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Optimal Taxation, Inflation and the Formal and Informal Sectors

Saif Asif Khan
Sajawal Khan

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Optimal Taxation, Inflation and the Formal and Informal Sectors

Saif Asif Khan
PhD Scholar, London School of Economics, Department of Economics, S.A.Khan@lse.ac.uk

Sajawal Khan,
Joint Director, State Bank of Pakistan, sajawal.khan@sbp.org.pk

Abstract

This paper adds to the literature concerning the relationship between inflation and the degree of informality observed in developing countries. The government's optimal response, vis-á-vis usage of an inflation tax and a consumption tax, to changes in the subsistence levels of labour and consumption faced by the formal and informal sectors of the model economy is studied. Very significantly, the paper finds that having a large informal sector does not always imply a high rate of inflation for the economy; and depending on what factors explain the size of the informal sector, we may get differing results for the optimal rate of inflation.

JEL Codes: E2; E5; H2

Keywords: Optimal Taxation; Inflation; Informal Sector

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Contact of author for correspondence
Saif Asif Khan
PhD scholar
London School of Economics,
Department of Economics,
Houghton Street, London WC2A

Optimal Taxation, Inflation and the Formal and Informal Sectors

1 Introduction

> Price inflation, inadequate wages, and an increasing surplus to the requirements of the urban labour market have led to a high degree of informality in the income generating activities of the urban proletariat. Consequently, income and expenditure patterns are more complex than is normally allowed for in the economic analysis of poor countries.[1]

In 1973, economic anthropologist Keith Hart, who is credited with introducing the concept of the informal economy (Bekkers and Stoffers, 1995), made the above observation about the economy of Ghana, a developing country on the coast of West Africa. More than thirty years on, the analysis remains just as valid for describing the situation in many Third World countries, and the problems associated with studying their economies. Indeed, high prices are a persistent feature of the economies of several developing countries. This claim is supported by numerous studies; for instance, Koreshkova (2006) finds a high difference between the mean inflation rates for rich versus poor countries (4.4%, against 19.1%). Similarly, Campillo and Miron (1996) also find a strong negative relationship between inflation and per capita GDP, and significantly higher rates of inflation for poorer countries; while Agenor and Montiel (1999) contend that high inflation has been a central problem confronting policy makers in several developing countries.

At the same time, the economies of many developing countries are also characterised by the presence of a large informal sector (see Koreshkova, 2006; Saracoğlu, 2008; and Schneider and Enste, 2000). In light of these unique features of developing countries, it is possible to propose a public finance rationale for inflation, where the government uses monetary policy to tax a large, pervasive informal sector. This paper, then, will employ the public finance motive for inflation to study the response of the government's optimal taxation policy[2] to changes in the subsistence levels of labour and consumption faced by the formal and informal sectors of the economy, which are elaborated subsequently.

[1] Hart, K. (1973). Informal income opportunities and urban employment in Ghana. *The Journal of Modern African Studies*, *11*(1), 61-89.

[2] By optimal taxation policy, we mean a policy which maximizes the government's utility, which is what the Ramsey problem approach we employ is all about: a benevolent planner who is solely interested in maximizing the utility of the households, while meeting the government's budget constraint. We assume away all negative utility fallouts of a high inflation rate.

There are numerous definitions of what exactly constitutes informal activity in an economy, ranging from illegal transactions to traditional, small-scale industry in developing countries. For instance, Feige (1990) defines informal firms as: "those engaged in activities (that) circumvent, escape, or are excluded from the institutional system of rules, rights, regulations and enforcement penalties that governs those agents engaged in formal production and exchange." For most developing countries, the concept of informal enterprise is also associated with having fewer barriers to entry as opposed to formal enterprise, and relying on indigenous resources, family ownership, labour intensiveness, and unregulated, perfectly competitive markets (Bromley, 1978). Mead and Morrisson (1994) liken the informal sector to an elephant: one might not be able to characterise its true nature exactly while being blindfolded, but upon observation, there remains no doubt about what stands in front.

The use of inflation to tax the informal economy has been a standard argument in the literature (see Bailey, 1956; Phelps, 1973 and Nicolini, 1997). According to Friedman (1969), in a first-best environment, where the government has access to lump-sum taxes, monetary policy should follow a standard rule: nominal interest rates should be set equal to zero, corresponding to a zero inflation tax. Correia and Teles (1999) provide the logic behind Friedman's argument: since the marginal cost of supplying money is negligible, and the marginal benefit should equal marginal cost, therefore, the nominal interest rate should equal zero. In monetary economics, this is known as the Friedman rule of taxation.

However, the literature is more inconsistent in establishing the optimality of the Friedman rule in second-best environments, where taxes are distortionary. Phelps (1973) contends that if alternative sources of revenue create distortions, then liquidity should be taxed, just as any other good, and the nominal interest rate should be greater than zero. This result is refuted by Lucas and Stokey (1983), who illustrate that it is still possible to satisfy the Friedman rule within a second-best environment, when certain conditions on the households' utility function are satisfied. Correia and Teles (1999) also find that the Friedman rule can be optimal in the presence of distortionary taxes, as long as the cost of producing money equals zero.

On the other hand, when the cost of producing money is no longer negligible, then the Friedman rule will not be optimal. This might happen in an economy with a large informal sector that operates independently of the formal sector. The crux of the argument rests on the fact that transactions in the informal sector are largely cash-based (Nicolini, 1997). As a result, money is no longer produced for free; because money equals the nominal value of informal consumption, while informal goods are produced at a cost. This enables the government to use a positive nominal interest rate as a tax on cash purchases (Koreshkova, 2006). The optimality of a non-zero nominal interest rate in the presence of imperfect taxation has been verified by Vegh (1989) and Roubini and Sala-i-Martin (1995).

Therefore, this paper attempts to find an optimal taxation policy using both monetary and fiscal tools for a model economy with a large cash-based informal sector. The country that has been chosen as a unit of analysis is Pakistan. The economy of Pakistan

Table 1; *Currency in circulation as a percentage of GNP in South Asia*					
	Bangladesh	*Bhutan*	*India*	*Pakistan*	*Sri Lanka*
2001	5.27%	6.77%	9.85%	10.33%	5.31%
2002	5.04%	6.34%	10.37%	10.90%	5.45%
2003	4.88%	6.24%	10.61%	11.29%	4.74%
2004	5.22%	6.83%	10.33%	11.37%	4.81%
2005	5.64%	8.01%	10.42%	11.03%	4.71%
2006	7.10%	7.05%	10.54%	11.25%	4.66%
2007	6.89%	8.87%	10.49%	11.42%	4.16%

Source: Haver Analytics

is a good candidate for such analysis firstly because it is characterised by a large informal sector. According to statistics for 2009-10 from the Federal Bureau of Statistics,[3] the informal sector constitutes up to 72% of the non-agricultural labour force, and 40% of the total labour force in Pakistan (FBS, 2008). At the same time, Pakistan also has one of the highest currencies in circulation as a percentage of GNP in the South Asian region, which indicates that a significant part of the economy is cash-based (see Table 1).

Of recent, the economy of Pakistan has also been characterised by high rates of inflation, in excess of 20%, as indicated by statistics from the State Bank of Pakistan (SBP, 2009).[4]As a consequence, it displays features that will be crucial elements of our model, i.e., the ability to use inflation as a tax on a large informal sector, in an economy where conventional fiscal tools' outreach is limited to the formal sector. The paper will employ a standard Ramsey planner approach that is used for such problems (for instance, see Chari, Christiano, and Kehoe, 1991; Cavalcanti and Villamil, 2003; Koreshkova, 2006; and Lucas and Stokey, 1983). The economy comprises two sectors that use differing production processes to produce two distinct goods that the household consumes. Households supply labour, which is the only factor of production, to both the sectors.

However, this paper differs from earlier work in this area in two significant ways. Firstly, the modelling of consumers' preferences reflects a negative income effect property for the informal sector good. There is good reason to believe that this presents an accurate picture of the demand for formal and informal sector goods in developing countries. For instance, Saracoğlu (2008) discovers that households in Turkey devote smaller fractions of total expenditure to the consumption of informal and agricultural

[3]The Federal Bureau of Statistics is the primary statistical agency of the country, operating under the Ministry of Economic Affairs and Statistics of Pakistan. It is responsible for collecting statistics on the economy, the government, trade and other fields.

[4]The State Bank is the central bank of Pakistan.

goods as incomes rise. Similarly, Tybout (2000) finds that there are Engel effects for goods produced by cottage industries. Similar results are also reported by Irz and Roe (2005), and Echevarria (1997, 2000). A negative income effect for the informal good is incorporated using a subsistence level of consumption for the informal good, which is retained as a variable in the model.

The second unique feature which we add to this study is the presence of an unavoidable sunk cost that the formal firm faces. This would amount to the cost of formality that firms in the informal sector would not face; for instance, the registration fee paid to the government. This is the second key variable that we keep as an instrument of choice in the model. More specifically, we analyse changes in the tax policy resulting from changes in the level of sunk cost faced by the formal sector, and informal subsistence faced by households. This might allow us to comment on the implications of having a high cost of formality (for example, a high registration fee), or a high dependence on informal sector consumption for taxation in the model economy.

Very significantly, these two features will have serious repercussions for the relationship that exists between informality and the inflation rate in the model economy. There is some evidence for a positive relationship between inflation and the size of the informal sector in literature. According to Koreshkova (2006), there is cross-country evidence to suggest that average inflation rates correlate positively with the size of a country's informal sector. These findings are also supported by Campillo and Miron (1996), who report that the inflation rate is higher in industrialized economies, which have a small informal sector, and lower in developing countries, which have large degrees of informality in their economies. On the other hand, we find that what explains the informal sector's large size will determine the magnitude of the optimal inflation rate for the economy, so that given the assumptions of the model, a larger informal sector does not necessarily imply at higher rate of inflation.

For the purpose of this paper, we will use the definition of the informal sector employed by the Federal Bureau of Statistics, which uses two criteria to define the informal sector: the enterprise must be private and unincorporated; and it must employ less than ten units of labour (FBS, 2010).[5]The first condition ensures that informal firms do not pay any taxes to the Government, while the second condition means that informal firms will inevitably be small. All enterprises engaged in agricultural activities are excluded from the definition of informality, as is standard for such studies.

The paper will proceed hereon as follows: Section 2 will describe the environment in which households, firms and the Government make decisions, while Section 3 brings together the agents' interaction in the model and solves for an equilibrium. Having arrived at an equilibrium, we then calibrate the economy in Section 4, and see how the optimal taxation policy changes in response to variation in the level of sunk cost and

[5]It is a standard practice to use both registration status, as well as threshold criteria, in measuring informality. For instance, the International Labour Organisation defines the informal sector as own-account workers, domestic workers and micro-enterprises with up to 5 workers (Zuin, 2004); while Perry et al (2007) report that labor informality in Latin America is primarily a small firm phenomenon, with the vast majority of workers who are unregistered with social security administrations found in firms of fewer than five workers.

subsistence expenditure, in Section 5. Finally, Section 6 concludes the paper.

