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Estimating Output Gap for Pakistan Economy: Structural and Statistical Approaches

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Abstract

The objective of this study is to estimate potential output *vis-à-vis* output gap for Pakistan's economy. This paper reviews six commonly used techniques to estimate potential output and from that the output gap. The results suggest that while measures of output gap are not identical they nonetheless do show some degree of association among each other. Therefore, a composite output gap is calculated for 1950 to 2007. The composite output gap depicts that Pakistan economy has been observing a cyclical episode of periods of excess supply followed by excess demand in the period of analysis. Furthermore, evidence suggests that Pakistan economy is currently experiencing rising demand pressures since FY05. These demand pressures show a high degree of correlation with the rising inflation as shown in the temporal correlation between inflation and composite of output gap measures.

JEL Classification: C22, C53, E37

Keywords: gross domestic product, potential output, output gap

1. Introduction

Assessing macroeconomic policies and identifying a sustainable non-inflationary growth remains one of the prime objectives of policy makers. Output gap[1] shows transitory movements from the potential output. The estimates of output gap provide key information to judge inflationary or contractionary pressures and the cyclical position of the economy. If the actual output is greater than the potential output it reflects that an economy is experiencing demand pressures. This situation is often considered as a source of inflationary pressures and requires a reduction in aggregate demand linked with reduced government spending or tightening of monetary policy. The reverse, which indicates excess capacity, may require easing of monetary conditions or other policies to stimulate demand. Thus the estimation of potential output *vis-à-vis* output gap is an important subject for policy makers.

The idea of "potential output" is not new, but not as well-structured in the literature as one may guess. In this backdrop, therefore should the concept of "potential" refer to the maximum achievable level of production as has been echoed in the past, or should it refer to a sustainable level of production in the sense that production can continue at this level without major constraints? The literature reveals that the potential output is the maximum possible output to the current observed one.[2] Broadly, the literature makes two distinctions on the definition of potential output [Scacciavillani and Swagel (1999, pp. 5–6)].

> *"In the first, more along the Keynesian tradition, the business cycle results primarily from movements in aggregate demand in relation to a slow moving level of aggregate supply. In business cycle downswings, there exist factors of production that are not fully employed…. A measure of potential output is thus crucial for the setting of demand management policy—both monetary and fiscal—and represents a principal guide for economic policy…. In the second approach—more along the neoclassical tradition—potential output is driven by exogenous productivity shocks to aggregate supply that determine both the long run growth trend and, to a large extent, short term fluctuations in output over the business cycle.… potential output in the neoclassical framework is synonymous with the trend growth rate of actual output. The key measurement problem is thus to distinguish between permanent movements in potential output and transitory movements around potential."*

[1] In general, output gap represents the difference between the actual and the potential output or the transitory movements from the potential output, measured as a share of potential output.

[2] Laxton and Tetlow (1992)

In the literature, measuring potential output and output gap is frequently connected with business cycle decomposition methods. These methods allow separating the permanent component or trend of a series from its cyclical or transitory component.[3] Therefore, potential output is the trend or permanent component while output gap is the transitory or cyclical component. Pagan (2003) however, points towards this practice as unrepresentative of business cycle. Infact, the potential output and output gap are never directly observable. They must be derived from some set of observable variables or determinants. Therefore, various techniques have been developed to measure potential output and output gap.[4] Many researchers, however, have shown little confidence over these series after observing from different methods of estimations. This is manifested in many empirical studies showing that different methodologies and assumptions for estimating a country's potential output and output gap may produce different results.[5] In connection with the propositions above and for policy making estimating potential output vis-à-vis output gap with some degree of precision is nonetheless desirable.

For Pakistan no previous study has attempted to estimate its potential output and output gap. Hence, this study attempts to measure Pakistan's potential output and output gap by applying six various methods. These are Linear trend, Hodrick-Prescott (HP filter) method, Band-Pass (BP) of Baxter-King method, Structural Vector Autoregressive (SVAR) method, Production Function (PF) method, and the Unobserved Component (KALMAN filter) method. The results derived from a sample of 1950 to 2007 suggest that though the measures of Pakistan's output gap are not close to each other yet they exhibit some degree of association. Therefore on the basis of these results, we calculate a benchmark output gap for the identification of demand/supply pressures in Pakistan economy. This estimate depicts that Pakistan economy has been observing varying episodes of excess supply and demand pressures from 1950 to 2007. The estimate also suggests that the economy is experiencing rising demand pressures since 2005.

We proceed as follow. Section 2 reviews the empirical studies. Methods of estimations and their limitations are discussed in Section 3. Empirical findings are presented and discussed in Section 4. Section 5 carries the concluding remarks.

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[3] See, for example, Beveridge and Nelson (1981), Blanchard and Quah (1989), King et al. (1991), Hodrick and Prescott (1997), and Evans and Reichlin (1994).

[4] See, for example, Laxton and Tetlow (1992) for a historical account.

[5] See for discussion, de Brouwer (1998), Dupasquier, Guay, and St-Amant (1999), Scacciavillani and Swagel (1999), Conway and Hunt (1997), Cerra and Saxena (2000), Butler (1996), Laxton and Tetlow (1992), Nelson and Plosser (1982), and Watson (1986).

2. Empirical Literature

The potential output and output gap generated from different techniques, does not distinguish clearly into the intellectual frameworks of Keynesian and neoclassical traditions. Consequently, a wide variety of measures has been taken into account. These may be classified into the economic (production function) and the statistical (time series) approaches.

The economic approach is essentially referred to the use of a production function to determine potential output and output gap.[6] Moreover, this approach may be utilized including relatively simple Cobb-Douglas function [Scacciavillani and Swagel (1999)] to a detailed simultaneous equation model [Adams and Coe (1990)]. On the other side, the statistical or time-series approaches may be used to generate potential output and output gap by applying the univariate and multivariate techniques.

The most frequently used univariate technique is the HP filter. Similar to the other univariate methods, the HP filter utilizes information appeared only in the actual output series to derive the potential output series. Other univariate techniques for example may include the Beveridge-Nelson (1981) method, the Band-Pass filter proposed by Baxter and King (1995), and the so-called "wavelet filters" [Scacciavillani and Swagel (1999)].

Dupasquier et al. (1997) describe that these univariate techniques, however, have been put to criticism and questioned for their ability to appropriately distinguish between the underlying permanent and transitory components of the time series considered. In response to such like limitations of univariate techniques, a variety of multivariate methods have been proposed. For example, the multivariate extensions of the Beveridge-Nelson method (MBN), unobserved-components model, the multivariate (MV) model and the extended multivariate filter (EMV) are main developments in this regard.[7]

Therefore, we have selected a wide variety of empirical literature as a review for this study. It includes empirical evidence mostly available (such as case studies analyses) for different countries. For this purpose the literature is distinguished and presented in Table 1.

[6] This approach has widely been used; including by institutions such as the IMF [Artus (1977) and De Masi (1997)] and the OECD [Giorno et al. (1995)].

[7] The discussion may be seen in Evans and Reichlin (1994), Watson (1986), Laxton and Tetlow (1992), and Butler (1996).

A number of researchers in recent years have made use of multivariate, structural vector autoregressive models along with other production function models to determine potential output and output gaps. These studies may differ in specification of the techniques, in terms of data frequency selection or some other dimensions considering their results. It is also observed that the empirical literature could not build a common opinion on any of the single measure of potential output and output gap for respective economies. It is because the results deduced from different measures have seldom shown similarities in the estimates. It is also observed from the empirical literature that some of the studies have just estimated the potential output by using any of the single technique but improving that technique by different methods.

For example, Filho and da Silva (2002) estimated output gap by using the extended production function approach for Brazil economy and presented the straight analysis of demand/supply pressures during 1980-2000. Similarly, the aggregate production function has been estimated by several studies [Gounder and Morling (2000), CBO (1995), Gosselin and Lalonde (2002), Bank of Japan (2003), Gradzewicz and Kolasa (2005), among many others].

