share of Capital in Production

We do understand that the estimation of the country specific capital share is the best way forward when it comes to modeling, however, the lack of relevant data and other empirical literature has resulted in exploring other avenues.

Table 2: Capital Share of Production for Developing Countries (Liu, 2008)

Country	**Capital Share**	**Country**	**Capital Share**
Argentina	0.529	Brazil	0.512
Bulgaria	0.525	**Mexico**	**0.565**
Chile	0.372	Slovakia	0.358
Columbia	0.455	Poland	0.274
Cote d' Ivoire	0.489	Tunisia	0.354
Czech Republic	0.459	Mozambique	0.554
Dominican Republic	0.395	**Philippines**	**0.46**
Estonoa	0.342	Portugal	0.313
Hungary	0.403	Ukraine	0.297
Kazakhistan	0.566	Moldova	0.42
Kyrgyzstan	0.258	Nicaragua	0.438
Latvia	0.38	**Average**	**0.43**
Lithuania	0.414	**Range**	**0.258-0.566**

To calibrate the share of capital in production α, we chose the value to be 0.50

which is quite close to the average of capital shares of other less developed countries as reported by Liu (2008) in the table above. The average capital shared for a group of 19 developing countries came out to be 0.43. The values for the Philippines and Mexico provide a range between 0.46 and 0.56. These two countries are important because of their comparison with the Pakistani economy in terms of the large informal sector. Both these countries, like Pakistan, have large populations as well. We have calibrated the share of capital to be 0.5 in our model which is closer to the same in these two countries as well as close to the overall average of 0.43. Important to note is that we are also within the sample's empirical range as well as above the average value of the sample of developed country whose value is 0.35 (see table 2 for details).

1.10 δ- Depreciation Rate

The depreciation rate δ has been set at 0.15 which is in line with values used by other authors in the literature on DSGE models for developing countries such as $\delta = 0.1255$ used by Garcia, et al. (2006). In addition, balance sheet analysis of joint stock companies listed at Karachi Stock Exchange reveals that overall depreciation rate has been close to 10 percent (refer to table 3 below for year wise aggregate range of the depreciation rates of joint stock companies.).

Table 3: Annual Depreciation Rates of Joint Stock Companies

Year	Minimum	Maximum	Year	Minimum	Maximum
1999	4.6	11.6	**2005**	3.5	17.6
2000	4.3	11.7	**2006**	3.8	14.7
2001	4.4	12.3	**2007**	3.5	13.5
2002	4.3	15	**2008**	3.9	12.5
2003	4	14.2	**2009**	4.1	13.3
2004	4	12.8	**Average** [6]	**4**	**13.6**

Most of the sectors exhibit annual depreciation rates between 8 and 10 percent.

[6] We did try by weighing the manufacturing sectors according to their contribution in the GDP and computing the average depreciation rate. It was found to be 9 percent both across firms and over time which was almost similar to the simple average of 8.8 percent.

The empirical range is in between 3.5 and 17.6 percent, however, the few sectors which exhibit depreciation rates below 8 percent have small shares in the overall manufacturing sector. On the other hand, one of the biggest sectors, i.e., transport, exhibits depreciation rates up to 15 percent in a couple of years from 1999 to 2009. This also makes it reasonable to have a range between 8-17.6 percent where 17.6 percent is the maximum value in the data set from the balance sheet analysis of joint stock companies. Since less well capitalized firms are expected to have higher depreciation rates and joint stock companies only represent a fraction of the manufacturing sector which is better capitalized in general, therefore we have adjusted the aggregate average estimate upwards, i.e., from 13.6 percent to 15 percent, but still within the range mentioned above.