2 The environment

2.1 Households

Consider an economy that uses the representative household framework à la Lucas and Stokey (1983). The household has a choice between goods from the formal and informal sectors. To simplify matters, we use a standard cash-versus-credit good distinction here for goods from the two sectors. More specifically, consumption of goods from the informal sector c_{it} can only be financed using cash, whereas consumption of goods from the formal sector c_{ft} may be financed using cash or credit. This is a reasonable assumption that finds support in the case of both developing, as well as developed, economies (Schneider and Enste, 2000; Nicolini, 1997). The informal/cash good is paid for at the time of purchase in period t, while the formal/credit good is paid for in period $t+1$(interest free). [6]The assumption of no interest is enforced to ensure that the household is indifferent between using cash and credit in buying goods from the formal sector.

Besides consuming, the household also supplies labour to firms in the two sectors. Labour supply to both sectors is separate: in each unit of time, the household's total endowment of time is normalised to 1 unit, out of which l_{ft} is supplied to the formal sector and l_{it} is supplied to the informal sector.[7] The remainder of the time endowment is spent on leisure. Households' preferences over a stream of consumption goods c_{ft}, c_{it} and leisure $1-l_{ft}-l_{it}$, t=0,1,2,... are given by:

$$\sum_{t=0}^{\infty} \beta^t u(c_{ft}, c_{it}, 1-l_{ft}-l_{it}) \tag{1}$$

where $\beta \in (0,1)$ is the stochastic discount factor. The utility function that we employ is:

$$u(c_{ft}, c_{it}, 1-l_{ft}-l_{it}) = a \ln c_{ft} + b \ln(c_{it}-\bar{p}) + \frac{(1-l_{ft}-l_{it})^{1-\theta}}{1-\theta} \tag{2}$$

which allows for negative income effects for the informal sector good. This is accomplished by utilising Stone Geary preferences for the household's consumption, where $\bar{p}$ represents the household's unavoidable expenditure on the informal good. Note that we must impose a non-negativity condition $c_{it}-\bar{p} \geq 0$ on informal consumption. Using Stone Geary preferences for the informal sector good implies the existence of a minimum subsistence level informal consumption, which would make demand for the informal good relatively inelastic. This formulation of the utility function has also been used

[6]The timing of purchase of formal and informal goods is fixed in this manner to maintain the indifference of the household between buying on credit versus making cash purchases. The unit of time t may typically be interpreted to mean one year.

[7]The idea that the household is indifferent between supplying labour to either the formal or the informal sector may theoretically lead to a situation where all the labour force is employed either in the formal or the informal sector. Given the assumptions of the model, which are not completely realistic, this should not be a problem.

by Saracoğlu (2008) to capture the Engel effects of consumption, whereby households spend less on informal and agricultural consumption as incomes rise.

The utility function is additively separable in leisure. Moreover, labour is perfectly substitutable between the formal and informal sectors. Besides consumption, households also choose an amount of money holdings, M_t, and bonds, B_t, which may be issued by the government and other households. Households earn income from working in both sectors, and the profits that the firms earn from production. Households maximise their utility from consumption subject to the following one-period budget constraint:

$$w_{f,t-1}l_{f,t-1} + w_{i,t-1}l_{i,t-1} + \Pi_{f,t-1} + \Pi_{i,t-1} + (1+i_{t-1})B_{t-1} + M_{t-1} \geq p_{f,t-1}(1+\tau^c_{t-1})c_{f,t-1} + p_{i,t-1}c_{i,t-1} + M_t + B_t \quad (3)$$

and a cash-in-advance constraint:

$$M_t \geq p_{it}c_{it} \quad (4)$$

which ensures that nominal expenditure in the informal sector does not exceed the household's cash balances.

2.2 Firms

Production uses labour as the sole factor of production. This is a simplifying assumption that is also used by other papers in this area, such as Koreshkova (2006) and Cavalcanti and Villamil (2003).[8] The production technology differs across sectors: production in the formal sector is characterised by the presence of a sunk cost, $\bar{k}$, that takes the form of a constraint on the utilisation of labour:

$$y_{ft} = \begin{cases} B(l_{ft} - \bar{k})^\gamma & \text{if } l_{ft} \geq \bar{k}, \\ 0 & \text{if } l_{ft} < \bar{k}. \end{cases} \quad (5)$$

According to Heijdra and van der Ploeg (2002), this formulation captures the notion that the formal firm must spend a minimum amount of labour ("overhead labour", $\bar{k}$) before it can produce any output at all. It is clear that formal firms face a cost of formality that firms in the informal sector do not have to pay; for instance, the cost of getting registered with the government, mentioned earlier. This is a sunk cost of formality that enters the model as labour units in the formal firm's production function. Perhaps one way of understanding this is a conversion of the sunk cost into labour units, given that labour is the only factor of production being employed by firms, and that the sole determinant of firm size is taken to be number of employees. In this setting, then, a larger value of k would imply a higher sunk cost, which is a realistic assumption for large firms facing correspondingly high sunk costs.

[8] There is no economic intuition for using a sole factor of production; the assumption that production employs just labour to produce output is made purely for simplification purposes, as is standard in such models. Adding two types of capital, or even one, would unnecessarily complicate the model. Since the focus of the paper is on determining optimal tax rates, this can be done by using different types of labour alone.

Other interpretations of $\bar{k}$ could be extra hiring of workers that firms are required to carry out in order to function in the formal sector. Formal firms may be forced to hire more than the number of workers involved in production to account for poor infrastructure in developing countries. For example, governments in poor countries often provide a poor quality of law enforcement, so that firms may be forced to hire more workers than those needed in production to provide security to the firm's property. These are issues a small informal firm would not be faced with. Formal firms might also be legally obliged to comply with certain laws; e.g., abiding by hiring quotas from different areas, ethnicities or genders, which might not match with the free market allocation.

On the other hand, production in the informal sector is not constrained by any sunk costs. However, we assume that technology in the informal sector is inferior to that of firms in the formal sector. In the model, this would imply that the productivity parameter, A, in the informal firm's production function is smaller than the corresponding parameter, B, in the formal firm's production function:

$$y_{it} = Al_{it}^{\delta} \tag{6}$$

There is evidence to suggest that this is a valid assumption for the informal sector in developing countries in general, and Pakistan in particular. For instance, Burki and Terrell (1998) in their study on the manufacturing sector of Pakistan discover that small firms, having lesser access to credit, use old/outdated technology, as opposed to the latest technology that is used by large firms. In their results, they show that most small firms are inefficient (as compared to other small firms), since they lie below the efficiency frontier for the industry. Nasir (2001) claims that the informal sector in Pakistan suffers from low productivity, and has little use for sophisticated technology that large firms use. These findings are also supported by Kristiansen (2003), who studied formal and informal production in Indonesia, and found that the rural cottage industry makes goods using an inferior technology as compared to the urban sector, which is typically formal.[9]

2.3 Government

In our model economy, we integrate the monetary and fiscal authority into a single entity known as "Government". The government issues bonds and money, and earns income from consumption taxes that are imposed on the formal sector, and an inflation tax on the informal sector. The government spends an exogenous amount $p_{ft}g_t$ on purchases from the formal sector. The government's period budget constraint is:

$$\tau_t^c p_{ft} c_{ft} + M_{t+1}^g - M_t^g + B_{t+1}^g - (1 + i_t)B_t^g \geq p_{ft} g_t \tag{7}$$

where M^g is the money supply and B^g is the government debt.

[9]The evidence found in the literature provided to support the presence of an inferior technology in the informal sector might be with reference to poorer quality of capital, or labour resources in the informal sector, which we have not factored for explicitly. However, the reader is reminded that technology in this model is treated as an exogenous variable. Therefore, the effect of poorer capital, manpower, working environment, or other factors will filter into the model through the technology parameters, A and B.

According to Lucas and Stokey (1983), tax policies that are optimal in the Ramsey sense are prone to suffer from a problem of time inconsistency. In an economy such as our's, with no capital, this time inconsistency would manifest itself in the government's incentive to default on outstanding debt obligations (Albanesi, 2006). Therefore, in order to circumvent this problem, we will assume that the government never defaults on its debt obligations. This is the standard approach used in the literature (for instance, see Koreshkova, 2006).

Why inflation acts as a tax on informal consumption should be fairly obvious. Dividing both sides of the government's binding budget constraint by p_{it}, we get:

$$\tau_t^c \frac{p_{ft}}{p_{it}} c_{ft} + \frac{1}{p_{it}}(M_{t+1}^g - M_t^g) + \frac{1}{p_{it}} B_{t+1}^g - (1+i_t)\frac{1}{p_{it}} B_t^g = \frac{p_{ft}}{p_{it}} g_t \tag{8}$$

Seigniorage, or the government's revenue derived from issuing currency in this economy, may be expressed as $\frac{\Delta M^g}{p_{it}}$. Therefore, we have seigniorage in the equation above. According to Drazen (1985), we may decompose seigniorage into:

$$\frac{\Delta M^g}{p_{it}} = \Delta m + \pi_{it} m \tag{9}$$

where m refers to real money balances (using informal sector prices) and π_{it} is the rate of growth of prices in the informal sector. According to the cash-in-advance constraint that households are subject to, $p_{it} c_{it} = M_t$, while we know that in equilibrium, $M_t^g = M_t$. This allows us to rewrite the expression for seigniorage as:

$$\frac{\Delta M^g}{p_{it}} = \Delta m + \pi_{it} c_{it} \tag{10}$$

In a steady state equilibrium, the government's revenue expressed in units of the informal good should equal:

$$\pi_i c_i + \tau^c \frac{p_f}{p_i} c_f \tag{11}$$

so that the rate of increase of prices in the informal sector acts as a tax on informal sector purchases, while the consumption tax earns revenue from the formal sector.

Conversely, we can also express the government's revenue in units of the formal good, which would involve dividing the government's binding budget constraint by p_{ft}. The rest of the steps are the same, and we obtain a value for government revenue equal to $\pi_i \frac{p_i}{p_f} c_i + \tau^c c_f$.