Moreover, the statistical methods have also been used equally to gauge the potential output. HP filters and simple time trend methods are frequently used in studies along with other structural methods. For example, SVAR has been used by Gounder and Morling (2000), Dupasquier et al. (1999) with long-run restrictions, Gosselin and Lalonde (2002), Rennison (2003), and Menashe and Yakhin (2004) and many other studies. State-space models and the unobserved Component method are alternative names of the same method of Kalman filtering and have been used in the estimates of potential output [Gradzewicz and Kolasa (2003), Kichian (1999)]. Scacciavillani and Swagel (1999) have also used wavelets filters to estimate the potential output for Israel economy. The wavelet filters are considered some kind of flexible method of estimation and it combines the linear time trend method with the HP filter method.

Despite these controversies, output gap is considered as a best manifestation to measure the supply/demand pressures in the overall economic analysis from a policy judgment point of view. Therefore, one point agenda that emerges clearly from this discussion is that the conventional methods should be improved to make them flexible in terms of capturing more information to estimate potential output and the output gap. Furthermore, the methods that have been commonly used are linear time trends, Hodrick-Prescott filters, Band-Pass filter method, Production Function, the Structural Vector Autoregressive method, and Unobserved Component methods.

Table 1: Empirical Literature Review

Authors	Empirical Approach	Variables	Data	Findings
Bjørnland et al. (2005)	Hodrick-Prescott filter (HP), Band-pass filter (BP), Univariate "unobserved component" methods (UC) and Production function method (PF), Multivariate unobserved component method (MVUC), and SVAR model.	GDP, domestic inflation and unemployment, potential levels of work hours, total factor productivity, capital, unemployment gap	Norway (1982-2004)	The different methods show a consistent pattern for the output gap, but there are also important differences. Assessments of the output gap must therefore also be based on professional judgment and supplementary indicators.
Njuguna et al. (2005)	Hodrick-Prescott filter and the unobserved components methods, linear method, structural vector autoregression (VAR) method and production function method.	GDP, private consumption, time trend, labor employed, capital stock	Kenya (1972-2001)	The estimation results for the values of potential output level and its growth, and the output gap vary from method to method, however results from most methods seem to be consistent with one another, which means that a consensus may be built on how the Kenyan economy has been performing in terms of its potential capacity and growth.
Cayen and Norden (2005)	The univariate and multivariates methods including Deterministic Trends, Mechanical Filters, the Beveridge-Nelson Decomposition, Unobserved Component Models, Unobserved Component Models with a Phillips Curve and the Structural VAR Approach.	Real GDP, consumer price index and interest rate	Canada (1972-2003)	This study has assembled and analyzed a new database of real-time estimates of Canadian output. Results from a variety of measures and a broad range of output gap estimates suggest that measurement error in Canadian data may be more severe than previously thought. Further analysis of output gap forecasts and of model risk is not conclusive and results vary considerably from model to model.
Barbosa-Filho (2005)	It presents the basic definitions used in growth accounting and the methods used for measuring labor, capital and the output gap. Then it merges theory and econometrics in a comparative analysis of recent estimates of the potential growth rate of Brazil.	GDP(gross and net), intermediate consumption, labor estimates and labor productivity estimates, capital and capital productivity estimates, unemployment, inflation rate, interest rate, capacity utilization, total imports, total exports, input-output estimates, average years of schooling, per capita income, TFP estimates, and non accelerating inflation rate of capacity utilization.	Brazil (1947-2003)	The main conclusions are: (i) the annual potential growth rate of Brazil's GDP varies substantially depending on the method and hypotheses adopted and, what is most important, potential GDP is not separable from effective GDP in the long-run; (ii) aggregate measures of potential output do not carry much information about the economy and, therefore, they should be complemented by sectoral estimates of capacity utilization to identify the bottlenecks in inter-industry flows and the corresponding demand pressures on inflation.
Gradzewicz and Kolasa (2005)	Two factor dynamic production function (estimated in the cointegrated VECM system)	GDP, labor and capital as inputs	Poland (1995-2002)	The development of the gaps and the analysis of their impact on inflation show the lack of any inflationary pressure from the demand side, which may be the case till the end of 2003. In view of relatively strong assumptions made during the estimation process and time relationships analysis, caution is recommended while drawing any conclusions.
Menashe and Yakhin (2004)	The production-function method and SVAR, both structural methods.	Bussiness sector product, estimates of TFP, capital input, labor input, utilization of capital, inflation rate, inflation expectations, time dummy and import prices.	Israel (1986-2001)	The results of the estimate give rise to several conclusions: (i) the annual rate of growth of potential output in the second half of the 1990s declined by about one percentage point from the rate in the first half. (ii) Estimates of the output gap including start-ups do not differ significantly from estimates excluding them. (iii) It is clear that the business cycle at the beginning of the 1990s derived mainly from supply shocks (in particular the influx of immigrants), while the recession that started

				in 1996 was due to demand shocks.
Cotis et al. (2003)	This study provides a critical review of variety of methods used in the literature.			Although it is difficult to give a universal ranking of the methods, the statistical methods (trend and univariate filters) seem to be having more shortcomings than the economic methods (particularly, multivariate filters and production function approaches).
Bank of Japan (2003)	The benchmark output gap is estimated using the method of production function. HP filter and time varying NAIRU.	Capital, labor, TFP and domestic inflation rate.	Japan (1983-2002)	Looking at the estimated potential growth rate in Japan, the study noted that the rate stood at around 4 percent throughout the 1980s. The output gap expands when the actual growth rate falls below the potential growth rate.
Rennison (2003)	The HP filter and two multivariate techniques: the Blanchard-Quah (1989) SVAR approach and the multivariate extensions of the HP filter (MVF). This study also considers an estimator that weighs a portfolio of inputs to estimate the output gap.	Core CPI inflation, GDP deflator, real exchange rate, slope of the yield curve, long-term nominal interest rates.	Canada	This Study indicates that the output-gap estimates from the SVAR and the HP-based filter are in many cases complementary. Results appear quite robust to alternative realistic assumptions about the DGP. It shows that the favorable results for the combined approach at the end-of-sample are due in part to misspecification and parameter uncertainty in the SVAR. Two additional results have been reported: (i) relative to other estimation methodologies, the SVAR is surprisingly robust to violations in its identifying assumptions, and (ii) in terms of the absolute accuracy of an estimator at the end-of-sample, the costs associated with imposing an arbitrary smoothing restriction can be high.
Changy and Pelgrin (2003)	This paper assesses the statistical reliability of different measures of the output gap - the multivariate Hodrick-Prescott Filter, the multivariate unobserved components method and the structural vector autoregressive model - in the Euro area.	GDP real, inflation rate (consumer price deflate), unemployment rate, capacity utilization, relative import price and NAIRU estimates.	Euro Area (1970-2002)	The results show that (i) additional economic information may be useful for the estimation of the output gap, (ii) economic interpretation may differ across different methods and within a given method (when different specifications are used), (iii) all multivariate detrending models performs less than an autoregressive process in terms of inflation prediction and (iv) multivariate UC models perform better than HPMV models in relative terms in order to reduce the filtered, smoothed uncertainty or quasi-real time estimates. However, it is difficult to conclude that a multivariate detrending method outperforms the others.
Gosselin and Lalonde (2002)	The eclectic approach is used to decompose potential output through the components of full employment labor input and average labor productivity at equilibrium. SVARs methods were applied for estimation.	Trend productivity, trend labor input, population, participation rate under-25 cohort trend participation rate, women's trend participation rate, men's trend participation rate, non-farm trend productivity.	U.S.A.	It shows an acceleration in the pace of potential output growth during the period 1995–99, peaking at 4.0 per cent in 1997. Currently, it hovers slightly above 3.0 per cent. The vigour observed over the course of the second half of the 1990s is attributable to a fall in the NAIRU and a notable acceleration in the pace of growth of trend productivity.
Denis et al. (2002)	Cobb-Douglas production function is used as the basic methodology to extract the potential output finally.	GDP, population of working age, structural unemployment, investment, capital stock.	EU15, Euro Zone and U.S. (1981-2003)	When comparing the growth contributions of labour, capital and TFP in the EU15 / Euro Zone over the last two decades with the experience of the US over the same period, there are striking differences.
Filho and da Silva (2002)	Aggregate Production Funtions	Actual GDP, labor force, capital stock, technology, capacity utilization, natural rate of unemployment.	Brazil (1980-2000)	In the 1980-2000 period, most of the time, the Brazilian economy was below its potential. The years of strongest economic activity were 1980, 1986 and 1987, when the economy was above its potential, and the years of 1989 and 1997, when the output gap was nearly zero.
Cerra and Saxena (2000)	The HP filter, Beveridge-Nelson decomposition, Univariate unobserved components model, the structural VAR approach The production function	GDP, GDP (Private and public), domestic inflation, unemployment, real exchange rate, relative output, relative price level, private capital	Sweden (1971-1998)	Each method has advantages and disadvantages. Although the various methods produce a range of results for the output gap, the overall evidence suggests that the large output gap--most pronounced in 1993--has either closed in 1998 or will close in the next 1-2 years if current trends continue.