1.11 $\bar{\pi}$- *Gross Inflation; and Taylor Rule's Response Parameters (Inflation* ψ_t^{π} *and Output* ψ_t^{Y}*)*

Our model's monetary policy is based on a Taylor Rule where the policy maker targets both inflation and output, however, with different degree of priorities. The Taylor Rule's responses of inflation and output have been estimated by regressing nominal interest rate on deviations of inflation and output from their steady states. To obtain the response of policy interest rate to deviations of inflation and output from the steady state, following Ireland (2004), we regress log of interest rate on deviations of inflation and output from their trend values. We use average call money rate, GDP deflator and per capita real GDP for interest rate, inflation and output respectively. The data is used in annual frequency from 1981 to 2010 and is easily available from various sources including SBP. Deviation of inflation from steady state is measured using the residuals μ_t^{π} of the following equation:

$$\ln \hat{\pi}_t = c + \hat{\mu}_t^{\pi} \tag{2}$$

The estimated equation is:

$$\ln \hat{\pi}_t = \underset{(0.007)}{0.088} + \hat{\mu}_t^{\pi}$$

Result for the estimation of the constant term $c = 0.088$ with $S.E = 0.007$ imply steady state gross inflation $\bar{\pi} = 1.09$ when rounded off.

For steady state output we regress log of per capita real GDP on the constant c and the trend t through the following equation where μ_t^{Y} is the output's residual:

$$\ln \hat{Y}_t = c + \gamma(t) + \hat{\mu}_t^Y \tag{3}$$

The estimated equation is expressed as:

$$\ln \hat{Y}_t = \underset{(0.01)}{9.7} + \underset{(0.001)}{0.02}(t) + \hat{\mu}_t^Y$$

Furthermore, following Ireland (2004), to estimate the response of interest rate to deviations in inflation and output, we estimate the following equation:

$$\ln \hat{r}_t = c + \upsilon^\pi(\hat{\mu}_{t-1}^\pi) + \upsilon^Y(\hat{\mu}_{t-1}^Y) \tag{4}$$

where μ_{t-1}^π and μ_{t-1}^Y are the lagged residuals from the two previous equations. The estimated equation is as follows:

$$\ln \hat{r}_t = \underset{(0.003)}{0.08} + \underset{(0.09)}{0.324}(\hat{\mu}_{t-1}^\pi) + \underset{(0.11)}{0.345}(\hat{\mu}_{t-1}^Y)$$

Estimated responses to inflation and output deviations are then normalized as $\psi^\pi = \frac{\upsilon^\pi}{\upsilon^\pi + \upsilon^Y}$ and $\psi^y = \frac{\upsilon^Y}{\upsilon^\pi + \upsilon^Y}$ and yield values of 0.48 for ψ^π and 0.52 for ψ^y. These values show that the Taylor Rule gives 48 percent weight to the inflation deviations from its steady state and 52 percent weight to the output deviations from its steady state.

Table 4: Structural Parameters

Parameter	**Value**	**Parameter**	**Value**
β	0.99	η	0.29
χ	0.25	ϑ	2
ϕ	1.5	σ	0.25
$\frac{P^F}{P}$	0.53	α	0.50
$\frac{P^I}{P}$	0.47	δ	0.15
$\frac{P^I}{P^F}$	0.89	ψ^π	0.48
ω	0.55	ψ^y	0.52
μ	0.7	$\bar{\pi}$	1.09

1.12 Calibration of Shock related Parameters[7]

We have 3 shocks in total. They are the technology shock, the government spending/fiscal shock and the interest rate shock. The shocks are calibrated for two parameters each which determine their degree of persistence and level of dispersion from the steady state. The standard procedure in this regard is to regress the variable of interest, i.e., the shock variable, on its lag. This method follows King and Rebelo (1999). The data for the estimations of shock related parameters has been acquired from Federal Bureau of Statistics (FBS) and State Bank of Pakistan (SBP).

1.12.1 Technology Shock (ρ_A,σ_A)

The technology shock is defined in terms of total factor productivity (TFP) which is denoted by A below. The TFP series is obtained by using residuals of estimated neo-classical production function through the following regression:

$$\ln Y_t = \alpha \ln K_t + (1 - \alpha) \ln L_t + \ln A_t \tag{5}$$

where K and L represent capital and labor respectively with α and $(1 - \alpha)$ representing their proportions in producing output Y. Earlier we have already pinned down share of capital $\alpha = 0.5$ spliting the contribution of both the factors of production equally at 50 percent each. Hence the production function depends upon capital, labor and TFP. The time series of the capital stock has been obtained from Ahmed (2011). He uses perpetual inventory method with the depreciation rate of capital fixed at 15 percent to estimate the capital stock. The labor series, in terms of number of hours worked and adjusted for education in order to differentiate between formal and informal labor, has also been provided by Ahmed (2011). The estimation of the above equation without TFP explicitly returns the residual which is then treated as TFP for Pakistan denoted by A. In order to estimate the persistence ρ_A in the TFP series we regress A_t on its lag, i.e., follow an AR-1 process as specified by the following equation:

$$\ln \hat{A}_t = c + \rho_A(\ln \hat{A}_{t-1}) + \hat{\mu}_t^A \tag{6}$$

The estimated equation takes the following values:

$$\ln \hat{A}_t = c + \underset{(0.07)}{0.88}(\ln \hat{A}_{t-1}) + \hat{\mu}_t^A$$