3 Model equilibrium

3.1 Competitive equilibrium

We start by solving for the competitive equilibrium, which is a set of allocations $\{c_{ft}, c_{it}, l_{ft}, l_{it}, B_t, M_t\}_{t=0}^{\infty}$, a set of firm choices $\{l_{ft}^d, l_{it}^d\}_{t=0}^{\infty}$ and a sequence of prices and taxes, $\{p_{ft}, p_{it}, i_t, \tau_t^c\}_{t=0}^{\infty}$, such that:

- given the sequence of prices and taxes, the allocations maximise (1) subject to (3) and (4), together with a no-Ponzi games condition.
- given the sequence of prices and taxes, the firms' allocations maximise profits; and
- market clearing:

$$M_t = M_t^g \tag{12}$$

$$B_t = B_t^g \tag{13}$$

$$l_{ft}^d = l_{ft} \tag{14}$$

$$l_{it}^d = l_{it} \tag{15}$$

Firms' profits do not affect the households' first order conditions, since these profits are obtained by firms using profit maximisation and contain the optimal amount of labour supplied. We simplify the first order conditions from the household's maximisation problem to obtain five conditions that must be satisfied in equilibrium:

$$w_{ft} = w_{it} \tag{16}$$

$$\frac{u_c(f,t)}{p_{ft}(1+\tau_t^c)} = \frac{-u_l(t)}{w_{ft}} \tag{17}$$

$$\frac{u_c(i,t)}{p_{it}(1+i_t)} = \frac{-u_l(t)}{w_{it}} \tag{18}$$

$$\frac{u_c(f,t)}{p_{ft}(1+\tau_t^c)} = \frac{u_c(i,t)}{p_{it}(1+i_t)} \tag{19}$$

$$1+i_{t+1} = \frac{u_c(f,t)p_{f,t+1}(1+\tau_{t+1}^c)}{\beta u_c(f,t+1)p_{ft}(1+\tau_t^c)} \tag{20}$$

in addition to the budget constraint (3) and the cash-in-advance constraint (4), for all t. Here, $u_c(j,t)$ denotes the period t marginal utility of consumption for sector j for $j \in (f,i)$, while $u_l(t)$ indicates the period t marginal utility for labour. Since the leisure component of the utility function is linear in the two kinds of labour, the marginal utility does not depend on what sector the worker works in. Consequently, the first result (16) indicates that the nominal wage in both sectors will be equal.[10] While this appears odd at first glance, yet there is some evidence to suggest that wages in the informal sector in Pakistan are similar to those earned in the formal sector (see Burki and Afaqi, 1996).

The rest of the equations are standard results. Eq.(17) describes the intra-temporal utility trade-off between an extra unit of formal consumption and an extra unit of working in the formal sector, while eq.(18) does the same thing for the informal sector. Eq.(19) compares the intra-temporal trade-off between consumption from either sector. Finally, eq.(20) determines the intertemporal utility trade-off in terms of formal consumption in

[10]There is no reason to believe that the real wage will also be the same in both sectors, if the price level in both sectors is not equal.

period t and $t+1$. Moving on to the firms, profit maximisation in a perfectly competitive setup implies that optimal labour demand functions will occur where $p_{jt}MPL_{jt} = w_t$ for $j \in (f, i)$ and MPL being the marginal product of labour. This allows us to work out both firms' maximised profits, Π_{ft} and Π_{it}:

$$\Pi_{ft} = p_{ft}B(l_{ft} - \bar{k})^{\gamma}\left[1 - \frac{\gamma l_{ft}}{(l_{ft} - \bar{k})}\right] \tag{21}$$

$$\Pi_{it} = p_{it}Al_{it}^{\delta}(1 - \delta) \tag{22}$$

These profits will enter the household's budget constraint as an additional source of income.

3.2 Ramsey equilibrium

Assume the government body chooses an optimal policy plan $s = \{\tau_t^c, M_t, B_t\}_{t=0}^{\infty}$ that maximises the household's welfare subject to the constraints necessary for a competitive equilibrium. The nominal stock of money and bonds, $M_0 + B_0$ is set equal to zero for simplicity.

The Ramsey equilibrium, then, is a policy plan s and a sequence of allocations $\{c_{ft}, c_{it}, l_{ft}, l_{it}\}_{t=0}^{\infty}$ such that:

- s maximises (1) subject to the resource and implementability constraints for the economy; and
- the sequence of allocations solves the household's problem, given the policy plan s.

The government's Ramsey problem is hence given by:

$$\max_{c_{ft}, c_{it}, l_{ft}, l_{it}} \sum_{t=0}^{\infty} \beta^t u(c_{ft}, c_{it}, 1 - l_{ft} - l_{it}) \tag{23}$$

subject to the two resource constraints;

$$y_{it} = c_{it} \tag{24}$$

$$y_{ft} = c_{ft} + g_t \tag{25}$$

and the implementability constraint:

$$\sum_{t=0}^{\infty} \beta^t \{u_l(t)l_{ft} + u_l(t)l_{it} + u_l(t)\frac{1}{\gamma}\left(l_{ft}(1-\gamma) - \bar{k}\right) + u_l(t)\frac{1}{\delta}l_{it}\left(1-\delta\right) + u_c(f,t)c_{ft} + u_c(i,t)c_{it}\} = 0 \tag{26}$$

Eq.(24) states that all goods produced by the informal sector are consumed by households, while eq.(25) states that goods produced by the formal sector are consumed by

households and the government. Finally, eq.(26) determines the trade-off between consumption and leisure, and is obtained by extending the household's one-period budget constraint to an infinite time horizon and substituting out all prices, wages, taxes and the interest rate using the results from the competitive equilibrium. Note that firms' maximised profits have been included in the implementability constraint.

Now, for purposes of convenience, we write the Ramsey problem in Lagrangian form. Let us define:

$$\begin{aligned} W(t) = u(c_{ft}, c_{it}, 1 - l_{ft} - l_{it}) \\ + \lambda\{u_l(t)l_{ft} + u_l(t)l_{it} + u_l(t)\frac{1}{\gamma}\left[l_{ft}(1-\gamma) - \bar{k}\right] \\ + u_l(t)\frac{1}{\delta}l_{it}\left(1-\delta\right) + u_c(f,t)c_{ft} + u_c(i,t)c_{it}\} \end{aligned} \tag{27}$$

Then, using the Lagrange method, where λ denotes the Lagrange multiplier on (26), we maximise $\sum_{t=0}^{\infty}\beta^t W(t)$ subject to (25) and (24). The function W simply incorporates the implementability constraint into the maximand. We obtain the following results:

$$W_{c,i}(t)MPL_{it} + W_{l,i}(t) = 0 \tag{28}$$

$$W_{c,f}(t)MPL_{ft} + W_{l,f}(t) = 0 \tag{29}$$

where $W_{j,k}(t)$ represents the partial derivative of $W(t)$ with respect to $j_{k,t}$. Note that the results above are obtained after simplifying the first order conditions, so that the Lagrange multipliers on the two resource constraints are eliminated.

Substituting for the $W_{j,k}(t)$ terms, we get two equations in terms of c_{ft}, c_{it}, l_{ft} and l_{it}:

$$u_c(f,t)\left\{1+\lambda\left[1+\frac{u_{cc}(f,t)}{u_c(f,t)}c_{ft}\right]\right\}MPL_{ft} = -u_l(t)\left\{1+\lambda\left[\frac{1}{\gamma}+\frac{u_{ll}(t)}{u_l(t)}\left(\frac{l_{it}}{\delta}+\frac{l_{ft}-\bar{k}}{\gamma}\right)\right]\right\} \tag{30}$$

$$u_c(i,t)\left\{1+\lambda\left[1+\frac{u_{cc}(i,t)}{u_c(i,t)}c_{it}\right]\right\}MPL_{it} = -u_l(t)\left\{1+\lambda\left[\frac{1}{\delta}+\frac{u_{ll}(t)}{u_l(t)}\left(\frac{l_{it}}{\delta}+\frac{l_{ft}-\bar{k}}{\gamma}\right)\right]\right\} \tag{31}$$

From the household's FOCs (17) and (18), we know that $-u_c(i,t)MPL_{it}/u_l(t)$ equals $1+i_t$ and $-u_c(f,t)MPL_{ft}/u_l(t)$ equals $1+\tau_t^c$. Solving for the relevant $W_{j,k}(t)$ and substituting back into the equations derived above, we obtain the following results:

$$1+\tau_t^c = 1+\lambda\left[\frac{1}{\gamma}+\frac{\theta(l_{ft}-\bar{k})}{\gamma(1-l_{ft}-l_{it})}+\frac{\theta l_{it}}{\delta(1-l_{ft}-l_{it})}\right] \tag{32}$$

$$1+i_t = \frac{1+\lambda\left[\frac{1}{\delta}+\frac{\theta(l_{ft}-\bar{k})}{\gamma(1-l_{ft}-l_{it})}+\frac{\theta l_{it}}{\delta(1-l_{ft}-l_{it})}\right]}{1-\frac{\lambda\bar{p}}{c_{it}-\bar{p}}} \tag{33}$$

Moreover, once we have the nominal interest rate, we may use this to calculate the rate of inflation. To calculate the rate of inflation for the entire economy, we would

need some kind of price index that combines the prices of both sectors. On the other hand, calculating the rate of inflation in the informal sector is much more simple, and simply involves combining eq.(19) with eq.(20), and imposing a steady state condition, to obtain:

$$\beta(1+i)-1=\pi_i$$

Notice that we drop the time subscript in lieu of imposition of steady state. Henceforth, we will use this π_i as a measure of the rate of taxation on the informal sector. The rate of inflation in the formal sector is not relevant to our discussion because the formal sector does not rely on cash, and is instead taxed via the consumption tax, τ^c.

4 Calibration

The way the quantitative exercise for this paper will be carried out is as follows. The economy of Pakistan is chosen as a base of analysis and the set of parameters are estimated so that the characteristics of the base country are delivered by a Ramsey equilibrium in the model economy.We calibrate different scenarios, depending on what values the level of subsistence informal consumption, $\bar{p}$, could assume. Following the calibration, the amount of sunk cost of formality, $\bar{k}$, will be increased gradually and the effects on the optimal taxation policy will be discussed.

4.1 Households

The discount factor is fixed at 0.95, which is a fairly standard approximation of β in the literature. While calibrating the model, we also tried using other values of β, ranging between 0.95 and 1. The results do not change significantly for these values of β.

The utility function contains four parameters: a, b, θ and $\bar{p}$. The parameter a may be interpreted as the fraction of supernumerary expenditures spent on the formal sector good. Similarly, b denotes the fraction of supernumerary expenditures devoted to the informal good. We attempt to calibrate these parameters for Pakistan using two distinct approaches.

The first approach uses data from the Federal Board of Revenue[11] from 1999-2010 to establish the list of 920 industrial and consumer goods and services on which the Government of Pakistan collects sales tax. If the Government earns tax from a commodity, it is classified as a formal good, whereas commodities with zero tax collection are classified as informal goods.[12]Next, we look at the list of 374 commodities that is used in calculating the consumer price index for Pakistan, and the weights assigned to each of

[11]The Federal Board of Revenue is the supreme federal agency responsible for enforcing and collecting revenue for the Government of Pakistan.

[12]There are some exceptions to this rule: certain goods, such as medicines, are produced by formal firms, but are legally exempted from taxes. Also, there is tax collection on certain commodities, such as wheat, or fresh meat, but the extremely low collection of revenue from such goods in relation to their consumption means that we have to adjust their classification as "formal" or "informal" accordingly.

these commodities.[13]We tally both these lists in order to obtain weights for the formal and informal sectors. This allows us to determine the weight allocated by the average household buying a representative basket of goods and services to purchases from both the formal and informal sectors.