	approach and system estimates of potential output and the NAIRU	stock, estimates of trend labor input, TFP estimates, NAIRU estimates, time dummies and import prices.		
Kichian (1999)	The general form of the State Space Framework is used and as a by-product liklihood function is constructed and finally the quasi optimal Kalman filter is applied.	Quarterly; real output, inflation rate, expected inflation rate, nominal trade weighted exchange rate, and nominal oil prices.	Canada (1961-1997)	There have been three important periods of excess supply in Canada around the dates of 1977, 1982 and 1991, the second being the most pronounced. As for periods of excess demand, the three major ones are mid to late 1960s, from 1972 to about 1974, and from around mid-1987 to about 1990.
Scacciavillani and Swagel (1999)	Methodologies used to estimate potential output are; i) Aggregare production function, ii) Univariate filters a)HP filter b)Running medium smoothing c)Wavelits fiters, iii) Structural Vector Autoregression.	GDP, price level, stock of physical capital and the labor force and TFP estimates.	Israel (1986-1998)	Using five different approaches to measure potential output in Israel, the annual estimates vary somewhat from year to year, but each methodology indicates that annual potential output growth accelerated during the 1990s to reach around 7 percent by 1995. The output gaps likewise vary by methodology, but most imply that output was above potential for a lengthy period in the early or mid 1990's.
Dupasquier et al. (1997) and (1999)	The multivariate Beveridge-Nelson methodology (MBN), Cochrane's methodology (CO), and the structural VAR methodology with long-run restrictions applied to output (LRRO).	Quarterly GDP, real consumption comprising non-durables and services and the federal funds rate when a third variable is added, money and inflation is also used as a proxy to federal fund rate.	U.S.A. (1963-1997)	The results show that the LRRO estimates provide significant evidence that permanent shocks have more complex dynamics than the random walk assumed in CO and MBN approaches. As in other studies, estimates of the out-put gap remain imprecise.
de Brouwer (1998)	Linear time trends, Hodrick-Prescott (HP) filter trends, multivariate HP filter trends, unobservable components models and a production function model.	Real GDP, inflation, import prices, expected inflation rate, unemployment rate, NAIRU estimates, capacity utilization, labor force, capital stock.	Australia (1980-1997)	The gap estimates at any particular point in time are imprecise; the broad profile of the gap is similar across the range of methods examined.
De Masi (1997)	Cobb-Douglas approach is used for industrial countries; univariate detrending techniques over the production function and HP filter for developing countries; and endogenous growth models for countries in transition.	GDP, labor, capital and TFP estimates	(1980-2002)	Over the medium term, potential output growth for the seven major industrial countries are projected to be in the range of 2 to 2.5 percent. The growth rate of potential output is expected to pick up slightly to 2.25 to 2.5 percent in the United Kingdom and Canada. In Italy, the growth rate of potential is expected to remain at about 2 percent, and in the United States to remain at about 2.5 percent.
CBO's (1995)	CBO uses production function approach.	Real GDP, labor, capital, inflation rate	U.S.A. (1950-2002)	Output generally falls below potential during recessions, remains below during recoveries and early expansions, and rises above potential during late expansions.

3. Review of Estimation Methods

This section reviews the empirical methodologies used for estimating potential output *vis-à-vis* output gap. In general, the different approaches to estimating potential output are classified into some of the detrending methods: the Hodrick-Prescott (HP) filter, the Band-Pass filter by Baxter-King and the Unobserved Components methods using the Kalman filter (univariate, bivariate, and common permanent and cyclical components). For estimating structural relationships the approaches include: the linear Time Trend method, Structural Vector Autoregressive (SVAR) method and Production Function method (PF).

3.1. The Linear Time Trend Method

The linear trend method is the simplest way to estimate the output gap and potential output. According to this method, it is assumed that potential output is a deterministic function of time and the output gap is a residual from the trend line.[8] This technique presumes that output is at its potential level on average, over the sample period[9]. Thus trend in output, which represents potential output, may be estimated as

$$Y_t^* = \hat{\alpha}_0 + \hat{\alpha}_1 \, TREND \qquad t = 1, 2 \ldots \qquad (1)$$

Where Y_t^* is potential output and $\hat{\alpha}_0$ and $\hat{\alpha}_1$ are estimated coefficients from the regression of the actual output on time trend variable (*TREND*); and, output gap is obtained using:

$$YGAP_t = Y_t - Y_t^* \qquad t = 1, 2 \ldots \qquad (2)$$

Where, $YGAP_t$ is the output gap and Y_t is the actual output.

One of the major shortcomings of this method is that the long-run evolution of the time series is perfectly predictable because it is deterministic. It is argued, however, that if the changes in economic series are a random process, then the deviation of the series from any deterministic path would grow without bound [Beveridge and Nelson (1981)]. Another criticism of this method is

8 This approach uses linear trend method as the optimal method considered among other trend methods, for example the polynomial trend methods up to degree 7.

9 This is contrary to the "through-the-peaks" method, which suggests that potential output is the maximum possible output; see, Laxton and Tetlow (1992) for more discussion on the latter method including its weaknesses.

that the estimate of the gap is found to be sensitive to the sample period used in the regression estimation.[10] According to de Brouwer (1998), the other limitation of the above method is that the assumption that potential output grows at a constant rate often does not hold.[11] Since output growth can be decomposed into growth of inputs, which in turn can be decomposed into changes in the population, labor participation and average hours worked, it is not justified to suppose that these components are not changing over time. This is particularly valid when an economy has undergone considerable structural reform, or when there are major changes in improvements in technological level.

3.2. The Hodrick-Prescott (HP) Filter Method

The Hodrick-Prescott (1997) filter method (HP) is a simple smoothing procedure. The main assumption of this method is that there is prior information, that growth component varies "smoothly" over time. In particular, a given time series, say Y_t (or output) may be expressed as the sum of a growth component or trend Y_t^* (or potential output) and a cyclical component or output gap $YGAP_t$:

$$Y_t = YGAP_t + Y_t^* \qquad t = 1, 2\ldots \qquad (3)$$

The measure of the smoothness of Y_t is the sum of the squares of its second difference. The average of the deviations of $YGAP_t$ from Y_t^* is assumed to be near zero over a long period of time. These assumptions lead to a programming problem of finding the growth components by minimizing the following expression:

$$Min\, L = \left\{ \sum_{t=1}^{T} YGAP_t^2 + \lambda \sum_{t=2}^{T} (\Delta Y_t^* - \Delta Y_{t-1}^*)^2 \right\} \qquad (4)$$

[10] For example, using Australian data, de Brouwer (1998) found that when the sample starts at the lowest point in a recession, the slope of the straight line fitting the series became steeper, making the gap between actual and potential output at the end of the sample smaller.