The persistence of the technology shock depends on the value of the AR coefficient ρ_A which is 0.88 and we rounded it off to 0.9 for our shock process. The

[7] Table 5 provides the summary of the values used for the shock related parameters.

extent of variations in the TFP series can be seen by the standard deviation σ_A of the error term $\hat{\mu}_t^A$ of the AR process which was found to be 0.02.

1.12.2 Fiscal Shock (ρ_G,σ_G)

We followed the standard procedure, as mentioned in case of the technology shock, to obtain the two parameters, i.e., persistence ρ_G and standard deviation of the residuals σ_G of the fiscal shock. The following equation specifies the AR-1 process:

$$\ln \hat{g}_t = c + \rho_G(\ln \hat{g}_{t-1}) + \hat{\mu}_t^g \tag{7}$$

Estimation results are as follow:

$$\ln \hat{g}_t = \underset{(0.88)}{2.34} + \underset{(0.112)}{0.78}(\ln \hat{g}_{t-1}) + \hat{\mu}_t^g$$

Using log of real per capita government consumption from 1981 to 2010 sourced from FBS, estimation of the above equation yields the value of 0.78 for ρ_G i.e., the persistence of the fiscal shock. The standard deviation of residuals $\hat{\mu}_t^g$ from the above regression yields the value for σ_G to be 0.14.

1.12.3 Interest Rate Shock (ρ_R,σ_R)

Since the monetary policy is operating through the Taylor Rule in our model (see equation 4) , the interest rate shock is also systematically transmitted through the same rule. The estimation of this equation generates the residual $\hat{\mu}_t^r$. This $\hat{\mu}_t^r$, i.e., the residual of the interest rate r_t (denoted below as R_t only) is then regressed on its lag following the standard AR-1 shock process.

$$\ln \hat{R}_t = c + \rho_R(\ln \hat{R}_{t-1}) + \hat{\mu}_t^R \tag{8}$$

Call money rate from 1981 to 2010,by SBP, has been used as the interest rate. The estimated equation follows:

$$\ln \hat{R}_t = \underset{(0.003)}{-0.001} + \underset{(0.184)}{0.284}(\ln \hat{R}_{t-1}) + \hat{\mu}_t^R$$

The persistence coefficient ρ_R comes out to be 0.28 and the standard deviation σ_R of the residual of the AR equation $\hat{\mu}_t^R$ is found to be 0.016.

Table 5: Shock Process Parameters

Sr.#	Parameter	Description	Value
1	ρ_A	Persistence of technology shock	0.9
2	ρ_G	Persistence of fiscal spending shock	0.78
3	ρ_R	Persistence of interest rate shock	0.28
4	σ_A	SD of technology shock	0.02
5	σ_G	SD of fiscal spending shock	0.14
6	σ_R	SD of interest rate shock	0.016

2 Conclusion

This document outlines the systematic process following which we calibrated the DSGE model for Pakistan with Informality. We have tried to proceed step by step to document all the empirical analyses we needed to conduct in order to make the model work effectively, i.e., to yield confident outcomes that match the stylized facts of the Pakistan's economy. In order to do so we have laid special emphasis on calibration of model parameters. We have tried to estimate as many of them as possible from the micro level data and have tried to restraint our reliance on secondary sources. However, such sources have been used when all feasible options failed. Secondly, calibrating the shock related parameters have been done in a consistent manner for all the three shocks, i.e., by following the standard AR-1 process. However, due to the unavailability of quarterly data our model outcomes are only as good as mimicking the stylized facts of the Pakistan economy. With the availability of the quarterly date we are hopeful to perform relevant policy analyses as well.

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