The issue of housing as an item of expenditure poses a potential problem because there is a substantial weight (23.43 units) associated with the House Rent Index in the CPI, while the nature of the housing market in Pakistan is somewhat ambiguous.[14]To circumvent this problem, we exclude housing from the representative basket of goods and services, so that the sum of weights for the CPI is 76.57 units instead of a 100 units. A division of weights between the two sectors based on tax collection, then, yields a value of 44.05% for the informal sector, and 55.95% for the formal sector.[15]This suggests that $a = 0.5595$ and $b = 0.4405$.

The second approach that we use to estimate a and b involves looking at data for average monthly per capita expenditure incurred by a representative household on food and other items across Pakistan for the year 2008. This information is captured by the Federal Bureau of Statistics' Household Integrated Economic Survey (HIES) for 2007-2008.[16]Using the same criteria for determination of formal versus informal goods mentioned earlier, all commodities in the HIES data are split into these two categories.[17]

Housing is again excluded from the total monthly expenditure, owing to its ambiguous nature. The total per capita monthly expenditure on all items excluding housing is Rs.1632.77, out of which expenditure on informal goods is Rs.753.16. This means that informal goods constitute 46.13% of per capita monthly consumption, whereas formal goods constitute 53.87%. This suggests that $a = 0.5387$ and $b = 0.4613$.[18]Based on these estimates, we set both a and b equal to averages of the numbers obtained from both approaches. Hence, a is set equal to 0.55, and b is set equal to 0.45. These values suggest that a higher proportion of income is spent by Pakistani households on purchases from the formal sector, than on the informal sector. This finding also applies to the Turkish economy, for which Saracoğlu (2008) finds that consumers spend 70% of their income on goods bought from the formal sector, and 30% of their incomes on informal and agricultural consumption.

[13]The list of CPI commodities and the associated weights were obtained from the Research Department of the State Bank of Pakistan.

[14]The housing market is officially formal, but there is considerable tax evasion, which makes it hard to place housing in the list of formal goods.

[15]If we include rent in the CPI, so that the weights sum to a hundred, and include housing as an informal good, then we get a sum of weights of 56.95% for the informal sector, and 42.58% for the formal sector.

[16]The HIES contains important data on household income, consumption expenditure and consumption patterns at national and provincial level, with urban/rural breakdowns.

[17]Commodities with tax collection in the FBR data are assigned a formal good status; commodities with zero tax collection are assigned an informal good status. There are certain exceptions to this rule, as before.

[18]Again, if we include housing in the monthly expenditures, and count housing as an informal good, we find that the share of the informal sector in monthly per capita consumption increases to 54.27%. This is similar to the value of 56.95% for the informal sector that we obtain using the first approach when we include housing in both the CPI, as well as the informal sector.

θ represents the negative inverse of the Frisch elasticity of leisure with respect to wage. There does not appear to be much consensus on the values of the Frisch elasticities of labour supply and leisure in literature. According to Domeij and Floden (2006), the Frisch elasticity of labour supply should be between 0.1 and 0.3. However, they admit that these estimates are biased downwards, and suggest that the true values should be larger. We set the value of θ equal to 2, so that the Frisch elasticity of leisure equals -0.5. This assumption is supported by N'Diaye, Zhang and Zhang (2010), and Carroll and Young (2010).[19]

The estimation of $\bar{p}$ is more problematic. We interpret $\bar{p}$ as the minimum necessary amount that the household must spend on the informal sector good. As mentioned earlier, informal subsistence expenditure is a realistic assumption for developing countries. As an economy transitions from low-income to high-income status, it would be expected that the subsistence expenditure would decrease, as consumers would prefer to buy less from informal markets such as street bazaars and outdoor markets, and more from supermarkets and shopping malls (Saracoğlu, 2008). In her study of the Turkish economy, Saracoğlu (2008) uses consumption data to estimate the subsistence level expenditures on agricultural and informal sector goods.

Following from Saracoğlu (2008), we use the HIES dataset to estimate subsistence expenditure for Pakistan. The HIES contains a list of commodities consumed by the average Pakistani household, as well as the monthly expenditure incurred on purchasing these commodities. The list of commodities is disaggregated into formal and informal goods, using the earlier approach. We next try to determine what commodities out of the informal commodities could be classified as "subsistence commodities". This poses various problems, since there is no universal definition of a "subsistence good"; what is considered essential by one individual may well be considered an avoidable luxury by another individual. At the same time, it is not compulsory that all subsistence goods be produced by the informal sector in the real world. For instance, allopathic medicines are often cited as an example of a necessity in economic literature, but in Pakistan, as elsewhere, the pharmaceutical industry is largely formal.

One way of resolving this would be to examine disaster relief goods distributed by relief organisations in times of crisis in Pakistan. Such goods commonly include wheat flour, tea, sugar, rice, pulses, basic spices, milk, and vegetable fat.[20]Adding basic clothing, fuel (firewood) and medical care to these items, and adjusting for the share of formal goods, we conclude that subsistence expenditure should be close to 54% of informal expenditure for Pakistan.[21]However, we admit that this is a rather subjective approach,

[19]In our calibrations, we also attempted using other values of θ, including 4 and 6. The impact on the results is not substantial. It is important to ensure that θ is strictly larger than 1 to keep the utility function non-linear in leisure. 2 is a fairly standard value in the literature. Also, 2 allows the model to be solved manually, for checking.

[20]A useful reference point could be the survival kits distributed by various organizations on occasion of the 2010 Floods. For instance, the World Food Programme ration pack included wheat flour, pulses, cooking oil, salt, biscuits, sugar, and tea. The UNHCR food basket contained tea, rice, sugar, salt, dates, spices, fruit juice, milk powder and flour. The Karachi Relief Trust ration pack contained sugar, rice, wheat flour, cooking oil, salt, red chili powder, powdered milk, pulses and tea, among other things.

[21]Again note that housing is excluded from the list of subsistence goods from Pakistan, which might

compounded by the fact that there are some quality issues with data from Pakistan (see Gennari, 2004). Therefore, we try to circumvent this problem by using different values for $\bar{p}$ ranging between 0 and 0.20. It will be observed that with every rise in $\bar{p}$, subsistence informal expenditure assumes a higher fraction of the equilibrium values for informal consumption. The values are chosen to progressively make informal subsistence expenditure higher and higher, so that it roughly varies between 0 to 80% of c_{it}. This should allow us to avoid the pitfall of working with any particular value of subsistence expenditure.

Without any sunk costs being incurred by the formal sector, $\bar{p}$ at 0.20 units is close to 84.53% of informal consumption, which is a reasonably high upper bound for subsistence expenditure. Beyond $\bar{p} = 0.30$, the model's convergence properties become very weak, since the non-negativity constraint on $c_{it} - \bar{p}$ is close to being violated.

4.2 Firms

The unknown parameters on the production side of the economy are B, A, γ, δ and $\bar{k}$. The value of B, which is the technology in the formal sector, is normalised to equal 1. We noted earlier that the technology in the informal sector is inferior to that possessed by the formal sector, which means that $A < 1$. Following Koreshkova (2006), who states that the ratio of informal to formal technologies for poor countries is typically 40%, we set $A = 0.4$.

γ refers to the returns-to-scale parameter on the formal firm's production function, while δ is the corresponding parameter for the informal firm. Even though we do not have capital in our model, yet we would realistically expect production in the formal sector to be relatively less labour-intensive than production in the informal sector, and this would be reflected in the returns-to-scale parameters for the two sectors.

While there haven't been any recent studies for factor shares in Pakistan across sectors, yet there has been substantial work on other developing economies that might be useful in helping us learn the factor shares for labour across the formal-informal divide. Verma (2008) has estimated labour factor shares for agriculture, industry and services in India, which she finds to be 58%, 45% and 56%, respectively. Saracoğlu (2008) calculates labour factor shares to be 26% and 58% for the formal and informal sectors in Turkey, respectively. Satchi and Temple (2009) report that the labour share for the formal sector in Mexico is 43%. Based on these studies, then, we set $\gamma = 0.45$ and $\delta = 0.6$ in our model. We feel these values should reflect the labour intensiveness of the informal sector adequately.

This leaves us with the last remaining parameter, $\bar{k}$, which is the sunk cost that the formal firm has to pay to operate in the formal sector. We retain $\bar{k}$ as the parameter of interest that we will vary to see how the optimal taxation policy changes.

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appear odd, since shelter is another widely cited example of a necessity good. This is to ensure consistency with the definition of the informal goods. If we were to include housing as both a subsistence and an informal good, then the share of subsistence in informal consumption rises to around 67%.

4.3 Government Expenditures

We look at figures for the public-private split in gross capital formation for Pakistan as a proxy for government spending in our model. According to data for gross fixed capital formation released by the Federal Bureau of Statistics, public sector investment was 28.79% of total investment in the economy for 2009-10 (FBS, 2010). Hence, the amount of government expenditure, g, is exogenously set equal to 0.08. This value of g is selected so that when $\bar{p}$ is realistically high (i.e, between 55% and 67% of informal output, as estimated earlier), then g as a fraction of total formal sector output is close to 30%.

5 Results

To recap the calibration, Table 2 reports the model parameters that deliver a Ramsey equilibrium with the properties specified above. We solve the model for different values of $\bar{k}$, ranging from 0 to 0.70. We consider five different cases, each with differing levels of informal subsistence, ranging from 0 to 0.20. Like the case of informal subsistence expenditure, there is a non-negativity constraint on the formal firm's effective labour: $l_f - \bar{k} \geq 0$. As $\bar{k}$ rises, the firm's effective labour usage, $l_f - \bar{k}$ gets smaller and smaller. Beyond $\bar{k} = 0.79$, the model's convergence properties become increasingly weak, implying that the non-negativity constraint is close to violation.