[11] As income level increases over time, the potential output grows at slower rates because of diminishing marginal returns to reproducible inputs, ceteris paribus.

The parameter λ is a positive number, which penalizes variability in the growth component series. The larger the value of λ, the smoother is the solution series. Moreover, as λ approaches infinity, the limit of the solutions for Equation (4) is the least squares of a linear time trend model. On the other hand, as the smoothing factor approaches zero, the function is minimized by eliminating the difference between actual and potential output that is making potential output equal to actual output. In most empirical works, the value of λ = 1000 is chosen when using annual data.

The HP method has been used in a number of empirical studies.[12] The popularity of this method is due to its flexibility in tracking the characteristics of the fluctuations in trend output. The advantage of the HP filter is that it renders the output gap stationary over a wide range of smoothing values and it allows the trend to change overtime.

The HP method, however, is far from ideal. The first weakness of the HP method relates to the smoothing weight (λ); as to how λ affects responsive potential output to movements in actual output. For high smoothing factor, the estimate indicates output above potential, but for moderate or low smoothing, the estimate suggests output below potential. Thus, an appropriate smoothing parameter (λ) is difficult to identify.

Another weakness of the HP method is the high end-sample biases, which reflect the symmetric trending objective of the method across the whole sample and the different constraints that apply within the sample and its edges. To counter this problem, however, researchers use output projections to augment the observations. The reliability of measured potential output and output gap would then depend on the accuracy of the forecasts used to avoid the end-sample bias.

Finally, for integrated or nearly integrated series, it has been shown that an arbitrary value of smoothing parameter could lead to spurious cyclicality and an excessive smoothing of structural breaks.

3.3. Baxter-King Method using Band-Pass (BP) Filter

We use another univariate approach known as BP-filter method to compute output gap. In this method the underlying time series is a weighted sum of varying cyclical frequencies. Thus the

[12] See for example, De Masi (1997), de Brouwer (1998), Scacciavillani and Swagel (1999), and Cerra and Saxena (2000).

correspondent cycles of varying frequencies are unassociated in the long run and the variance of a subjected time series depicted as the sum of its variances over all frequencies. In this way the function by decomposing the total variance by frequency is traditionally known as the spectrum density.

Moreover, the basic concept of this method is to extract the information from relevant frequencies of concern. Therefore, with reference to measuring the cyclical component of GDP, this would generally be the business cycle frequencies. Hence through this method, it would help by excluding all other frequencies and give a view on the cycle lengths for defining a business cycle. On the basis of this set up the volatility with a higher frequency are normally seen as irregular/seasonal, while fluctuations with a lower frequency are recognized to movements in the trend/potential GDP.

In this set up an optimal filter would pass through all frequencies in the specified frequency range with probability 1, depicting no concern with other frequencies. The optimal filters of this kind can be derived but these are of little use in practical work because it needs an infinite number of observations. Therefore, all the BP filters suggested in the literature are generalization to any of the optimal filter. In this study, we use the BP filter developed by Baxter and King (1995). Their filter takes the form of a 3-year moving average:

$$YGAP_t = \sum_{i=-3}^{3} \alpha_i \, y_{t-i} \qquad (5)$$

where 'α_i' are corresponding weights of the frequency response function. These weights are derived from the inverse fourier transformation. However, an apparent problem with this filter given in (5) is that we drop 3 years of observations for the output gap estimates at the start and end of the sample.

3.4. Structural Vector Autoregressive (SVAR) Method

In this section, a well known multivariate estimation technique, called structural vector autoregressive (SVAR) is used to develop an estimation procedure for potential output vis-à-vis output gap. The model setup extends bivariate model, originally proposed by Blanchard and Quah

(1989) to trivariate model (including variables[13]; output; unemployment; and inflation) consistent with Bjornland *et al* (2006). The basic reason of this extension is due to strong criticism on the bivariate SVAR model[14] of Blanchard and Quah (1989). In the process of model setup, it needs to identify and incorporate structural shocks primarily distinguishing between demand and supply shocks. With this trivariate model, one can easily identify three different structural shocks: two demand shocks and one supply shock. This procedure assumes that neither of the demand shocks can have a long-term effect on unemployment, but allow one of them; a real demand (or preference) shock to have a potential long-lasting effect on GDP. [15] The aggregate supply shock is allowed to have a long-term effect on GDP, unemployment and prices. Since the unemployment rate has increased in the course of our estimation period and is perceived to be nonstationary, it is reasonable to assume that the real (supply) shock can affect equilibrium unemployment over time. Inflation is perceived to be stationary, so none of the shocks by definition can affect inflation permanently. Lastly, estimation procedure follows Cesaroni (2007) to compute potential output vis-à-vis output gap.

SVAR Model Setup

Consider X_t be a vector with the three endogenous variables. u_t is unemployment rate, y_t is output and INF_t is inflation:

$$G(L)X_t = \varepsilon_t \quad \text{and} \qquad \text{var}(\varepsilon_t) = \Omega_{\mu_t} \tag{6}$$

Where *G(L)* is a function of lag operators and Ω_{μ_t} is an information set consisting of variance/covariance's of residual vector ε_t. The vector moving average (VMA) representation of above VAR is given below:

$$\Delta X_t = H(L) * \mu_t \tag{7}$$

The above representation (7) can also be translated into the structural vector moving average (SVMA) as:

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[13] This multivariate methodology uses information from a number of variables that have a high degree of correlation with GDP, such as unemployment in terms of labor force and domestic inflation or money supply, to estimate potential GDP and the output gap.
14 For further detail, see, Faust and Leeper (1997).
[15] See, Blanchard and Quah (1989) for further interpretation.

$$\Delta X_t = A(L) * \xi_t \tag{8}$$

where μ_t and ξ_t are reduced form and structural shocks, respectively. ξ_t is i.i.d. with mean zero and $\text{var}(\xi_t) = \Omega_{\xi_t}$ Comparing (7) and (8), the VMA of both reduced form and structural form would give:

$$A(L) * \xi_t = H(L) * \mu_t \tag{9}$$

and setting the polynomial at L=0:

$$A(0) * \xi_t = H(0) * \mu_t \tag{10}$$

Since *H(0)* is identity matrix; $\xi_t = A(0)^{-1} \mu_t$. This shows that structural shocks are related to reduced form shocks via *A(0)*, further implying that:

$$\Omega_{\mu_t} = A(0)A(0)' \tag{11}$$

The above representation of Ω_{μ_t} gives us some information about *A(0)*, however that information is not sufficient to identify *A(0)* since Ω_{μ_t} is a covariance matrix and number of non-redundant equations is less than the number of unknowns. We need to have some extra structural information to fully identify *A(0) as* $A(L) = H(L)A(0)$. This implies that the functional relationship between *A(L)* and *H(L)* is related via *A(0)*. The structural long run response to the levels of endogenous variables can be obtained by evaluating the polynomial lag operator at lag 1; $A(1) = H(1)A(0)$. The matrix *A(1)* can be used to identify *A(0)* if we know some elements of the *A(1)* matrix. Thus we impose some restrictions on some elements of *A(1)* matrix using long run information of economic structure.