$Table 2 - Calibration$

β	a	b	θ	A	B	γ	δ	g
0.95	0.55	0.45	2	0.4	1	0.45	0.60	0.08

Before we analyse the effects of raising the sunk cost for the formal sector in the model, let us discuss briefly the effect of introducing informal subsistence expenditure by households in a zero-sunk cost economy. Consider the information presented in Table 3. While the first few columns are self-explanatory, two deserve special mention: the information headed "% informal" is calculated by dividing informal output by the sum of formal and informal output produced in the economy, while the column titled $\frac{p_i}{p_f}$ contains the ratio of informal to formal sector prices. All values for government revenue are expressed in units of the informal good. As we mentioned earlier, these values may be expressed in either units of formal or informal goods. Hence, the value for "Revenue from τ^c" is the product $\tau^c c_f$; while "Revenue from π_i" is the product $\pi_i \frac{p_i}{p_f} c_i$. "Government Revenue from Tax" is the sum of the two revenues. This is the format for presenting tables that we adhere to for the rest of the paper.[22]

When we introduce a non-zero value for $\bar{p}$, the optimal rate of informal inflation declines from 7% to 6.45%. To understand why this happens, observe that a rise in $\bar{p}$

[22] If we were to construct tables where values for revenue are expressed in terms of the formal good, then "Revenue from τ^c" would be the product $\tau^c c_f$; while "Revenue from π_i" would be $\pi_i \frac{p_i}{p_f} c_i$.

increases c_{it}, since overall consumption of the good must rise if subsistence expenditure is higher (informal sector firms hire more labour, so that informal output rises; see rising values for l_{it} in Row 2, and beyond). For a fixed level of government spending, the government only needs to raise enough revenue to cover its budget. As a consequence, for any increase in $\bar{p}$, the rate of inflation will be lower, because the government is able to make higher revenue, despite having a lower rate of tax (Compare the values for revenue from inflation tax in Row 1 of Table 3, to the values in Rows 2 and 3).[23]On the other hand, c_f falls, because households allocate less labour to formal firms and more labour to informal firms.

The revenue from the inflation tax keeps rising initially with higher informal subsistence expenditure. But eventually (when $\bar{p} = 0.08$), the inflation tax rate and the revenue earned from inflation tax are both lower. The reason is that even though the inflation tax is a very important part of the government's revenue (with each rise in $\bar{p}$, it becomes a larger fraction of total government revenues than before); yet, the government's total tax revenue is getting smaller. The government is earning less revenue from taxation for each increase in p because it is spending lesser in real terms than before, so it needs to earn less. And the reason behind the lower real public expenditure is the government's restriction on buying only from the formal sector, which is shrinking in size with rising $\bar{p}$. This can be seen from the change in output for both sectors with rising p. Notice that as we increase $\bar{p}$, l_i rises, while l_f falls. This pattern is reflected in output in the two sectors; y_i rises, whereas y_f falls, so that the informal sector grows in size (as observed in Column 8 of Table 3).

We also try to analyse the relationship between the inflation tax and the consumption tax in this economy. These results, too, are presented in Table 3. For low levels of $\bar{p}$, the rate of inflation tax is much lower than the rate of consumption tax. But as $\bar{p}$ gets higher, inflation becomes a more important source of revenue (since the informal sector is growing) so that for all subsequent rows of Table 2, the rate of consumption tax falls steadily as $\bar{p}$ rises.

This trend is also reflected in the revenues earned from consumption taxes. Initially (for low $\bar{p}$), the revenue earned from inflation tax is lower than the revenue from τ^c. But as $\bar{p}$ gets higher, inflation tax revenue becomes larger as a fraction of total tax revenue for the government. Conversely, with each subsequent rise in $\bar{p}$, the revenue from τ^c is lower and lower.

Finally, when the amount of informal subsistence expenditure is increased, the ratio of informal to formal prices, $\frac{p_i}{p_f}$, increases. Intuitively, an increase in $\bar{p}$ indicates that there is a rightward shift of the demand curve for informal goods (more c_i consumed at each price). *Ceteris paribus*, this should lead to an increase in the price of informal goods; hence, the increase in magnitude of $\frac{p_i}{p_f}$. We only discuss the results for increasing the level of informal subsistence expenditure from 0 to 0.20 for an economy with zero sunk cost. However, it is easily verifiable that for any level of sunk cost, increasing the value of $\bar{p}$ from 0 to 0.20 holds qualitatively similar results for the size of the informal

[23]The revenue from inflation tax equals $\pi_i \frac{p_i}{p_f} c_i$. If c_i and $\frac{p_i}{p_f}$ are increasing, then a smaller value for π_i will still allow for the product to be higher, as long as the rates of increase are high enough.

economy, the inflation rate, and the rate of consumption tax. As a consequence, a situation where the informal sector is large owing to rising subsistence level expenditure for a given level of formal sunk cost will imply a lower rate of inflation.

5.1 Case 1: $\bar{p} = 0$

We begin with the simplest model where we set the informal subsistence consumption equal to zero. This is analogous to making the consumption component of the utility function a Cobb Douglas specification. The sunk cost is increased gradually from 0 to 0.70, and the results are reported in Table 4.

In an economy with $\bar{p} = 0$, when we introduce sunk costs, both the tax rates—the consumption tax, τ^c, and the rate of inflation in the informal sector, π_i—rise. The revenue from the inflation tax rises, while the revenue from the consumption tax falls. The revenue from τ^c falls because formal consumption, c_f, is getting smaller; as is the ratio of prices, $\frac{p_f}{p_i}$.[24]The amount of labour l_f hired by the formal sector is higher, but because of the sunk cost, the amount of *effective* labour used in production, $l_f - \bar{k}$, is smaller. As a consequence, the formal firm produces less, so that households consume less (c_f falls, so that the consumption tax base is smaller).[25]

The ratio of prices, $\frac{p_f}{p_i}$, falls because the sunk cost causes households' income to decline, so that the demand for both kinds of goods decreases (As $\bar{k}$ rises, the income from formal labour rises, but firms' profits in both sectors decrease, as less is produced. The economy on the whole is poorer). As demand decreases for both goods, prices fall in both sectors. However, the price of the formal good decreases by a greater magnitude than that for the informal good because the demand for the formal good falls by a larger proportion. This occurs because the decline in demand is a pure income effect, which is greater in the formal sector than the informal sector (the fraction of income spent on the formal good, a is larger than that spent on the informal good, b).

In fact, this relationship between the ratio of prices, $\frac{p_f}{p_i}$, and the sunk cost is monotonic, and holds true throughout the study; for any value of $\bar{p}$, as $\bar{k}$ rises, $\frac{p_f}{p_i}$ falls. Hence both rising $\bar{p}$ (across cases) and $\bar{k}$ (within cases) serve to decrease $\frac{p_f}{p_i}$ for different reasons. On the other hand, the revenue raised by the inflation tax is higher, even though informal consumption, c_i, falls as the sunk cost rises (households only have a fixed endowment of time. If formal firms are hiring more formal labour, then the household will supply less labour to the informal sector. Informal labour, l_i, falls, causing a decline in informal sector output. Hence, c_i decreases). However, the decline in c_i is smaller than the rise in π_i, so that revenue from the inflation tax, on the whole, still rises.

The inflation rate, and the rate of consumption tax, both rise because the Lagrange multiplier on the implementability constraint, λ, becomes larger when the sunk cost is

[24]Conversely, we may say that $\frac{p_i}{p_f}$ rises, as in our representation.

[25]If we were to express revenues in terms of formal goods, then revenue from the consumption tax would continue to rise, though very slightly (See Table 3.2). The reason for this is that even though c_f is shrinking, the tax rate is rising, so that the product $\tau^c c_f$ is still increasing.

increased (See eqs.((32)) and ((33)). A rise in λ automatically causes both taxes to rise, for $\bar{p} = 0$). Mathematically, λ rises because the household's consumption of leisure declines as $\bar{k}$ increases (since l_f rises). In effect, formal firms are hiring more labour than before, so that households get to enjoy less leisure. On the other hand, some of this extra hiring is not used in producing output, since the labour spent as sunk cost does not contribute towards production directly. At the same time, labour hired by the informal sector actually falls with rising $\bar{k}$. Overall consumption in the economy is lower, since less is being produced by both sectors.

What does it mean for the Lagrange multiplier on the implementability constraint to increase in size? Correia and Teles (1997) note that λ measures the marginal excess burden of government deficits in the second-best environment. Intuitively, as we mentioned, the implementability constraint describes the household's trade-off between consumption and leisure. Therefore, a positive value for λ indicates that the implementability constraint is binding, suggesting that the trade-off between consumption and leisure is equalised. According to Albanesi (2006), the Lagrange multiplier on the implementability constraint is only positive if the present discounted value of government spending obligations is positive. Also, Gorostiaga (1999) notes that a positive Lagrange multiplier implies that the funding of public spending constrains the decisions taken by the agents in the economy. Therefore, we may infer that adding a non-zero sunk cost to the model makes households and firms more constrained in their utility and profit maximisation decisions. Firms respond by producing less.

Consequently, the government needs a higher tax rate in both sectors in order to meet its budget.

For this value of $\bar{p}$, the informal sector does not grow larger with a rising sunk cost. On the contrary, it actually shrinks when $\bar{k}$ rises. This happens because the demand side forces favouring informality are not strong enough to prevent the informal sector from becoming smaller when a supply side shock (the formal sunk cost) hits the system. This is expected to change once the level of subsistence, $\bar{p}$, is substantially high.

5.2 Cases 2 and 3: $\bar{p} = 0.04$ and $\bar{p} = 0.06$

The results for increasing $\bar{p}$ to 0.04, and subsequently 0.06, are somewhat similar, so they are discussed together. The results themselves are presented in Tables 5 and 6, and are similar to those obtained for the case of $\bar{p} = 0$. With rises in the sunk cost, π_i increases from its $\bar{k} = 0$ level. Revenue from the inflation tax rises, while revenue from the consumption tax falls, owing to the aforementioned reasons.

There is a change in the results for the consumption tax, however. More specifically, as $\bar{k}$ rises, not only the revenue from the consumption tax falls, as before, but also, the *rate* of taxation, τ^c, itself starts to fall after a certain threshold value of $\bar{k}$ has been surpassed! (See the last row of Table 5, and the last two rows of Table 6.)

What could explain this eventual fall in the rate of consumption tax? Again, we turn to look at the value of the Lagrange multiplier on the implementability constraint to provide clues for the change in pattern of the consumption tax rate. In fact, we may attribute this decrease to the change in the values of λ for higher values of $\bar{k}$. In earlier

cases (when $\bar{p}$ was zero) the Lagrange multiplier on the implementability constraint was rising rapidly, but now, since $\bar{p}$ is non-zero, λ does not rise a lot when $\bar{k}$ rises; and for a high enough $\bar{k}$, λ actually falls (see the last row of Table 5, and the last two rows of Table 6). Mathematically, it is possible to calculate the value of λ from eq.((30)) as:

$$\lambda = \frac{\frac{a}{c_f}\gamma(l_f - \bar{k})^{\gamma-1}(1 - l_f - l_i)^{\theta} - 1}{\frac{1}{\gamma} + \frac{\theta}{1-l_f-l_i}\left(\frac{l_i}{\delta} + \frac{l_f - \bar{k}}{\gamma}\right)} \tag{34}$$

and substituting the steady state values for the allocations allows us to see that λ begins falling for high values of $\bar{k}$ due to the large decrease in leisure that households now enjoy in this scenario when both $\bar{k}$ and $\bar{p}$ are non-zero.

Intuitively, a declining value for λ indicates that agents in the economy are less constrained by the funding of public spending (or equivalently, the implementability constraint is less binding). In the presence of a high subsistence level of informal consumption, a large sunk cost imposed on the formal sector will shrink the household's consumption of leisure so much so, that households are not willing to pay a high tax on the formal good and the increase in τ^c is minuscule, and eventually, negative.