The MA process can easily be defined if *A(0)* is identified. Now consider the three uncorrelated structural shocks as a system: $\mu_t = [\mu_t^{AS}, \mu_t^{RD}, \mu_t^{ND}]$, where μ_t^{AS} is an aggregate supply shock, μ_t^{RD} is a real demand shock, and μ_t^{ND} is the remaining demand (i.e. nominal demand) shock. The system array of long run multipliers can be defined as;

$$\begin{bmatrix} \Delta u \\ \Delta y \\ INF \end{bmatrix}_t = \begin{bmatrix} H_{11}(1) & H_{12}(1) & H_{13}(1) \\ H_{21}(1) & H_{22}(1) & H_{23}(1) \\ H_{31}(1) & H_{32}(1) & H_{33}(1) \end{bmatrix} \begin{bmatrix} \mu^{AS} \\ \mu^{RD} \\ \mu^{ND} \end{bmatrix}_t \tag{12}$$

where $H(1) = \sum_{j-0}^{\infty} H_j$ show the long run multi dimensional array of *H(L)*. Hence, one can impose restrictions in such a way that neither of the demand shocks can affect the unemployment rate permanently. This further implies that: $H_{12}(1) = H_{13}(1) = 0$. In line with these two restrictions, the third restriction can be imposed is that nominal demand shocks can not affect output permanently. This is defined symbolically as; $H_{23}(1) = 0$. Hence three structural restrictions are imposed in SVAR system.

It has been shown that if the structure of long run response is recursive we can exploit a type of Cholesky transformation. This involves recognizing that if *A(1)* is recursive, than it is triangular and we can compute *A(1)* as Cholesky decomposition of long run covariance matrix for ΔX_t. The long run covariance matrix is given by:

$$\Omega_{\Delta X_t}(1) = H(1)\Omega_{U_t} H(1)' \tag{13}$$

Now *A(1)* can be computed as Cholesky decomposition of $\Omega_{\Delta X_t}(1)$ and could be used to identify *A(0)* that will be used to compute structural shocks.

Table2. Structural Block Restrictions

Dependent Block	Independent Block		
	u	y	inf
u	*	**0**	**0**
y	*	*	**0**
inf	*	*	*

The three long run restrictions as discussed earlier are defined in Table 2. From this table one can easily visualize that output can now potentially be divided into two different parts; first component determined by shocks that have a permanent effect on the supply side of all the variables in the economy, and a component determined by shocks that affect demand in the short term.

Cesaroni (2007) defined first component as potential output. It will consist of the accumulated supply shocks. The second component is defined as the output gap and will consist of the accumulated aggregate demand shocks. [16]

3.5. Production Function (PF) Method[17]

Output can be described by a production function. The production function models the supply side of the economy where output is determined by available technology with input factors as labor and capital. Potential output may be perceived as the resulting output level if the input factors are neither exposed to strong pressures nor partially unutilized. The difference between actual output and estimated potential output can then be interpreted as the output gap.

The aggregated production function for the economy can be expressed as a Cobb-Douglas production function differentiating with respect to time:

$$y_t = tfp_t + \alpha_1 l_t + (1-\alpha_1)k_t \qquad t = 1, 2 \ldots \qquad (14)$$

where y is output, l is labor, k is capital stock, tfp is total factor productivity. All variables are measured as natural logarithms. The coefficients α_1 and $(1-\alpha_1)$ are the factor shares for labor and capital. Since the share of capital income is one minus the share of labor income under the assumption of constant returns to scale, the growth rate of output is decomposed into TFP growth and the weighted sum of the growth of capital and labor.

The potential levels of labor, capital and total factor productivity are then used to estimate potential output, y^*.

$$y_t^* = tfp_t^* + 0.56 l_t^* + 0.44 k_t^* \qquad (15)$$

[16] Bjornland et al (2006) defined the real demand shock as it can potentially affect output in the long run. We assume that it contributes to the output gap the first two years (business cycle frequencies), whereas any effect above that will contribute to developments in potential output. Assuming instead that real demand shocks can have a long run effect on the unemployment rate will not change the results.
[17] This sub-section is based on the description in Frøyland and Nymoen (2000) and as followed by Bjornland *et al* (2006).

Following Khan (2006), for Pakistan we assume values for the factor income shares to be 0.56 for labor and 0.44 for capital.

Potential use of employed labor depends on the potential level of the labor force. Potential capital stock is assumed to be the same as actual capital stock since it is difficult to determine to what extent capital stock is used in the production process. The potential level of total factor productivity is calculated using the HP filter.

There is some psychological advantage of this method that it is based on a theoretical foundation and intuitively seems plausible. It is, however, based on one of many possible types of production functions. The underlying data may also cause problems; measuring the capital stock is particularly uncertain.

3.6. Unobserved Component Method using KALMAN Filter

This method is based on the premise that an observable variable is composed of two or more components that are not observable. The basic idea is that the unobservable variables can be identified by assuming that they affect the variable that can be observed. In addition, we must specify the underlying processes that are behind the unobservable variables over time. Both the unobservable and the observable variables are modeled and estimated as a "maximum likelihood" system using the Kalman filter.

We adopt model specification consistent with Harvey (1985), Watson (1986), Clark (1987) and Vineet (2004). In this model setup output Y_t is decomposed into a trend Y_t^* and a cycle C_t. For simplicity, the trend component is assumed to follow a random walk with drift and the cyclical component is assumed to follow an *AR (2)* process. Thus, (natural logarithm of) output is specified as:

$$\left.\begin{aligned} &Y_t = Y_t^* + C_t && \\ &y_t^* = \delta_{t-1} + y_{t-1}^* + \eta_t && \eta_t \sim N(0,\sigma_\eta^2) \\ &\delta_t = \delta_{t-1} + \nu_t && \nu_t \sim N(0,\sigma_\nu^2) \\ &C_t = \rho_1 C_{t-1} + \rho_2 C_{t-2} + \varepsilon_t && \varepsilon_t \sim N(0,\sigma_\varepsilon^2) \end{aligned}\right\} \qquad (16)$$

Following Kuttner (1994), we also incorporate backward looking Phillips curve with model specification (15).

$$\left.\begin{array}{ll} INF_t = INF_t^* + \varphi C_{t-1} + \xi_t + \lambda_1 \xi_{t-1} + \lambda_2 \xi_{t-2} + \omega_t & \omega_t \sim N(0, \sigma_\omega^2) \\ INF_t^* = INF_{t-1}^* + \tau_t & \tau_t \sim N(0, \sigma_\tau^2) \end{array}\right\} \qquad (17)$$

Simultaneous Equations (16) and (17) can be conveniently translated as a State-Space model, facilitating estimation of the variables by Maximum Likelihood using Kalman Filter. Further details of the estimation can be found in Harvey (1993).

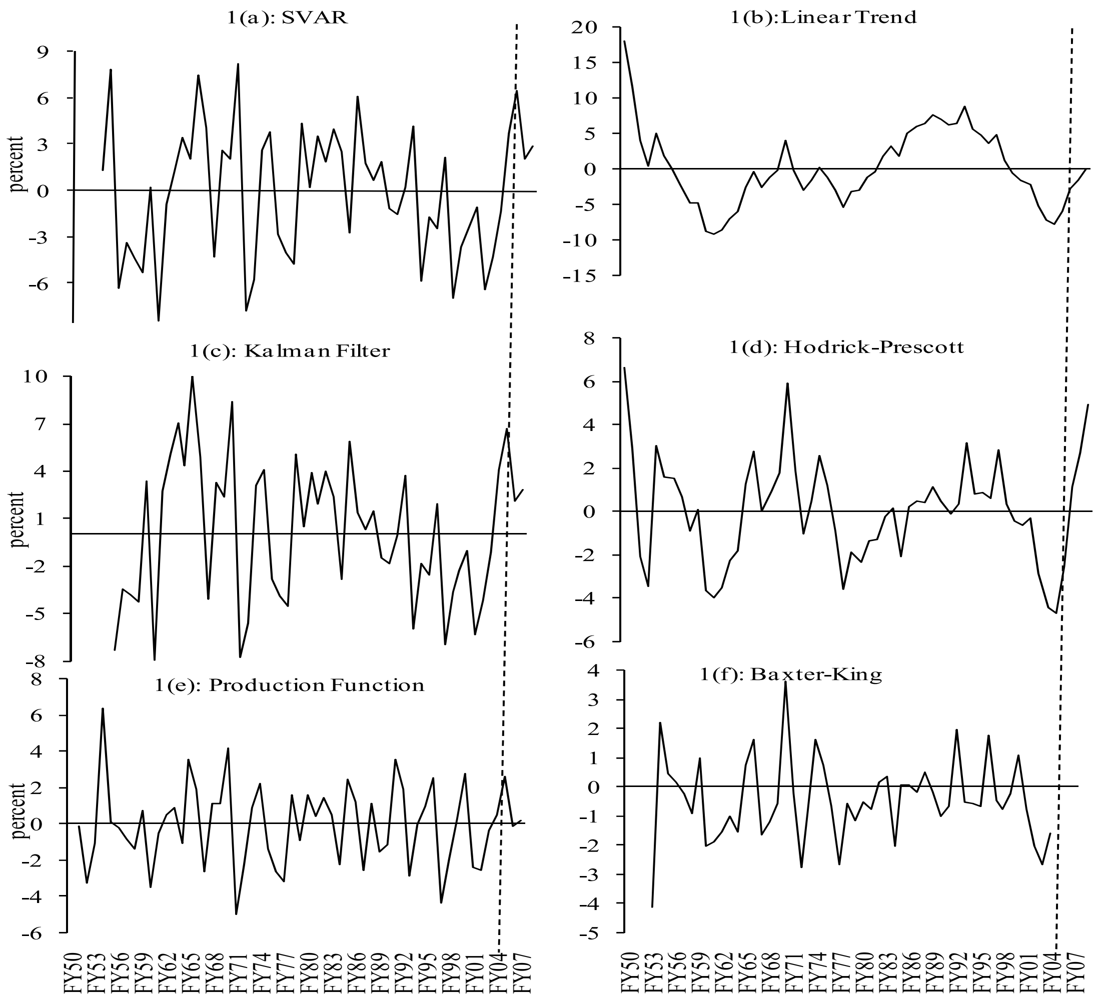
4. Empirical Findings

The six measures of the output gap are constructed by utilizing annual data series of capital, GDP, inflation, labor force and unemployment spanning over 1950 to 2007. The dataset is collected from published sources of both Ministry of Finance and State Bank of Pakistan.[18] The results are shown in Figure 1 through Figure 1(a) to 1(f). Looking at the graphical block of different figures, it appears that the computed output gap could not be observed as an identical outcome of the six methods.[19] It also appears that the volatility in terms of standard deviation and magnitude differs from method to method. Furthermore, it can be observed that sometimes the direction (i.e. the sign of output gap values) of the results also varies diametrically by changing the method of estimation. Through these observations, the casual observer may perceive that none of the method is reliable otherwise, and perhaps there are no practical implications of output gap. These observations, however, are in line with the available literature.

[18] Pakistan Economic Surveys, SBP Annual Reports, and SBP Inflation Monitor.

[19] Therefore, we also have presented the distribution of output gap with respect to each method in Table 4 for point to point explanation of turning points during the whole period of analysis.

Figure 1: Output Gap Calculated from Different Methods



Literature, mentions that the diversification of results of output gap with respect to different methods is not unusual and attributes it to a set of reasons. Among these reasons, it is important to note as pointed out earlier, that the potential output is not directly observable. Therefore, it depends on how it is defined and estimated. The statistical methods are sometimes quite different to the structural methods in their specification and may produce different estimates of the potential output and the output gap. Another reason for this can be limitations of these methods, illustrated under each method respectively in the preceding section.

Despite these justifications of varying results, output gap is still considered as the best measure to gauge the supply/demand pressures in the overall economic analysis from the policy judgment point of view. For this purpose, the methods of estimations (both structural and statistical) have been modified by making them either flexible or expanding them to incorporate more information for better judgment of the potential output. Such developments can be observed from the estimates of CBO, International Monetary Fund (IMF), Organization for Economic Cooperation and Development (OECD) and several other central banks over time.[20] Moreover, the controversies relevant to computation of potential output remain at their level at the end of the day.

These curiosities lead this study to estimate all commonly used methods for the estimation of potential output for Pakistan's economy to observe demand/supply pressures over time. Hence we compare the estimates of output gap with each other through the common sample correlation and statistical summary analysis in Table 3. The volatility (standard deviation) is different between the methods, with the SVAR method being the most volatile and the Baxter-King method the least. The magnitude of the volatility also varies between the six methods with the SVAR method having the highest magnitude and Baxter-King method having the least magnitude. Nonetheless, a close look at the graphs shows that the absolute trend of the results presents somewhat identical behavior of movement over the time. Table 4 provides a clear picture of the turning points with reference to each method during analysis. Since the different measures depict different turning points and different degrees of slack in the economy, they also signal the need for substantially different policy responses, both in terms of the timing and magnitude of policy changes. Table 4 helps to recognize the periods of economy with excess or deficient characteristics over the time period of 57 years.[21]

Table 3. Different Estimates of Output Gap (summary statistics)

	Baxter-King	Kalman-Filter	Hodrick-Prescott	Production function	Linear Trend	SVAR
Correlation Coefficient						
Baxter-King	1	0.38	0.83	0.58	0.53	0.47
Kalman-Filter		1	0.20	0.78	0.04	0.97
Hodrick-Prescott			1	0.42	0.70	0.32
Production function				1	0.25	0.83
Linear Trend					1	0.21
SVAR						1
Descriptive Statistics						
Mean	-0.43	0.09	-0.26	-0.05	-0.41	-1.78
Median	-0.55	0.11	0.10	0.20	-0.55	-2.30
Maximum	3.60	6.50	5.90	6.30	8.80	8.02
Minimum	-4.10	-7.56	-4.70	-5.00	-9.30	-11.28
Std. Dev.	1.39	2.50	2.19	2.26	4.79	4.98
Observations	*52*	*52*	*52*	*52*	*52*	*52*

[20] See Section 2 for references.
[21] This is accompanied with Figure 2 which distinguishes the economy between the years of excess capacity and deficient capacity utilization.

Table 4. Identification of Deficient/Excess Capacity (1950-2007)

	FY50-FY59	FY60-FY69	FY70-FY79	FY80-FY89	FY90-FY99	FY00-FY07
Years Identified with the Deficient Capacity						
Linear Trend	FY50-FY55		FY70	FY82-FY89	FY90-FY97	
Hodrick-Prescott	FY50, FY51 FY54-FY57 FY59	FY60-FY64	FY70-FY71, FY73-FY75	FY83 FY85-FY89	FY91-FY97	FY05-FY07
Baxter-King	FY54-FY56, FY59	FY65, FY66	FY70, FY74, FY75	FY82, FY83, FY85, FY86, FY88	FY92-FY96	FY00
SVAR	FY53, FY54 FY59	FY62-FY66, FY68, FY69	FY70, FY73, FY74, FY78, FY79	FY80-FY83, FY85-FY88	FY91, FY92, FY96	FY04-FY07
Kalman Filter	FY59	FY61-FY66, FY68, FY69	FY70, FY73, FY74, FY78, FY79	FY80-FY83, FY85-FY88	FY92, FY96	FY04-FY07
Production Function	FY54, FY55, FY59	FY62, FY63, FY66, FY68, FY69	FY70, FY73-FY77, FY79	FY80-FY83, FY85, FY86, FY88	FY91, FY92, FY95, FY96, FY99	FY00, FY04, FY05, FY07
Structural Methods	**FY59**	**FY61-FY66, FY68, FY69**	**FY70, FY73, FY74, FY78**	**FY80-FY83, FY85, FY86, FY88**	**FY91, FY92, FY96**	**FY00, FY04-FY07**
Statistical Methods	**FY54-FY56**	**FY66-69**	**FY70, FY71, FY74, FY75**	**FY82, FY83, FY85-FY89**	**FY90-FY97**	**FY06, FY07**
Benchmark	**FY51, FY54, FY59**	**FY63, FY65, FY66, FY68, FY69**	**FY70, FY73, FY74, FY78**	**FY80-FY83, FY85-FY89**	**FY90-FY92, FY94, FY96**	**FY05-FY07**
Years Identified with the Excess of Capacity						
Linear Trend	FY56-FY59	FY60-FY69	FY71-FY79	FY80, FY81	FY98, FY99	FY00-FY07
Hodrick-Prescott	FY52, FY53, FY58	FY65-FY69	FY72, FY76-FY79	FY80, FY82-FY84	FY90, FY98, FY99	FY00-FY04
Baxter-King	FY57, FY58	FY60-FY64, FY67-FY69	FY71-FY73, FY76-FY79	FY80, FY81, FY84, FY87, FY89	FY90, FY91, FY95, FY97-FY99	FY01-FY04
SVAR	FY55-FY58	FY60, FY61, FY67	FY71, FY72, FY75-FY77	FY84, FY89	FY90, FY93-FY95, FY97-FY99	FY00-03
Kalman Filter	FY55-FY58	FY60, FY67	FY71, FY72, FY75-FY77	FY84, FY89	FY90, FY91, FY93-FY95, FY97-FY99	FY00-FY03
Production Function	FY51-FY53, FY56-FY58	FY60, FY61, FY64, FY67	FY71, FY72, FY78	FY84, FY87, FY89	FY90, FY93, FY94, FY97, FY98	FY01-FY03, FY06
Structural Methods	**FY55-FY58**	**FY60-FY67**	**FY71, FY72, FY75-FY77, FY79**	**FY84, FY87, FY89**	**FY90, FY93-FY95, FY97-FY99**	**FY01-FY03**
Statistical Methods	**FY53, FY57-FY59**	**FY60-FY65, FY67, FY68**	**FY72, FY73, FY76-FY79**	**FY80, FY81, FY84**	**FY98, FY99**	**FY00-FY05**
Benchmark	**FY52, FY53, FY55-FY58**	**FY60-FY62, FY64, FY67**	**FY71, FY72, FY75-FY77, FY79**	**FY84**	**FYY93, FY95, FY97-FY99**	**FY00-FY04**