Therefore, for a high enough $\bar{k}$, τ^c falls—and c_f is lower still, because less is being produced by formal firms; while $\frac{p_f}{p_i}$ is falling continuously—so that the revenue raised from the consumption tax falls.

Why does the inflation rate, π_i, continue to rise throughout, if λ is falling? Note that even though λ is decreasing, yet $\bar{p}$ is now high enough to drive up the inflation rate. Observe ((33)) where it may be seen that the nominal interest rate, and hence the rate of inflation in the informal sector, depends positively on λ, as well as $\bar{p}$. So even though λ is not too high for large values of $\bar{k}$, yet the high value of $\bar{p}$ offsets any dampening effect that λ might have on the inflation rate. This does not mean, however, that the fall in λ has no effect on π_i—rather, it is clear that the variation in π_i has been reduced significantly (it only rises from 6.44% to 21.27% in Table 5, and 6.02% to 13.98% in Table 6 as k rises, as opposed to case 1). This also has an eventual effect on the revenue raised from π_i, which falls in the last two rows of Table 6.

Intuitively, with a rising sunk cost, households are increasingly faced with a decreasing amount of leisure to enjoy. Therefore, households are certainly not willing to allow higher rates of taxation on the formal sector good. On the other hand, the rising sunk cost is also accompanied by a very high informal subsistence expenditure that households cannot avoid. The government knows that households will consume at least $\bar{p}$ units of the informal good certainly, and is able to impose a higher rate of tax on the informal sector. To summarise, the household is consuming less leisure, but since it has to consume a certain amount of c_i, the government will still be able to impose a high tax on the informal sector.

As for the case of $\bar{p} = 0$, the informal sector does not become larger when the formal sunk cost rises (with the exception of the last two rows of Table 6). The reason for this is as mentioned earlier; $\bar{p}$ is not large enough to sustain a high value for c_i, so that the sunk cost of formality causes the informal sector's output to shrink more than the formal sector's output falls.

5.3 Case 4: $\bar{p} = 0.08$

The results for the case of $\bar{p} = 0.08$ are displayed in Table 7. Using the earlier argument, as $\bar{k}$ rises above 0, the inflation rate in the informal sector rises, and the revenue from the inflation tax initially rises as well. With the rise in sunk cost, the formal sector's hiring of labour, l_f rises, and even though l_i falls, yet the rise in l_f is high enough to offset the negative impact, so that the household's consumption of leisure decreases. Therefore, both c_i and l_i are lower, so that the government is forced to increase the rate of inflation, π_i, in order to meet its budget. These results are analogous to those obtained for the earlier cases.

However, we now witness that when $\bar{k}$ is high enough, π_i also decreases (see last row of Table 7). This is just a more extreme version of what was already witnessed in Cases 2 and 3, when the variation in π_i was reduced for large values of $\bar{k}$. The household's consumption of leisure is very low. Both $(l_f - \bar{k})$ and l_i are very low, so that λ becomes low, which explains the slow rise of, and eventual decline of π_i. So π_i is rising less (or declining), while informal consumption c_i is falling. As a consequence, the revenue from π_i starts falling.

Very crucially, the ratio of informal consumption to total output, "% informal", is now higher for any increase in $\bar{k}$. Even though output is lower in both sectors, yet, the formal sector suffers more, owing to the sunk cost being borne entirely by the formal firm, while the informal sector is buffered by the high amount of unavoidable consumption that must be incurred as subsistence. As a result, the informal sector is larger for a higher sunk cost of formality, as is the optimal rate of inflation.

5.4 Case 5: $\bar{p} = 0.10$

With $\bar{p} = 0.10$, the inflation rate in the informal sector rises and then falls, while the revenue from inflation taxation follows the same trend. Both the rate of, and the revenue raised from, the consumption tax fall. The results are presented in Table 8.

τ^cfalls because the rise in λ for increases in $\bar{k}$ is very small, and actually a decline in λ occurs when $\bar{k}$ gets high enough (See row 5 of Table 8). The inflation rate also declines for an earlier value of $\bar{k}$ now, than in Case 4. The reasons for this have already been discussed above.

5.5 Case 6: $\bar{p} = 0.20$

The last case we consider is that where informal subsistence consumption is made very high (in excess of 84% of informal consumption). Any higher values of $\bar{p}$ cause a violation of the non-negativity constraint on c_i. We consider only five non-zero values for the sunk cost, because any higher values for $\bar{k}$ lead to very weak convergence for the model. The results for this specification of the economy are presented in Table 9. As $\bar{k}$ rises, the rate of consumption tax continues to fall throughout; as does the rate of inflation. The revenue from both taxes falls unambiguously as $\bar{k}$ rises.[26]Intuitively, households find their

[26]Again, we know this is only true if we express revenues in terms of the informal good.

loss of leisure extremely painful, and since the extra time they are sparing from leisure as a sunk cost is not utilised in production, output is also very low. The government's tax policy is a response to this decline in output and consumption.

6 Summary of Results

To summarise the results briefly, for any given level of $\bar{k}$, if the level of informal subsistence rises, then the optimal rate of inflation and the consumption tax rate are both lower. In this case, a large informal sector will be associated with having a lower rate of inflation. Intuitively, rising informal subsistence causes less to be produced in the whole economy, so that the government needs to earn less to meet its expenditures. As a consequence, the government can afford to lower taxes on both sectors.

On the other hand, for low levels of subsistence, $\bar{p}$, both the inflation rate and the consumption tax rate increase as sunk cost increases. As $\bar{p}$ rises further (from 0.06 onwards), the consumption tax rate eventually declines for increasing sunk cost. For the next three values of $\bar{p}$ (0.08, 0.10 and 0.20, the highest level of informal subsistence expenditure), as sunk cost rises, both tax rates eventually decline. Mathematically, why this is happening is that the Lagrange multiplier on the implementability constraint is getting exceedingly smaller. This is pushing down the tax rate. Within the expression for the Lagrange multiplier, the household's consumption of leisure is falling, which is why the multiplier is decreasing so rapidly.

Intuitively, if the household is consuming less leisure, and consumption in both sectors is falling (because output in both sectors is falling), then it makes sense that the government lowers taxes. The impact on the inflation tax is not immediate because of the presence of low quantities of subsistence expenditure in the earlier cases. Once $\bar{p}$ is high enough, the government can afford to have a lower tax on both sectors and still meet its budget.

For a given level of $\bar{p}$, a higher sunk cost causes output in both sectors to move in the same direction (i.e., to become lower). However, when $\bar{p}$ is small, the informal sector's output is not large enough to dominate in the fraction of informal to total output. As a consequence, for low levels of $\bar{p}$, a higher sunk cost does not cause the informal economy to become larger. However, for higher (and more realistic) levels of $\bar{p}$, a higher sunk cost in the formal sector will result in the ratio of informal to total output becoming larger. In this case, we will observe a positive relationship between inflation and the size of the informal sector: a high degree of informality will be associated with a high rate of inflation.

7 Conclusion

This paper uses a quantitative general equilibrium analysis to describe an economy where households choose to consume two goods, of which one has a minimum unavoidable consumption level. On the other hand, the production of the other good involves an unavoidable sunk cost in terms of labour resources that must be incurred. The economy

is calibrated for varying amounts of the sunk cost and informal subsistence, and the implications for optimal taxation policy are analysed.

The key findings are: (1) If the informal sector is large owing to the presence of escalating levels of unavoidable informal subsistence expenditure, then it is optimal to have a low rate of inflation and consumption tax. (2) In economies where informal subsistence consumption is small, a higher sunk cost on formal production causes the government to raise taxes on both sectors. Revenue from the inflation tax rises, while revenue from the consumption tax falls. (3) In economies where the informal sector is large and pervasive – as indicated by large amounts of unavoidable informal consumption – a higher sunk cost on formal production causes the informal sector to expand. Government will find it optimal to set a higher inflation tax, but a low consumption tax. Revenue raised by both kinds of taxes falls for the most extreme case considered.[27]

Most crucially, findings (1) and (3) seem to suggest that a large informal sector does not necessarily make a high rate of inflation inevitable, for the informal sector could be large due to two different reasons. We show that a large degree of informality could exist either due to persistent, unavoidable subsistence expenditure in the informal sector, or because of a large sunk cost faced by the formal sector. In the first case, the model dictates that the optimal rate of inflation should be lower, for higher subsistence levels, while in the latter, it dictates that the optimal rate of inflation should be higher, for higher sunk costs. This non-monotonic relationship between informality and inflation is an unprecedented result of the model that does not find a parallel in the literature on informality. It is difficult to draw any kind of general policy directives for poor countries from this exercise, since different countries are expected to have different levels of informal subsistence and sunk costs. Clearly, Case 1 is quite unrealistic for Third World countries, because it is hard to imagine any developing country for which informal subsistence consumption does not exist. We have estimated informal subsistence for Pakistan to be around 54% of informal consumption, which could swell to up to 67% if we include housing as an informal, subsistence commodity. Most developing countries, including Pakistan, are expected to have essential informal expenditure in the range of 50-70% of total informal output, as estimated by Saracoğlu (2008), which corresponds with Cases 4 and 5 of the model.

What are the implications for policy towards the level of sunk cost faced by the formal sector in developing countries? The model indicates that a higher sunk cost in the formal sector leads to lower output in both the formal and informal sectors, and lower tax revenue for the government. This would imply that the government should make attempts to help firms overcome this obstacle that they face to utilizing labour hired fully. For instance, if there is a law that requires formal firms to employ $\bar{k}$ workers

[27] It is not completely evident at this stage which of the two features; the persistent, unavoidable subsistence expenditure, or the large sunk cost, are more applicable to the case of Pakistan, because not much is known about either of these factors. In our sensitivity analysis, we tried using different values of $\bar{p}$ and $\bar{k}$ within the bounds of the non-negativity constraints. Very crucially, the results do not change direction for extremes. So, for instance, very high $\bar{p}$ gives similar results as the moderately high $\bar{p}$ that we have used in this paper for different values of $\bar{k}$. Fixing $\bar{p}$ at a particular value and varying $\bar{k}$ does not change direction of the results, but it does affect magnitudes.

without making full use of their productive abilities, then there is the need to address this inefficient legislation. Reducing the cost of formality would allow all firms to produce more output, lead to higher levels of consumption of goods and leisure, and raise more revenue for the government. Here, we have assumed that no one in society gains from the sunk cost that is paid by formal firms. However, if the cost of formality is paid to the government in the form of a registration fee, or it enters households' utility functions, then these inferences will not be meaningful.

Despite its simplifications, the paper certainly underscores the relevance of using seigniorage financing for poor countries, which are often confronted with numerous problems associated with the collection of conventional taxes. Of course, there are the adverse impacts of inflation on welfare which are excluded from our analysis completely. Indeed, the welfare costs of inflation may change some of the results, if accounted for, in a richer model. This is an area identified for possible future research.