The correlation coefficient shows how closely the results are associated with each other. The closest correlation appears to be between the linear Trend, HP method and the Baxter-King filter methods. These methods are commonly known as statistical methods. Similarly, we observe the strong correlation among all of the three structural methods. These methods include the SVAR method, Production Function and Kalman filter method. There appears to be two classifications of methods comprising each of the three methods which show reasonable correlation for some meaningful analysis. Therefore, we estimate a composite of output gap for each classification taking into account the statistical and the structural methods and present the results in Figure 2. For composite output gap we adopt simple average methods of calculations. Question arises that: by this approach, have we resolved the problem of authenticity of output gap for its practical implication for economic analysis? Certainly, it needs more justification or investigation for some satisfactory answer to this question. For this purpose, we compare the degree of association between these two broader and independent categories of composite output gap with each other. Therefore, we compute correlation coefficient between statistical and structural methods with their corresponding composite output gaps. We observe that the independent coefficient of correlation between these two composite output gaps is substantially low at only 27 percent.

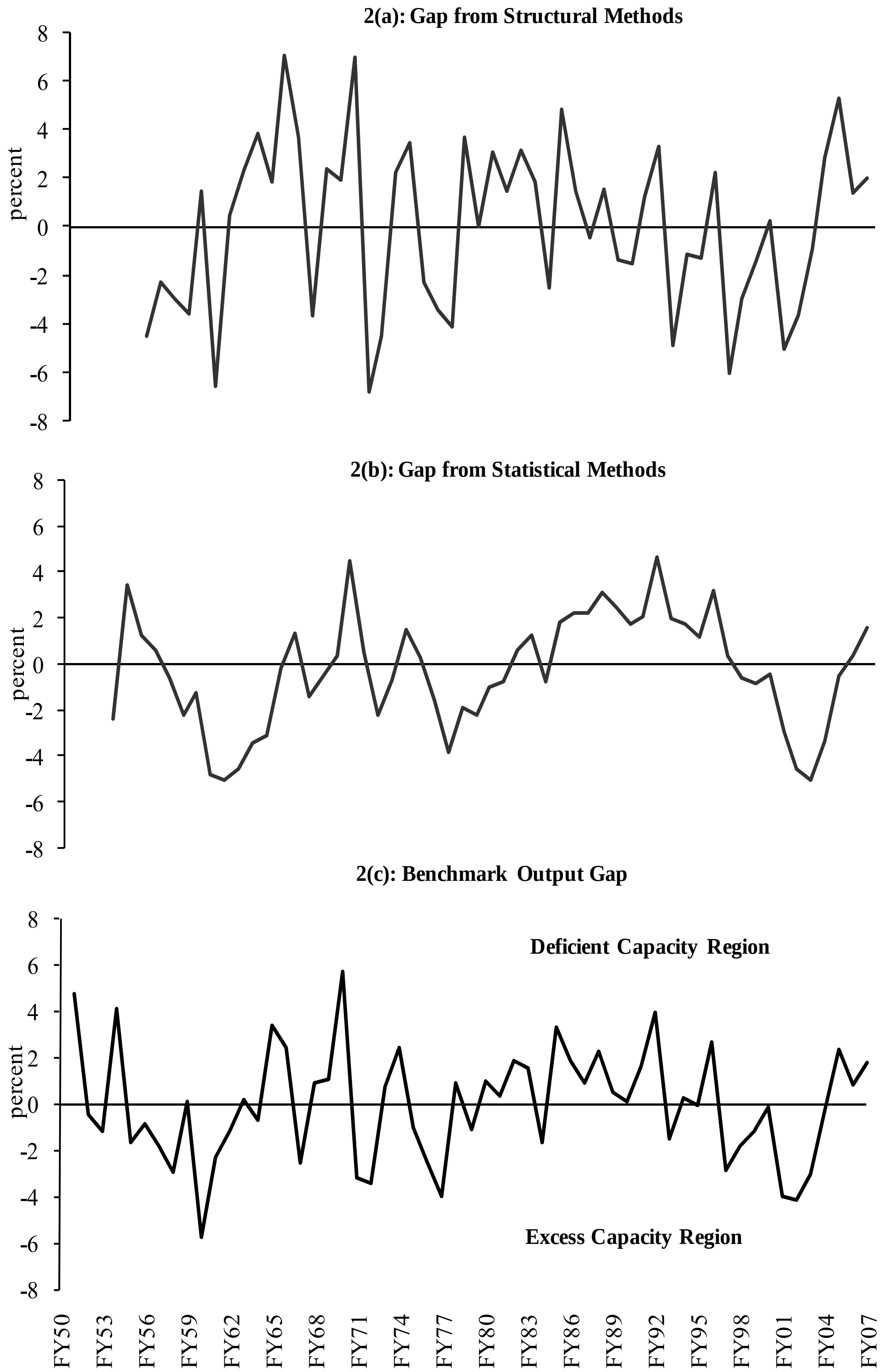
In the next step we estimate another composite output gap which can be called as the benchmark output gap of the above six methods (Figure 2). The benchmark output gap is computed by the simple average of the six different methods by bringing each method on the same scale without disturbing their dimensions. This method is adopted to avoid the biasedness of the benchmark output gap towards any of the extreme values of any of the method. This output gap should be representative of all the six methods in terms of its characteristics. Thus we compare the benchmark output gap with the recently estimated two different composites of output gaps. We estimate a coefficient of correlation between the benchmark output gap and composite of statistical and structural output gaps.[22] The benchmark output gap shows higher degree of correlation with both structural and statistical methods. It is 87 percent with structural and almost 71 percent with the statistical composite output gap. This implies that different estimates of potential output have in one way or the other some similarities in their statistical behavior. After distinguishing this sense of association among these identified methods, we also find some other statistical similarities among all of these estimates of different output gaps.[23] Thus we deduce that this benchmark output gap may be utilized to identify demand/supply pressures in the Pakistan economy.

[22] Appendix provides a historical picture of percent annual growth of GDP and Potential GDP along with the benchmark output gap.

[23] For instance, the mean value extracted from each of the method is with the negative sign except the one with the Kalman filetr. It implies that except this method all other techniques can be placed in one cohort for analysis. It is also noted from the descriptive statistics that the minimum values are negative but with the positive standard deviations. It should also be noted that the *sign* (-, +) with the output gap distinguishes between supply/demand pressures and the magnitude tells about the severity of the pressure.

The benchmark output gap has shown a cyclical pattern of excess/deficient scenarios over time. It has depicted six different cycles of excess supply following demand pressures during time period of 58 years of the analysis. The period of FY55-FY64 can be labeled as with the excess capacity except FY63. Moreover, it turned around and depicted some frequent fluctuations of demand/supply pressures during FY65-FY74 and the demand pressure was observed dominating during this time. Unlike this observation the next cycle appears with the supply pressures and persisted in the second half of 1970s. Going ahead, in the decade of 1980s, the benchmark output gap reflects supply pressures throughout the decade except for FY84. In the decade of 90s we observe that the supply pressures were dominating in its later half as compared to demand pressures observed at the start of this decade. The supply pressures continued even in the 2000s, till FY04. Unlike the historical pattern the output gap started rising, which points to building of demand pressures in the economy since FY04 till FY07. The benchmark output gap has revealed one thing interesting and it is the shrinking of supply rise with the frequent appearance of demand pressures as evidenced since FY97 to FY07.

Figure 2: Summary of Output Gaps



As a supplementary note to estimating output gap, we compute temporal cross correlation between inflation and output gap measures. The individual output gap measures, however, portray a relatively complex scenario by showing large variation in the degree of correlation across each other during this sample range. But as shown in Table 5, all the composites of output gap measures do reflect a degree of correlation between inflation and output gap during the sample range of 1973-07. Correlation coefficient with the negative/positive sign distinguish between supply/demand pressures in the economy. The high degree of correlation also implies that the movement of output gap may assist in the prediction of inflation pressures in the economy. For example the movement of output gap towards zero may demonstrate the signaling of stability in inflation rate. The positive increasing output gap indicates forthcoming inflationary pressures. In other words, the evidence of demand pressures indicates the inflationary pressures in the economy. Therefore, all of the composite output gap measures depict demand pressures as a source of rising inflation during 1973-2007. A common benchmark of structural and statistical output gap measure has shown highest degree of correlation with inflation during 2001-07.

Table 5. Temporal Correlation Between Inflation and Output Gap Measures
(decade wise 1950-2007)

	1951-60	1961-70	1971-80	1981-90	1991-00	2001-07	1973-07
Linear Trend	-0.53	0.34	0.46	-0.77	-0.53	-0.77	-0.40
HP Filter	-0.64	0.29	0.55	0.36	0.67	0.62	0.55
BP Filter	-0.55	0.04	0.74	0.39	0.23	0.97	0.58
SVAR	0.04	-0.28	0.27	-0.02	0.04	0.74	0.26
Kalman Filter	-0.26	-0.48	0.25	0.30	0.65	0.59	0.45
Production Function	-0.14	-0.29	0.35	0.62	0.51	-0.34	0.28
Statistical Composite	-0.37	-0.37	0.28	0.92	0.92	0.97	0.77
Structural Composite	-0.33	-0.07	0.46	0.85	0.95	0.92	0.80
Benchmark	-0.39	0.27	0.60	0.57	0.81	0.84	0.70

5. Concluding Remarks

This paper reviewed six commonly used methods and attempted to estimate potential output and output gap for Pakistan economy. These methods categorized as statistical and structural estimation techniques, include linear time trends, Hodrick-Prescott filters, Band-Pass filter, Production Function, the Structural Vector Autoregressive, and Unobserved Component methods. The performance of these methods has been discussed critically in terms of its limitations and advantages as well.

The results suggest that measures of output gap produced different outcomes that are not identical to each other. But there has been some association among the results achieved from these methods. In addition a high degree of correlation was observed within the statistical and structural methods. Therefore, all of the six methods were divided into the classification of statistical and structural to measure the composite output gaps for each group. We observed a low degree of correlation between these two composite output gaps. Therefore, a benchmark output gap has been calculated by combining the outcome of each method. Interestingly, the benchmark output gap observes reasonable coefficient of correlation with both composites output gaps of statistical and structural methods. The benchmark output gap demonstrates a cyclical episode of demand pressure followed by excess supply, implying different degree of slack in the economy over a time period. In the recent past, since FY05, it has been observed that the demand pressures are overriding in Pakistan economy. As a part of these concluding remarks, a high degree of temporal cross correlation has been observed between inflation and the composite measures of output gap. Furthermore, a common composite of structural and statistical output gap measure has shown highest degree of correlation with inflation during 2001-07.

As observed, the results propose a considerable caution when constructing output gaps and using them for policy analysis in developing countries such as Pakistan.

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Appendix

Real GDP, Potential GDP and Output Gap
(percentage annual change)

Years	GDP	P.GDP	GAP	Years	GDP	P.GDP	GAP	Years	GDP	P.GDP	GAP
FY51	-0.4	2.2	-1.2	FY71	1.2	6.3	-0.8	FY91	5.6	4.9	0.1
FY52	-1.8	2.2	-1.8	FY72	2.3	4.0	-0.4	FY92	7.7	5.3	0.5
FY53	1.7	2.8	-0.4	FY73	6.8	4.7	0.4	FY93	2.3	5.6	-0.6
FY54	10.2	3.9	1.6	FY74	7.5	5.5	0.4	FY94	4.5	4.3	0.1
FY55	2.0	6.3	-0.7	FY75	3.9	5.4	-0.3	FY95	4.1	4.7	-0.1
FY56	3.5	3.9	-0.1	FY76	3.3	4.9	-0.3	FY96	6.6	4.5	0.5
FY57	3.0	4.4	-0.3	FY77	2.8	5.2	-0.4	FY97	1.7	5.5	-0.7
FY58	2.5	3.9	-0.4	FY78	7.7	5.3	0.5	FY98	3.5	4.1	-0.1
FY59	5.5	4.1	0.3	FY79	5.5	6.4	-0.1	FY99	4.2	4.4	0.0
FY60	0.9	4.8	-0.8	FY80	7.3	5.7	0.3	FY00	4.9	4.3	0.1
FY61	4.9	4.2	0.2	FY81	6.4	6.1	0.0	FY01	2.0	4.7	-0.6
FY62	6.0	5.5	0.1	FY82	7.6	5.9	0.3	FY02	3.1	4.4	-0.3
FY63	7.2	5.9	0.2	FY83	6.8	6.2	0.1	FY03	4.7	4.8	0.0
FY64	6.5	6.7	0.0	FY84	4.0	6.1	-0.3	FY04	7.5	5.4	0.4
FY65	9.4	6.1	0.5	FY85	8.7	5.5	0.6	FY05	9.0	5.9	0.5
FY66	7.6	6.6	0.2	FY86	6.4	6.4	0.0	FY06	6.6	5.8	0.1
FY67	3.1	6.1	-0.5	FY87	5.8	6.1	0.0	FY07	7.0	4.5	0.6
FY68	6.8	5.0	0.3	FY88	6.4	5.5	0.2				
FY69	6.5	5.7	0.1	FY89	4.8	5.6	-0.1				
FY70	9.8	5.4	0.8	FY90	4.6	5.2	-0.1				

Source: Authors' estimation
Notes:

- *The real gross domestic product (GDP) is based on the constant prices of 1999-00.*
- *P.GDP: Potential GDP is the average of six different methods explained in Section 6.*
- *GAP: The benchmark output gap is the outcome of average of six different methods corresponding to the potential GDP. It is computed as:* $GAP = [(GDP_t - P.GDP_t) / P.GDP_t] * 100$
- *The numbers for output gap in this table may differ to those of the figures in this study in terms of magnitude. It does not, however, change its meaning because it is computed from the annual average growth rate of GDP and Potential GDP.*