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References

[1] Agenor, P., & Montiel, P.J. (1999). Development macroeconomics. New Jersey: Princeton University Press.

[2] Albanesi, S. (2006). Ramsey allocation problem in a real economy without capital. (Lecture notes for G6222: Advanced Macroeconomic Analysis at Columbia University, Spring 2006).

[3] Bailey, M.J. (1956). The welfare cost of inflationary finance. *Journal of Political Economy, 64*(2), 93-110.

[4] Bekkers, H., & Stoffers, W. (1995). Measuring informal sector employment in Pakistan: Testing a new methodology. *International Labour Review, 134*(1), 17–36.

[5] Bromley, R. (1978). The urban informal sector: Why is it worth discussing? *World Development, 6* (9-10), 1033-1039.

[6] Burki, A.A., & Afaqi, U. (1996). Pakistan's informal sector: Review of evidence and policy issues. *Pakistan Journal of Applied Economics, 12*(1), 1–30.

[7] Burki, A.A., & Terrell, D. (1998). Measuring production efficiency of small firms in Pakistan. *World Development, 26*(1), 155–169.

[8] Campillo, M., & Miron, J.A. (1996). Why does inflation differ across countries? NBER Working Paper No. 5540.

[9] Carroll, D.R., & Young, E.R. (2010). The long run effects of changes in tax progressivity. Working Paper No 913, Federal Reserve Bank of Cleveland.

[10] Cavalcanti, T.V.V., & Villamil, A.P. (2003). Optimal inflation tax and structural reform. *Macroeconomic Dynamics, 7*(3), 333–362.

[11] Chari, V.V., Christiano, L.J., & Kehoe, P.J (1991). Optimal fiscal and monetary policy: some results. *Journal of Money, Credit and Banking, 23*(3), 519–539.

[12] Correia, I., & Teles, P. (1999). The optimal inflation tax. *Review of Economic Dynamics, 2*(2), 325–346.

[13] Domeij, D., & Floden, M. (2006). The labor-supply elasticity and borrowing constraints: Why estimates are biased. *Review of Economic Dynamics,9*(2), 242–262.

[14] Drazen, A. (1985). A general measure of inflation tax revenues. *Economics Letters, 17*(4), 327–330.

[15] Echevarria, C. (1997). Changing sectoral composition associated with economic growth. *International Economic Review, 38*(2), 341–452.

[16] Echevarria, C. (2000). Non-homothetic preferences, growth, trade and land. *Journal of International Trade and Economic Development, 9*(2), 151–171.

[17] Federal Bureau of Statistics. (2008). *Household Integrated Economic Survey (HIES) 2007-08.*

[18] Federal Bureau of Statistics. (2010). *Percentage distribution of employed persons 10 years of age and over by major sectors of employment, sex, area, province and Pakistan, 2007—8.*

[19] Federal Bureau of Statistics. (2010). *Gross fixed capital formation in private, public and general government sectors by economic activity at current prices.*

[20] Feige, E.L. (1990). Defining and estimating underground and informal economies: The new institutional economics approach. *World Development, 18*(7), 989–1002.

[21] Freidman, M. (1969). The optimal quantity of money. In M. Freidman (Ed.), *The optimal quantity of money and other essays* (1–50). Chicago: Aldine.

[22] Gennari, P. (2004). *The estimation of employment and value added of informal sector in Pakistan.* United Nations Economic and Social Commission for Asia and the Pacific, Bangkok.

[23] Gorostiaga, A. (1999). *Optimal fiscal policy with rationing in the labour market.* Working Paper No. 9908, Center for Monetary and Financial Studies [CEMFI].

[24] Hart, K. (1973). Informal income opportunities and urban employment in Ghana. *The Journal of Modern African Studies, 11*(1), 61–89.

[25] Heijdra, B.J., & van der Ploeg, F. (2002). *Foundations of modern macroeconomics.* Oxford: Oxford University Press.

[26] Irz, X., & Roe, T. (2005). Seeds of growth? Agricultural productivity and the transitional dynamics of the Ramsey model. *European Review of Agricultural Economics, 32*(2), 143–165.

[27] Karachi Relief Trust. (2010). *30 days ration.* Retrieved from http://www.karachirelief.org/30daysr.php.

[28] Koreshkova, T.A. (2006). A quantitative analysis of inflation as a tax on the underground economy. *Journal of Monetary Economics, 53*(4), 773–796.

[29] Kristiansen, S. (2003). *Linkages and rural non-farm employment creation: Changing challenges and policies in Indonesia* (ESA Working Paper No. 03-22). Retrieved from ftp://ftp.fao.org/docrep/fao/007/ae055e/ae055e00.pdf.

[30] Lucas, R.E., & Stokey, N.L. (1983). Optimal fiscal and monetary policy in an economy without capital. *Journal of Monetary Economics, 12*(1), 55–93.

[31] Mead, D.C., & Morrisson, C. (1996). The informal sector elephant. *World Development, 24*(10), 1611–1619.

[32] Nasir, Z. M. (2001). Poverty and labour market linkages in Pakistan (MIMAP Technical Paper Series No. 7, Pakistan Institute of Development Economics (PIDE)). Retrieved from http://www.research-matters.net/uploads/user-S/10281436810mimap22.pdf.

[33] N'Diaye, P., Zhang, P. & Zhang, W. (2010). Structural reform, intra-regional trade, and medium-term growth prospects of East Asia and the Pacific: Perspectives from a new multi-region model. *Review of Economic Dynamics, 21*(1), 20–36.

[34] Nicolini, J.P. (1998). Tax evasion and the optimal inflation tax. *Journal of Development Economics, 55*(1), 215–232.

[35] Perry, G.E., Maloney, W.F., Arias, O.S., Fajnzylber, P. Mason, A.D. & Saavedra-Chanduvi, J. (2007). *Informality: Exit and Exclusion.* World Bank: Washington D.C.

[36] Phelps, E.S. (1973). Inflation in the theory of public finance. *Swedish Journal of Economics, 75*(1), 67–82.

[37] Roubini, N., & Sala-i-Martin, X. (1995). A growth model of inflation, tax evasion, and financial repression. *Journal of Monetary Economics, 35*(2), 275–301.

[38] Saracoglu, D. S. (2008). The informal sector and tax on employment: A dynamic general equilibrium investigation. *Journal of Economic Dynamics and Control, 32*(2), 529–549.

[39] Satchi, M., & Temple, J. (2009). Labour markets and productivity in developing countries. *Review of Economic Dynamics, 12*(1), 183–204.

[40] Schneider, F., & Enste, D.H. (2000). Shadow economies: Size, causes, and consequences. *Journal of Economic Literature, 38*(1), 77–114.

[41] State Bank of Pakistan. (2009). *Trends in Inflation* [Data File]. Retrieved from http://www.sbp.org.pk/ecodata/MPM.pdf.

[42] Tybout, J.R. (2000). Manufacturing firms in developing countries: How well do they do, and why? *Journal of Economic Literature, 38* (1), 11–44.

[43] United Nations High Commissioner for Refugees. (2010). *Pakistan: Ramadan's spirit is sharing.* Retrieved from http://www.unhcr.org/4c62d90e9.html.

[44] Vegh, C.A. (1989). Government spending and inflationary finance: A public finance approach. *International Monetary Fund Staff Papers, 36* (3), 657–677.

[45] Verma, R. (2008). The service sector revolution in India: A quantitative analysis (Working Papers RP2008/72, World Institute for Development Economic Research). UNU-WIDER.

[46] World Food Programme. (2011). *Pakistan emergency*. Retreived from http://www.wfp.org/crisis/pakistan.

[47] Zuin, V. (2004). *Business strategies of informal micro-entrepreneurs in Lima, Peru*. Discussion Paper 150, Decent Work Program, International Institute for Labour Studies, ILO, Geneva. Retrieved from http://www.ilo.org/public/english/bureau/inst/download/dp15004.pdf.

Table 3															
$\bar{p}$	$\frac{\bar{p}}{c_i}$	$\bar{k}$	$\frac{\bar{k}}{l_f}$	c_f	c_i	$\%\ Informal$	λ	l_f	l_i	$\frac{p_i}{p_f}$	π_i	Revenue from π_i	τ^c	Revenue from τ^c	Total Govt. Revenue
0	0	0	0	0.337	0.117	21.88%	0.0378	0.143	0.128	2.405	7.00%	0.0082	13.92%	0.0195	0.0276
0.04	29.25%	0	0	0.326	0.137	25.21%	0.0297	0.135	0.167	2.761	6.45%	0.0088	11.53%	0.0136	0.0224
0.06	40.64%	0	0	0.319	0.148	27.00%	0.0257	0.130	0.190	2.965	6.02%	0.0089	10.27%	0.0111	0.0199
0.08	50.30%	0	0	0.312	0.159	28.89%	0.0218	0.124	0.215	3.189	5.49%	0.0087	9.02%	0.0088	0.0175
0.10	58.50%	0	0	0.303	0.171	30.86%	0.0181	0.119	0.242	3.438	4.87%	0.0083	7.80%	0.0069	0.0152
0.12	65.48%	0	0	0.293	0.183	32.93%	0.0148	0.112	0.272	3.715	4.19%	0.0077	6.65%	0.0053	0.0129
0.14	71.43%	0	0	0.283	0.196	35.09%	0.0118	0.105	0.305	4.031	3.46%	0.0068	5.58%	0.0039	0.0107
0.16	76.48%	0	0	0.270	0.209	37.38%	0.0093	0.097	0.339	4.382	2.76%	0.0058	4.65%	0.0029	0.0086
0.18	80.83%	0	0	0.257	0.223	39.79%	0.0071	0.089	0.377	4.795	1.98%	0.0044	3.77%	0.0020	0.0064
0.20	84.53%	0	0	0.242	0.237	42.34%	0.0053	0.081	0.417	5.273	1.23%	0.0029	3.02%	0.0014	0.0043

Table 4															
$\bar{p}$	$\frac{\bar{p}}{c_i}$	$\bar{k}$	$\frac{\bar{k}}{l_f}$	c_f	c_i	$\%\ Informal$	λ	l_f	l_i	$\frac{p_i}{p_f}$	π_i	Revenue from π_i	τ^c	Revenue from τ^c	Total Govt. Revenue
0	0	0	0	0.337	0.117	21.88%	0.0378	0.143	0.128	2.405	7.00%	0.0081	13.92%	0.0195	0.0276
0	0	0.01	6.64%	0.334	0.116	21.83%	0.0383	0.151	0.126	2.409	7.10%	0.0082	14.05%	0.0194	0.0276
0	0	0.05	27.44%	0.322	0.111	21.61%	0.0402	0.182	0.118	2.425	7.49%	0.0083	14.56%	0.0193	0.0276
0	0	0.10	45.11%	0.308	0.105	21.32%	0.0428	0.222	0.108	2.447	8.01%	0.0084	15.25%	0.0192	0.0276
0	0	0.20	66.31%	0.277	0.093	20.67%	0.0492	0.302	0.088	2.495	9.29%	0.0086	16.94%	0.0188	0.0275
0	0	0.30	78.41%	0.246	0.081	19.93%	0.0578	0.383	0.070	2.548	10.97%	0.0089	19.18%	0.0185	0.0274
0	0	0.40	86.06%	0.212	0.069	19.05%	0.0698	0.465	0.053	2.608	13.26%	0.0091	22.24%	0.0181	0.0272
0	0	0.50	91.16%	0.176	0.056	17.96%	0.0877	0.549	0.038	2.674	16.62%	0.0093	26.74%	0.0176	0.0270
0	0	0.60	94.64%	0.138	0.043	16.59%	0.1175	0.634	0.025	2.739	22.12%	0.0096	34.15%	0.0172	0.0268
0	0	0.70	97.05%	0.097	0.030	14.65%	0.1762	0.721	0.014	2.798	32.72%	0.0099	48.49%	0.0168	0.0268

Table 5

$\bar{p}$	$\frac{\bar{p}}{c_i}$	$\bar{k}$	$\frac{\bar{k}}{l_f}$	c_f	c_i	$\%\ Informal$	λ	l_f	l_i	$\frac{p_i}{p_f}$	π_i	Revenue from π_i	τ^c	Revenue from τ^c	Total Govt. Revenue
0.04	29.24%	0	0	0.326	0.137	25.22%	0.0297	0.135	0.167	2.760	6.44%	0.0088	11.52%	0.0136	0.0224
0.04	29.50%	0.01	7.01%	0.323	0.136	25.18%	0.0300	0.143	0.165	2.770	6.54%	0.0089	11.60%	0.0135	0.0224
0.04	30.49%	0.05	28.64%	0.312	0.131	25.09%	0.0311	0.175	0.156	2.801	6.88%	0.0090	11.90%	0.0132	0.0223
0.04	31.87%	0.10	46.58%	0.297	0.126	24.96%	0.0325	0.215	0.145	2.848	7.33%	0.0092	12.26%	0.0128	0.0220
0.04	35.06%	0.20	67.61%	0.268	0.114	24.69%	0.0395	0.296	0.124	2.954	8.45%	0.0096	13.12%	0.0119	0.0215
0.04	38.99%	0.30	79.37%	0.237	0.103	24.44%	0.0395	0.378	0.104	3.081	9.81%	0.0101	14.05%	0.0108	0.0209
0.04	43.91%	0.40	86.67%	0.205	0.091	24.22%	0.0437	0.462	0.085	3.240	11.61%	0.0106	15.07%	0.0095	0.0201
0.04	50.25%	0.50	91.49%	0.172	0.080	24.04%	0.0479	0.547	0.068	3.451	13.99%	0.0111	16.02%	0.0080	0.0191
0.04	58.57%	0.60	94.74%	0.136	0.068	24.00%	0.0508	0.633	0.053	3.748	17.21%	0.0118	16.51%	0.0060	0.0178
0.04	69.44%	0.70	96.95%	0.100	0.058	24.29%	0.0485	0.722	0.040	4.214	21.27%	0.0123	15.45%	0.0036	0.0159

Table 6

$\bar{p}$	$\frac{\bar{p}}{c_i}$	$\bar{k}$	$\frac{\bar{k}}{l_f}$	c_f	c_i	$\%\ Informal$	λ	l_f	l_i	$\frac{p_i}{p_f}$	π_i	Revenue from π_i	τ^c	Revenue from τ^c	Total Govt. Revenue
0.06	40.64%	0	0	0.319	0.148	27.00%	0.0257	0.130	0.190	2.965	6.02%	0.0089	10.27%	0.0111	0.0199
0.06	40.94%	0.01	7.25%	0.316	0.147	26.99%	0.0258	0.138	0.188	2.975	6.09%	0.0089	10.31%	0.0110	0.0199
0.06	42.20%	0.05	29.41%	0.305	0.142	26.96%	0.0265	0.170	0.178	3.020	6.39%	0.0091	10.49%	0.0106	0.0197
0.06	43.89%	0.10	47.52%	0.291	0.137	26.93%	0.0275	0.210	0.167	3.079	6.79%	0.0093	10.72%	0.0101	0.0194
0.06	47.72%	0.20	68.49%	0.262	0.126	26.89%	0.0293	0.292	0.145	3.219	7.70%	0.0097	11.16%	0.0091	0.0188
0.06	52.30%	0.30	80.04%	0.231	0.115	26.92%	0.0311	0.375	0.125	3.394	8.79%	0.0101	11.54%	0.0079	0.0180
0.06	57.81%	0.40	87.17%	0.200	0.104	27.07%	0.0323	0.459	0.106	3.622	10.09%	0.0105	11.74%	0.0065	0.0169
0.06	64.46%	0.50	91.84%	0.166	0.093	27.43%	0.0325	0.544	0.088	3.933	11.59%	0.0108	11.55%	0.0049	0.0157
0.06	81.55%	0.70	97.15%	0.094	0.074	29.73%	0.0232	0.721	0.059	5.143	13.98%	0.0103	8.20%	0.0015	0.0118

Table 7

$\bar{p}$	$\frac{\bar{p}}{c_i}$	$\bar{k}$	$\frac{\bar{k}}{l_f}$	c_f	c_i	$\%\ Informal$	λ	l_f	l_i	$\frac{p_i}{p_f}$	π_i	Revenue from π_i	τ^c	Revenue from τ^c	Total Govt. Revenue
0.08	50.30%	0	0	0.312	0.159	28.88%	0.0218	0.124	0.215	3.189	5.49%	0.0087	9.02%	0.0088	0.0175
0.08	50.63%	0.01	7.54%	0.309	0.158	28.89%	0.0218	0.133	0.213	3.203	5.55%	0.0088	9.04%	0.0087	0.0175
0.08	52.02%	0.05	30.32%	0.298	0.154	28.93%	0.0222	0.165	0.203	3.259	5.79%	0.0089	9.12%	0.0083	0.0172
0.08	53.87%	0.10	48.65%	0.284	0.148	29.00%	0.0227	0.206	0.192	3.336	6.11%	0.0091	9.21%	0.0078	0.0169
0.08	57.99%	0.20	69.53%	0.254	0.138	29.21%	0.0234	0.288	0.170	3.517	6.80%	0.0094	9.31%	0.0067	0.0161
0.08	62.74%	0.30	80.88%	0.224	0.128	29.55%	0.0236	0.371	0.149	3.750	7.56%	0.0096	9.24%	0.0055	0.0152
0.08	68.23%	0.40	87.82%	0.192	0.117	30.11%	0.0231	0.455	0.129	4.059	8.34%	0.0098	8.89%	0.0042	0.0140
0.08	74.49%	0.50	92.35%	0.159	0.107	31.03%	0.0211	0.541	0.112	4.495	9.01%	0.0097	8.07%	0.0028	0.0125
0.08	81.46%	0.60	95.40%	0.123	0.098	32.60%	0.0171	0.629	0.096	5.160	9.27%	0.0091	6.58%	0.0016	0.0107
0.08	88.77%	0.70	97.47%	0.085	0.090	35.37%	0.0108	0.718	0.083	6.295	8.57%	0.0077	4.36%	0.0006	0.0083

Table 8

$\bar{p}$	$\frac{\bar{p}}{c_i}$	$\bar{k}$	$\frac{\bar{k}}{l_f}$	c_f	c_i	$\%\ Informal$	λ	l_f	l_i	$\frac{p_i}{p_f}$	π_i	Revenue from π_i	τ^c	Revenue from τ^c	Total Govt. Revenue
0.10	58.50%	0	0	0.303	0.171	30.86%	0.0180	0.119	0.242	0.0083	4.87%	0.0083	7.80%	0.0069	0.0152
0.10	58.85%	0.01	7.90%	0.300	0.170	30.89%	0.0181	0.127	0.240	0.0084	4.92%	0.0084	7.80%	0.0068	0.0151
0.10	60.29%	0.05	31.41%	0.289	0.166	31.00%	0.0182	0.159	0.231	0.0085	5.10%	0.0085	7.79%	0.0064	0.0148
0.10	62.19%	0.10	49.98%	0.275	0.161	31.18%	0.0183	0.200	0.219	0.0086	5.33%	0.0086	7.77%	0.0059	0.0145
0.10	66.33%	0.20	70.76%	0.246	0.151	31.64%	0.0182	0.283	0.197	0.0087	5.79%	0.0087	7.60%	0.0048	0.0136
0.10	70.97%	0.30	81.88%	0.215	0.141	32.32%	0.0175	0.366	0.176	0.0088	6.22%	0.0088	7.24%	0.0037	0.0125
0.10	76.12%	0.40	88.62%	0.183	0.131	33.32%	0.0159	0.451	0.156	0.0086	6.54%	0.0086	6.57%	0.0026	0.0112
0.10	81.74%	0.50	92.98%	0.149	0.122	34.84%	0.0132	0.538	0.139	0.0080	6.57%	0.0080	5.51%	0.0016	0.0096
0.10	87.62%	0.60	95.91%	0.112	0.114	37.25%	0.0093	0.626	0.124	0.0069	6.07%	0.0069	4.04%	0.0007	0.0077
0.10	93.33%	0.70	97.86%	0.072	0.107	41.27%	0.0048	0.715	0.111	0.0050	4.71%	0.0050	2.29%	0.0002	0.0053

Table 9

$\bar{p}$	$\frac{\bar{p}}{c_i}$	$\bar{k}$	$\frac{\bar{k}}{l_f}$	c_f	c_i	$\%\ Informal$	λ	l_f	l_i	$\frac{p_i}{p_f}$	π_i	Revenue from π_i	τ^c	Revenue from τ^c	Total Govt. Revenue
0.20	84.54%	0	0	0.242	0.237	42.34%	0.0053	0.081	0.417	5.274	1.27%	0.0030	3.04%	0.0014	0.0044
0.20	84.82%	0.01	11.23%	0.239	0.236	42.49%	0.0052	0.089	0.414	5.323	1.25%	0.0029	3.00%	0.0013	0.0043
0.20	85.96%	0.05	40.80%	0.227	0.233	43.11%	0.0049	0.123	0.405	5.531	1.17%	0.0027	2.82%	0.0012	0.0039
0.20	87.39%	0.10	60.73%	0.212	0.229	43.97%	0.0044	0.165	0.394	5.828	1.04%	0.0024	2.59%	0.0009	0.0033
0.20	90.26%	0.20	80.08%	0.179	0.222	46.09%	0.0034	0.250	0.374	6.588	0.69%	0.0015	2.09%	0.0006	0.0021
0.20	93.07%	0.30	89.25%	0.144	0.215	48.92%	0.0023	0.336	0.355	7.696	0.19%	0.0004	1.54%	0.0003	0.0007